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Note

Disproving a conjecture on planar visibility graphs ☆

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Abstract

Two vertices A and B of a simple polygon P are (mutually) *visible* if $\overline{AB}$ does not intersect the exterior of P. A graph G is a *visibility graph* if there exists a simple polygon P such that each vertex of G corresponds to a vertex of P and two vertices of G are joined by an edge if and only if their corresponding vertices in P are visible. No characterization of visibility graphs is available. Abello, Lin and Pisupati conjectured that every hamiltonian maximal planar graph with a 3-clique ordering is a visibility graph. In this paper, we disprove this conjecture.

Keywords: Computational geometry; Visibility problem; Visibility graph; Planar graph; Hamiltonian cycle

1. Introduction

Our terminology and notation in visibility problem are standard; see [14], except as indicated. Polygons discussed in this paper are assumed simple (i.e., with no holes and with no two edges crossing) and in general position (i.e., no three vertices collinear). A polygon P in the plane is specified by a cyclically ordered sequence of distinct points $V_1, V_2, \ldots, V_n$ $(n \geqslant 3)$ called the *vertices* of P. The *edges* of P are the line segments $\overline{V_1V_2}, \overline{V_2V_3}, \ldots, \overline{V_{n-1}V_n}$ and $\overline{V_nV_1}$. The *exterior* of P is the open region of the plane outside P. Two vertices A and B of a polygon P are (mutually) *visible* if $\overline{AB}$ does not intersect with the exterior of P. The *visibility graph* of a polygon P is the graph obtained by representing each vertex of P by a vertex of the graph and two vertices

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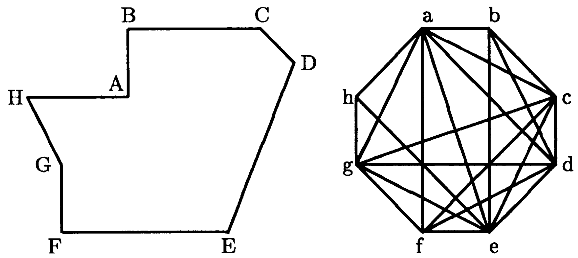

(a) a polygon (b) its visibility graph

Fig. 1.

of the graph are joined by an edge if and only if their corresponding vertices in P are visible. Suppose G is the corresponding visibility graph of P. Throughout this paper, we use upper-case letters to represent the vertices of P and use lower-case letters to represent the vertices of G. See Fig. 1 for an example. A graph G is a *visibility graph* if there exists a polygon P such that G is isomorphic to the visibility graph of P.

No characterization of visibility graphs is available [1–3, 5–20]. For a survey of the visibility problem, refer to [15].

Our terminology and notation in graphs are standard; see [4], except as indicated. Graphs discussed in this paper are assumed simple and finite. A graph is *planar* if it can be drawn in the plane with no two edges crossing. A graph is *maximal planar* if, for every pair of non-adjacent vertices a and b of the graph, adding the edge $\overline{ab}$ to the graph results in a non-planar graph. A graph is *hamiltonian* if it has a hamiltonian cycle. A *k-clique* is a complete graph with k vertices.

Suppose G is a graph and $[v_1, v_2, \ldots, v_n]$ is a vertex ordering of G. A_j is used to denote the set of vertices in $\{v_1, v_2, \ldots, v_{j-1}\}$ that are adjacent to v_j and $G[A_j]$ is used to denote the subgraph of G induced by A_j. A *k-clique ordering* of a graph is a vertex ordering such that the first k vertices form a k-clique and for any other vertex v, the subgraph of G induced by the vertices adjacent to v that precede v in the ordering contains a k-clique. More precisely, a *k-clique ordering* of a graph G is a vertex ordering $[v_1, v_2, \ldots, v_n]$ such that $\{v_1, v_2, \ldots, v_k\}$ forms a k-clique and for any other vertex v_j, $G[A_j]$ contains a k-clique. For example, in Fig. 2, $A_4 = \{v_1, v_2, v_3\}$, $A_5 = \{v_1, v_3, v_4\}$, $A_6 = \{v_1, v_2, v_4\}$; it is not difficult to verify that $[v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11}, v_{12}, v_{13}, v_{14}]$ is a 3-clique ordering of the graph in Fig. 2.

Coullard and Lubiw [5] proved that every 3-connected visibility graph has a 3-clique ordering starting from any 3-clique. Based on this result, Abello et al. [1] proved that every 3-connected planar visibility graph is maximal planar and every 4-connected visibility graph is non-planar. Abello et al. [1] then asked what are the necessary and sufficient conditions for a 3-connected planar graph to be a visibility graph? They conjectured that every hamiltonian maximal planar graph with a 3-clique ordering is a visibility graph. In this paper, we disprove this conjecture.

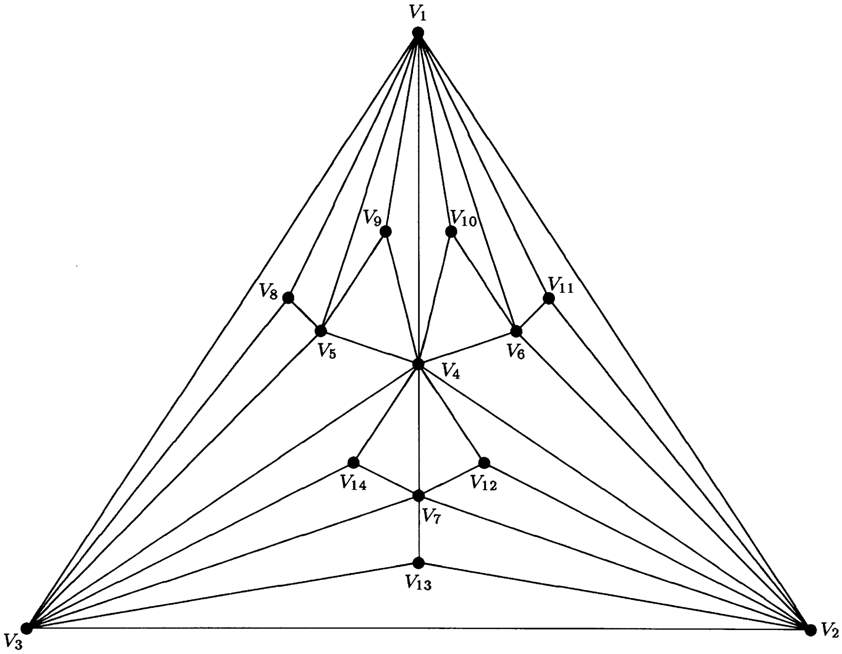

Fig. 2.

2. The main result

Given a polygon, we can traverse its boundary clockwise or counterclockwise. In this paper, we always assume the clockwise order. Let A and B be two vertices of a polygon P. *AB-chain* is the chain of vertices encountered after A but before B in a clockwise traversal around P. For example, in Fig. 1(a), EB-chain $= [F, G, H, A]$.

Theorem 1. *There exists a hamiltonian maximal planar graph with a* 3*-clique ordering which is not a visibility graph.*

Proof. Let G be the graph in Fig 2. G has a hamiltonian cycle $[v_1, v_{10}, v_6, v_{11}, v_2, v_{12}, v_4, v_{14}, v_7, v_{13}, v_3, v_8, v_5, v_9]$. G is maximal planar and has a 3-clique ordering $[v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11}, v_{12}, v_{13}, v_{14}]$. Hence G is a hamiltonian maximal planar graph with a 3-clique ordering. To prove that G is not a visibility graph, we first prove that G has only six hamiltonian cycles; then, we prove that none of the six hamiltonian cycles can form the polygon boundary.

Note that G has seven vertices of degree 3 and seven vertices not of degree 3. Also note that no two vertices of degree 3 are joined by an edge. Hence vertices of degree 3 and vertices not of degree 3 must occur alternately in any hamiltonian cycle of G. We shall use this property to show that G has only six hamiltonian cycles.

Let C be a hamiltonian cycle of G. Since v_5 has exactly two neighbors of degree 3 (i.e., v_8, v_9), $\overline{v_5v_8}$ and $\overline{v_5v_9}$ must appear in C. Since $\overline{v_5v_8}$ and $\overline{v_5v_9}$ appear in C, $\overline{v_1v_8}$ and $\overline{v_1v_9}$ cannot appear in C simultaneously. Since v_6 has exactly two neighbors of degree 3 (i.e., v_{10}, v_{11}), $\overline{v_6v_{10}}$ and $\overline{v_6v_{11}}$ must appear in C. Since v_2 has three neighbors of degree 3 (i.e., v_{11}, v_{12}, v_{13}), exactly two of $\overline{v_2v_{11}}$ and $\overline{v_2v_{12}}$ and $\overline{v_2v_{13}}$ appear in C. There are three cases:

Case 1: *$\overline{v_5v_8}$, $\overline{v_5v_9}$, $\overline{v_6v_{10}}$, $\overline{v_6v_{11}}$, $\overline{v_2v_{11}}$ and $\overline{v_2v_{12}}$ appear in C.* Since $\overline{v_2v_{11}}$ and $\overline{v_2v_{12}}$ appear in C, $\overline{v_{13}v_2}$ cannot appear in C. Since v_{13} is of degree 3 and $\overline{v_{13}v_2}$ cannot appear in C, $\overline{v_{13}v_7}$ and $\overline{v_{13}v_3}$ must appear in C. Note that v_1 has four neighbors of degree 3, i.e., v_8, v_9, v_{10}, and v_{11}. Since $\overline{v_6v_{11}}$ and $\overline{v_2v_{11}}$ appear in C, $\overline{v_1v_{11}}$ cannot appear in C. Since $\overline{v_1v_{11}}$ cannot appear in C and since $\overline{v_1v_8}$ and $\overline{v_1v_9}$ cannot appear in C simultaneously, $\overline{v_1v_{10}}$ must appear in C; moreover, exactly one of $\overline{v_1v_9}$ and $\overline{v_1v_8}$ must appear in C. To sum up, in Case 1, $\overline{v_{13}v_7}$, $\overline{v_{13}v_3}$, and $\overline{v_1v_{10}}$ must appear in C and exactly one of $\overline{v_1v_9}$ and $\overline{v_1v_8}$ must appear in C. There are two subcases:

Case 1.1: *$\overline{v_1v_9}$ appears in C.* Since v_8 is of degree 3 and $\overline{v_1v_8}$ cannot appear in C, $\overline{v_3v_8}$ must appear in C. Since $\overline{v_3v_8}$ and $\overline{v_{13}v_3}$ appear in C, $\overline{v_3v_{14}}$ cannot appear in C. Since v_{14} is of degree 3 and $\overline{v_3v_{14}}$ cannot appear in C, $\overline{v_{14}v_4}$ and $\overline{v_{14}v_7}$ must appear in C. Since $\overline{v_{14}v_7}$ and $\overline{v_{13}v_7}$ appear in C, $\overline{v_{12}v_7}$ cannot appear in C. Since v_{12} is of degree 3 and $\overline{v_{12}v_7}$ cannot appear in C, $\overline{v_4v_{12}}$ must appear in C. Therefore, in this subcase

$$C = [v_1, v_{10}, v_6, v_{11}, v_2, v_{12}, v_4, v_{14}, v_7, v_{13}, v_3, v_8, v_5, v_9].$$

Case 1.2: *$\overline{v_1v_8}$ appears in C.* Since v_9 is of degree 3 and $\overline{v_1v_9}$ cannot appear in C, $\overline{v_4v_9}$ must appear in C. Note that we cannot derive a hamiltonian cycle if $\overline{v_4v_{12}}$ appears in C. Since v_{12} is of degree 3 and $\overline{v_4v_{12}}$ cannot appear in C, $\overline{v_7v_{12}}$ must appear in C. Since $\overline{v_{13}v_7}$ and $\overline{v_7v_{12}}$ appear in C, $\overline{v_{14}v_7}$ cannot appear in C. Since v_{14} is of degree 3 and $\overline{v_{14}v_7}$ cannot appear in C, $\overline{v_3v_{14}}$ and $\overline{v_{14}v_4}$ must appear in C. Therefore in this subcase

$$C = [v_1, v_{10}, v_6, v_{11}, v_2, v_{12}, v_7, v_{13}, v_3, v_{14}, v_4, v_9, v_5, v_8].$$

Case 2: *$\overline{v_5v_8}$, $\overline{v_5v_9}$, $\overline{v_6v_{10}}$, $\overline{v_6v_{11}}$, $\overline{v_2v_{11}}$ and $\overline{v_2v_{13}}$ appear in C.* Since $\overline{v_2v_{11}}$ and $\overline{v_2v_{13}}$ appear in C, $\overline{v_{12}v_2}$ cannot appear in C. Since v_{12} is of degree 3 and $\overline{v_{12}v_2}$ cannot appear in C, $\overline{v_{12}v_7}$ and $\overline{v_{12}v_4}$ must appear in C. Note that v_1 has four neighbors of degree 3, i.e., v_8, v_9, v_{10}, and v_{11}. Since $\overline{v_6v_{11}}$ and $\overline{v_2v_{11}}$ appear in C, $\overline{v_1v_{11}}$ cannot appear in C. Since $\overline{v_1v_{11}}$ cannot appear in C and since $\overline{v_1v_8}$ and $\overline{v_1v_9}$ cannot appear in C simultaneously, $\overline{v_1v_{10}}$ must appear in C; moreover, exactly one of $\overline{v_1v_9}$ and $\overline{v_1v_8}$ must appear in C. To sum up, in Case 2, $\overline{v_{12}v_7}$, $\overline{v_{12}v_4}$, and $\overline{v_1v_{10}}$ must appear in C and exactly one of $\overline{v_1v_9}$ and $\overline{v_1v_8}$ must appear in C. There are two subcases:

Case 2.1: *$\overline{v_1v_9}$ appears in C.* Since v_8 is of degree 3 and $\overline{v_1v_8}$ cannot appear in C, $\overline{v_3v_8}$ must appear in C. Since v_{13} is of degree 3 and $\overline{v_2v_{13}}$ appears in C, exactly one of $\overline{v_{13}v_7}$ and $\overline{v_{13}v_3}$ appears in C. Note that we cannot derive a hamiltonian cycle if $\overline{v_{13}v_3}$ appears in C. Thus $\overline{v_{13}v_7}$ appears in C. Since $\overline{v_{13}v_7}$ and $\overline{v_{12}v_7}$ appear in C, $\overline{v_{14}v_7}$

cannot appear in C. Since v_{14} is of degree 3 and $\overline{v_{14}v_7}$ cannot appear in C, $\overline{v_{14}v_4}$ and $\overline{v_{14}v_3}$ must appear in C. Therefore in this subcase

$$C = [v_1, v_{10}, v_6, v_{11}, v_2, v_{13}, v_7, v_{12}, v_4, v_{14}, v_3, v_8, v_5, v_9].$$

Case 2.2: $\overline{v_1v_8}$ *appears in* C. Since v_9 is of degree 3 and $\overline{v_1v_9}$ cannot appear in C, $\overline{v_4v_9}$ must appear in C. Since $\overline{v_1v_8}$ and $\overline{v_5v_8}$ appear in C, $\overline{v_3v_8}$ cannot appear in C. Since $\overline{v_3v_8}$ cannot appear in C and v_3 has to be adjacent to two vertices of degree 3 to form a hamiltonian cycle, $\overline{v_3v_{13}}$ and $\overline{v_3v_{14}}$ must appear in C. Finally, $\overline{v_7v_{14}}$ must appear in C. Therefore in this subcase

$$C = [v_1, v_{10}, v_6, v_{11}, v_2, v_{13}, v_3, v_{14}, v_7, v_{12}, v_4, v_9, v_5, v_8].$$

Case 3: $\overline{v_5v_8}$, $\overline{v_5v_9}$, $\overline{v_6v_{10}}$, $\overline{v_6v_{11}}$, $\overline{v_2v_{12}}$ *and* $\overline{v_2v_{13}}$ *appear in* C. Since $\overline{v_2v_{12}}$ and $\overline{v_2v_{13}}$ appear in C, $\overline{v_2v_{11}}$ cannot appear in C. Since v_{11} is of degree 3 and $\overline{v_2v_{11}}$ cannot appear in C, $\overline{v_1v_{11}}$ must appear in C. Note that we cannot derive a hamiltonian cycle if $\overline{v_1v_{10}}$ appears in C. Since v_{10} is of degree 3 and $\overline{v_1v_{10}}$ does not appear in C, $\overline{v_{10}v_4}$ must appear in C. Since $\overline{v_2v_{12}}$ and $\overline{v_2v_{13}}$ already appear in C, $\overline{v_7v_{12}}$ and $\overline{v_7v_{13}}$ cannot appear in C simultaneously. Since v_7 has to be adjacent to exactly two vertices of degree 3 to form a hamiltonian cycle and since $\overline{v_7v_{12}}$ and $\overline{v_7v_{13}}$ cannot appear in C simultaneously, $\overline{v_7v_{14}}$ must appear in C; moreover, exactly one of $\overline{v_7v_{12}}$ and $\overline{v_7v_{13}}$ must appear in C. To sum up, in Case 3, $\overline{v_1v_{11}}$, $\overline{v_{10}v_4}$, and $\overline{v_7v_{14}}$ must appear in C and exactly one of $\overline{v_7v_{12}}$ and $\overline{v_7v_{13}}$ must appear in C. There are two subcases:

Case 3.1: $\overline{v_7v_{12}}$ *appears in* C. Then $\overline{v_{13}v_7}$ cannot appear in C. Since v_{13} is of degree 3 and $\overline{v_{13}v_7}$ cannot appear in C, $\overline{v_{13}v_3}$ must appear in C. Note that we cannot derive a hamiltonian cycle if $\overline{v_3v_{14}}$ appears in C. Since v_{14} is of degree 3 and $\overline{v_3v_{14}}$ cannot appear in C, $\overline{v_4v_{14}}$ must appear in C. Since $\overline{v_{10}v_4}$ and $\overline{v_4v_{14}}$ appear in C, $\overline{v_4v_9}$ cannot appear in C. Since v_9 is of degree 3 and $\overline{v_4v_9}$ cannot appear in C, $\overline{v_1v_9}$ must appear in C. Then $\overline{v_1v_8}$ cannot appear in C. Since v_8 is of degree 3 and $\overline{v_1v_8}$ cannot appear in C, $\overline{v_3v_8}$ must appear in C. Therefore in this subcase

$$C = [v_1, v_{11}, v_6, v_{10}, v_4, v_{14}, v_7, v_{12}, v_2, v_{13}, v_3, v_8, v_5, v_9].$$

Case 3.2: $\overline{v_7v_{13}}$ *appears in* C. Then $\overline{v_7v_{12}}$ cannot appear in C. Since v_{12} is of degree 3 and $\overline{v_7v_{12}}$ cannot appear in C, $\overline{v_4v_{12}}$ must appear in C. Note that we cannot derive a hamiltonian cycle if $\overline{v_4v_{14}}$ appears in C. Since v_{14} is of degree 3 and $\overline{v_4v_{14}}$ cannot appear in C, $\overline{v_3v_{14}}$ must appear in C. Since $\overline{v_{10}v_4}$ and $\overline{v_4v_{12}}$ appear in C, $\overline{v_4v_9}$ cannot appear in C. Since v_9 is of degree 3 and $\overline{v_4v_9}$ cannot appear in C, $\overline{v_1v_9}$ must appear in C. Then $\overline{v_1v_8}$ cannot appear in C. Since v_8 is of degree 3 and $\overline{v_1v_8}$ cannot appear in C, $\overline{v_3v_8}$ must appear in C. Therefore in this subcase

$$C = [v_1, v_{11}, v_6, v_{10}, v_4, v_{12}, v_2, v_{13}, v_7, v_{14}, v_3, v_8, v_5, v_9].$$

From the above discussions, G has only six hamiltonian cycles. Note that the hamiltonian cycle in Cases 2.1, 3.1, and 3.2 is isomorphic to the hamiltonian cycle in Cases 1.1, 2.2 and 1.2, respectively. Hence G has only three hamiltonian cycles up to

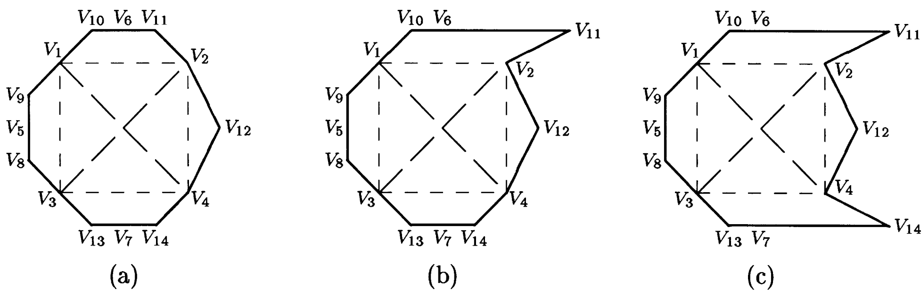

Fig. 3.

isomorphism. To show that G is not a visibility graph, we shall show that none of the hamiltonian cycles in Cases 1.1, 2.2 and 1.2 can form the polygon boundary.

We now show that the hamiltonian cycle in Case 1.1 cannot form the polygon boundary. Let $E(G)$ denote the edge set of G. Suppose G is a visibility graph and P is its corresponding polygon. Since v_1, v_2, v_4, v_3 form a 4-clique in G, V_1, V_2, V_4, V_3 form a quadrilateral subpolygon in P. Since $C=[v_1,v_{10},v_6,v_{11},v_2,v_{12},v_4,v_{14},v_7,v_{13},v_3,v_8,v_5,v_9]$, we have V_1V_2-chain $=[V_{10},V_6,V_{11}]$, V_2V_4-chain $=[V_{12}]$, V_4V_3-chain $=[V_{14},V_7,V_{13}]$, and V_3V_1-chain $=[V_8,V_5,V_9]$. See Fig. 3(a).

Consider where to put V_{11}: Since $\overline{v_{11}v_3}\notin E(G)$, V_{11} and V_3 are not visible in P. Since V_{11} and V_3 are not visible in P and V_{11} is on V_1V_2-chain, V_{11} lies either to the right of $\overrightarrow{V_3V_2}$ or to the left of $\overrightarrow{V_3V_1}$. We claim that it is impossible for V_{11} to lie to the left of $\overrightarrow{V_3V_1}$. Suppose this is not true and V_{11} lies to the left of $\overrightarrow{V_3V_1}$. Since $\overline{v_6v_4}\in E(G)$, V_6 and V_4 are visible in P. Since V_6 and V_4 are visible in P and V_6 is on V_1V_{11}-chain, V_6 must lie to the right of $\overrightarrow{V_4V_1}$. Since $\overline{v_{11}v_1}\in E(G)$, V_{11} and V_1 are visible in P. Since V_{11} and V_1 are visible in P, V_{11} must lie to the right of $\overrightarrow{V_1V_6}$. Since there is no vertex on $V_{11}V_2$-chain, V_{11} and V_4 are visible in P; this contradicts the fact that $\overline{v_{11}v_4}\notin E(G)$. Therefore

(*) V_{11} must lie to the right of $\overrightarrow{V_3V_2}$ (see Fig. 3(b)).

Consider where to put V_{14}: Since $\overline{v_{14}v_1}\notin E(G)$, V_{14} and V_1 are not visible in P. Since V_{14} and V_1 are not visible in P and V_{14} is on V_4V_3-chain, V_{14} lies either to the right of $\overrightarrow{V_1V_3}$ or to the left of $\overrightarrow{V_1V_4}$. We claim that it is impossible for V_{14} to lie to the right of $\overrightarrow{V_1V_3}$. Suppose this is not true and V_{14} lies to the right of $\overrightarrow{V_1V_3}$. Since $\overline{v_2v_7}\in E(G)$, V_2 and V_7 are visible in P. Since V_7 and V_2 are visible in P and V_7 is on $V_{14}V_3$-chain, V_7 must lie to the left of $\overrightarrow{V_2V_3}$. Since $\overline{v_{14}v_3}\in E(G)$, V_{14} and V_3 are visible in P. Since V_{14} and V_3 are visible in P, V_{14} must lie to the left of $\overrightarrow{V_3V_7}$. Since there is no vertex on V_4V_{14}-chain, V_{14} and V_2 are visible in P; this contradicts the fact that $\overline{v_{14}v_2}\notin E(G)$. Therefore

(**) V_{14} must lie to the left of $\overrightarrow{V_1V_4}$ (see Fig. 3(c)).

Consider where to put V_{12}: Since $\overline{v_{12}v_1} \notin E(G)$, V_{12} and V_1 are not visible in P. Since V_{12} and V_1 are not visible in P and V_{12} is on V_2V_4-chain, V_{12} lies either to the left of $\overrightarrow{V_1V_2}$ or to the right of $\overrightarrow{V_1V_4}$. Since $\overline{v_{12}v_3} \notin E(G)$, V_{12} and V_3 are not visible in P. Since V_{12} and V_3 are not visible in P and V_{12} is on V_2V_4-chain, V_{12} lies either to the left of $\overrightarrow{V_3V_2}$ or to the right of $\overrightarrow{V_3V_4}$. Therefore, V_{12} lies either to the left of $\overrightarrow{V_3V_2}$ or to the right of $\overrightarrow{V_1V_4}$. Since P is simple, by $(*)$, V_{12} cannot lie to the left of $\overrightarrow{V_3V_2}$. Since P is simple, by $(**)$, V_{12} cannot lie to the right of $\overrightarrow{V_1V_4}$. Therefore we have no place to put V_{12}. Hence P does not exist and G is not a visibility graph.

By similar arguments, we can prove that the hamiltonian cycle in Case 2.2 (or 1.2) cannot form the polygon boundary. Therefore G is not a visibility graph. □

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