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Order splitting under periodic review inventory systems

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Abstract

Most research on order splitting have focused on the reduction of safety stock in the multiple sourcing setting. Moreover, all works study the use of order splitting for the continuous review inventory systems. In this paper, we investigate the possibility of the multiple-delivery arrangement in the sole sourcing environment. In addition, we concentrate on the reduction of cycle stock for periodic review systems. We show that splitting an order into multiple deliveries can significantly reduce the total cost especially if the cost of despatching an order for an item is not small. Although the use of information technology such as EDI decreases the ordering cost and thus shortens the period length, order splitting remains a cost-effective approach as long as the cost of despatching an order is not close to zero.

Keywords: Inventory; Order-splitting; JIT; Periodic review; Information technology

1. Introduction

The use of order splitting during an order cycle seems to have received much attention recently. For example, Kelle and Silver [1] analyze the safety stock reduction by order splitting assuming the Weibull-distributed lead times, and Sculli and Shum [2] present numerical results on the effect of order splitting on the lead-time demand. Ramasesh et al. [3], Lau and Zhao [4], and Chiang and Benton [5] further develop a total cost model, respectively to obtain the optimal reorder point and order quantity jointly. However, these works have focused on the reduction of safety stock in the multiple sourcing setting. In comparision with cycle stock, safety stock is only a small portion of a company's inventory. In a recent paper, Chiang and Chiang [6] propose the arrangement of multiple deliveries during each order cycle, and consider the reduction of cycle stock in the sole sourcing environment. All of these studies have concentrated on the use of order splitting for continuous review inventory systems. In this paper, we investigate the use of order splitting during each period (also called order cycle thereafter) for periodic review (R, S) systems (without a reorder point). In addition, we focus on the reduction of cycle stock in the sole sourcing setting. Our research also provides a rationale for the just-in-time (JIT) frequent-delivery approach.

In a typical periodic review system, an order quantity which brings the inventory level to S is placed with a specific vendor whenever inventory is reviewed every period of length R. Such an operating policy, known as a replenishment cycle system, is often found in practice (see, e.g. [7]). Although the use of computer systems nowadays has made

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continuous review systems more popular, periodic review systems are still applied in many situations (see, e.g. [8]). Often, periodic systems are found to have the review periods which are longer than the supply lead times. For instance, a retailer may place regular replenishment orders biweekly while the supply lead time is of the order of one week.

It is generally assumed in a periodic system that the whole order quantity from the supplier arrives in a single delivery in each period. It is possible, however, that the buyer could have the supplier agree to order splitting so that portions of an order quantity arrive at the receiving point at different times of a period. For companies who work with their suppliers on a long-term relationship, this multiple-delivery approach is particularly feasible and useful. For example, Hotai Motor Co. Ltd., the distributor of Toyota products (and the largest auto distributor) in Taiwan, has recently adopted this multiple-delivery approach, which will be commonly used in the future by other auto distributors. Hotai Motor Co. Ltd. orders thousands of service parts for domestically manufactured cars (such as Toyota Corona Exsior and Tercel) monthly from six major suppliers (not including the Toyota Motor Company in Japan) (manufacturing of these cars is in a different company). Each of the six suppliers makes at least 10 deliveries (low-usage items are shipped less frequently than high-usage items) per month to the central warehouse of Hotai Motor Co. Ltd.

Apparently, the benefit of this multiple-delivery approach is the reduction in the average cycle stock, while the disadvantage of this approach is that ordering costs (which include transportation and inspection costs) may increase. The major goal of this research is to develop a multiple-delivery model and evaluate this tradeoff. In addition, if it benefits the buyer to arrange multiple deliveries with the supplier, does there exist an optimal number of deliveries per order cycle? This research also investigates this issue.

We assume that both R and S are decision variables. In addition, we assume that lead time is constant, demand is non-negative and independently distributed in disjoint time intervals, and that demand during a time interval of length τ is normally distributed with mean $\mu\tau$ and variance $\sigma^2\tau$. Note that in practice R is often predetermined by the firm. For example, a retailer may coordinate a group of items to a distribution center weekly or biweekly. Also, vendors in a department store often make routine visits to customers to take fresh orders [8]. There may exist other practical or organizational considerations (see, e.g. [9]). In this paper, however, we assume that R is a variable inside the model.

In addition, as the supply chain management of materials has been a trend in industry, some firms have invested in information technology to reduce the communication and transaction time among trading partners. The use of electronic data interchange (EDI) in inventory control systems has been particularly noteworthy and the benefits of reduced logistics and order processing costs are reported [10]. In this paper, we also discuss the impact of the reduction of ordering costs on the shortening of the review periods and on the performance of the multiple-delivery model, after the firm and its supplier(s) have decided to invest in EDI (i.e., the decision of establishing an EDI-based inventory system has been made and is not considered inside the model).

This paper is organized as follows. First, we briefly review the ordinary single-delivery approach under periodic review systems. Then we present a two-delivery model, which is followed by some computational results. Next, we generalize the analysis to the multiple-delivery model. Finally, this paper ends with the conclusion.

2. Review of the single-delivery approach

We first review the ordinary single-delivery approach. Let L_1 be the constant lead time and $h_1(Y_1)$ the probability density function (PDF) for the demand Y_1 over a period of length R. Then $h_1(Y_1)$ is $N(\mu R, \sigma^2 R)$. Suppose that we review inventory at the time point t and Y_1 is the demand during the preceding period $(t - R, t)$. Then we will order Y_1 and raise the inventory position up to S which should be large enough to meet the demand for the upcoming time interval of length $R + L_1$. Let $B(L_1)$ be the average backorder that might build up before the next order arrives at time

$t + R + L_1$. Then,

$$B(L_1) = \sigma\sqrt{R + L_1}\,G(k_1), \tag{1}$$

where

$$k_1 = \frac{S - \mu(R + L_1)}{\sigma\sqrt{R + L_1}} \tag{2}$$

and $G(\cdot)$ is the partial expectation function tabled in Brown [11] or Silver and Peterson [7]. Instead of having to estimate the backorder cost, we use a service level (SL) constraint for the objective function (as in [6]). Service level is defined as the percentage of demand to be served directly from stock. For the single-delivery approach, service level is given by

$$\mathrm{SL} = 100 - 100\frac{B(L_1)}{\mu R}. \tag{3}$$

Let D be the average annual demand, A the fixed ordering cost, J the review cost, and h the annual carrying cost per unit. To simplify the notation, we incorporate the review cost into the ordering cost, i.e., A also includes the review cost. The ordering cost A, as described by Lau and Zhao [4], consists of two components. One is the cost of despatching an order for an inventory item each time, denoted by O, such as administrative and processing costs (note that the review cost is included in O). The other is the cost of receiving an incoming procurement, denoted by I, such as costs of transportation, handling and inspection of the procurement after it arrives, etc. O and I togeter constitute the fixed ordering cost A (i.e., $A = O + I$). The use of information technology such as EDI will decrease O and thus shorten the period length R. In Section 4, we will investigate this issue. As the average amount ordered per period is μR and the safety stock is $S - \mu L_1 - \mu R$, the decision problem for the single-delivery model can be expressed by

$$\text{Min } C(R,S) = [D(O + I)/\mu R] + h\left(S - \mu L_1 - \frac{\mu R}{2}\right) \tag{4}$$

s.t.

$$100 - 100\frac{B(L_1)}{\mu R} \geqslant \psi, \tag{5}$$

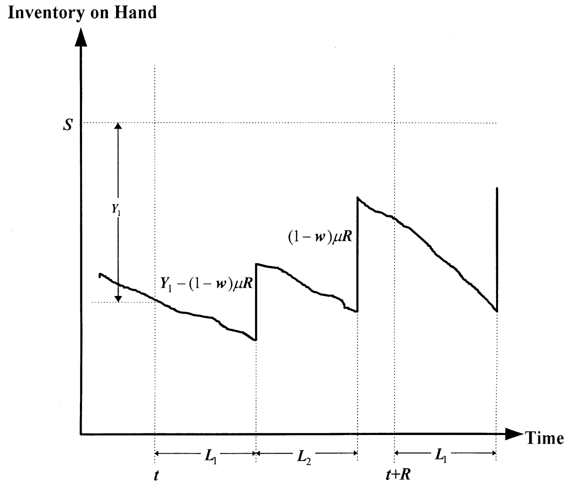

Fig. 1. A two-delivery model.

where ψ is a preassigned service level. Note that the expression for average on-hand inventory is only an approximation, i.e., it assumes that the average backorder level is quite small (see, e.g. [12]). Given the nature of the problem, the optimal solution will automatically satisfy constraint (5) at equality. To find the optimal R and S, we use (5) to find the optimal S for a given value of R. Then we tabulate the total cost as a function of R to determine the optimal R [12].

3. A two-delivery model

We now present a two-delivery model for the periodic review (R, S) system. Let L_2 (which is smaller than R) be the inter-arrival time between the first and second shipments. We suppose that we order Y_1 (i.e., demand during the preceding period) at the review epoch t and the supplier agrees to deliver part of the order quantity after time L_1 and the remaining part after time $L_1 + L_2$ (see Fig. 1). (Notice that $L_1 + L_2$ need not be less than R as shown in Fig. 1.) Let $(1 - w)\mu R$ be the size of the second shipment and thus $Y_1 - (1 - w)\mu R$ is the size of the first shipment (note that the average size of the first shipment is $w\mu R$). The idea is to raise the inventory up to $S - (1 - w)\mu R$ in the first shipment. It is assumed that $Y_1 > (1 - w)\mu R$. If

Y_1 should be less than $(1-w)\mu R$, there is only one shipment of size Y_1 delivered after $L_1 + L_2$. Y_1 is at least $(1-w)\mu R$ if L_2 is not too short so that w is very small (the fact that w depends on L_2 is shown in the following computation). Intuitively, to make the two-delivery arrangement attractive, the inter-arrival time L_2 should not be too short, since otherwise the reduction in the average cycle stock (to be explained below) would be very small and the two-delivery approach may only result in an increase in the material handling cost (part of the ordering cost). It is assumed for the time being that L_2 is fixed. Later we will explore how L_2 can have an impact upon the total cost of the buyer.

Let $h_2(Y_2)$ be the PDF for demand Y_2 during the upcoming time interval $(t, t+L_1+L_2)$. Then $h_2(Y_2)$ is $N(\mu(L_1+L_2), \sigma^2(L_1+L_2))$. In the single-delivery approach, it is generally assumed that the order placed at the review epoch t would clear all the shortages (if any) at the time of arrival (see, e.g. [12]). In the two-delivery model, shortages may not be all cleared at time $t+L_1$ (since part of the order arrives at time $t+L_1+L_2$). More importantly, shortages can occur between time $t+L_1$ and $t+L_1+L_2$. Thus, we need to compute the shortages that might build up before the receipt of the second shipment. Note that there is the possibility of double-counting the same shortages as long as a shipment could not clear all the shortages at the time of arrival. Although this should rarely happen (since the average backorder level is quite small), we assume that we are willing to accept possible double-counting [6]. Let $B(L_2)$ denote the average backorder that might build up before the recipt of the second shipment. For the two-delivery approach, the service level is given by

$$\mathrm{SL} = 100 - 100\frac{B(L_1)+B(L_2)}{\mu R} \tag{6}$$

and $B(L_2)$ is expressed as

$$B(L_2) = \int_{S-(1-w)\mu R}^{\infty} Y_2 - [S-(1-w)\mu R]h_2(Y_2)\,\mathrm{d}Y_2$$
$$= \sigma\sqrt{L_1+L_2}\,G(k_2), \tag{7}$$

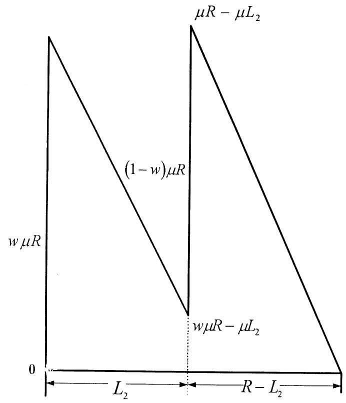

Fig. 2. Average cycle stock of the two-delivery model.

where

$$k_2 = [S-(1-w)\mu R - \mu(L_1+L_2)]/\sigma\sqrt{L_1+L_2}. \tag{8}$$

Also, if we let the arrival of the first shipment of an order initiate a cycle, then a cycle consists of two time intervals of length L_2 and $R-L_2$ (see Fig. 1). The average cycle stock is $w\mu R - \mu L_2/2$ for the time interval of length L_2 and $\mu R/2 - \mu L_2/2$ for the time interval of length $R-L_2$ (see Fig. 2). Thus, the overall average cycle stock is $\mu R/2 - (1-w)\mu L_2$. By splitting an order into two deliveries, we see that the average cycle stock is reduced by $(1-w)\mu L_2$ (if R remains the same as in the single-delivery model).

On the other hand, when two deliveries of an order are arrenged with the supplier, the ordering cost may increase. While the cost of despatching an order is unchanged, the cost of receiving incoming procurements may nearly double. We assume that when two deliveries of an order are arranged with the supplier, the ordering cost becomes $O+2I$.

It follows that the decision problem for the two-delivery model discussed above is expressed by

$$\mathrm{Min}\ C(R,S,w) = [D(O+2I)/\mu R] + h\left[S - \mu L_1 - \frac{\mu R}{2} - (1-w)\mu L_2\right] \tag{9}$$

Table 1
Effect of the inter-arrival time L_2 on the performance of the two-delivery model. Data: $\mu = 10$ units/day, $\sigma = 2$ units, 1 year = 250 days, $A = \$2(O = I = \$1)$, $\psi = 99.90$, $h = \$0.5$/unit/year

L_2	S	w	$C(R,S)$	L_2	S	w	$C(R,S,w)$
(A) $L_1 = 10$ days, $R = 20$ days				(B) $L_1 = 10$ days, $R = 25$ days			
8	321	0.3774	73.1	10	372	0.3792	72.5
9	321	0.4301	72.4	11.5	372	0.4422	71.4
9.5	321	0.4564	72.2	12	372	0.4632	71.3
10.0	321	0.4827	72.1	12.5	372	0.4841	71.3
10.5	321	0.5090	72.2	13	372	0.5051	71.3
11	321	0.5353	72.4	13.5	372	0.5260	71.5
12	321	0.5878	72.3	15	372	0.5888	72.7
(C) $L_1 = 5$ days, $R = 20$ days				(D) $L_1 = 5$ days, $R = 25$ days			
8	269	0.3719	71.9	10	320	0.3766	71.3
9	269	0.4250	71.1	11.5	320	0.4400	70.3
9.5	269	0.4515	70.9	12	320	0.4611	70.2
10.0	269	0.4779	70.9	12.5	320	0.4821	70.1
10.5	269	0.5044	71.0	13	320	0.5032	70.2
11	269	0.5308	71.2	13.5	320	0.5243	70.4
12	269	0.5836	72.0	15	320	0.5874	71.6

s.t.

$$100 - 100\,\frac{B(L_1) + B(L_2)}{\mu R} \geqslant \psi$$

or

$$B(L_1) + B(L_2) - \frac{(100 - \psi)\mu R}{100} \leqslant 0, \tag{10}$$

$$0 < w < 1. \tag{11}$$

Note that (10) can be easily shown to be convex with respect to S and w (for a given R). Given the nature of the problem, the optimal solution will always have constraint (10) held at equality. By formulating the Lagrangian

$$[D(O + 2I)/\mu R] + h\left[S - \mu L_1 - \frac{\mu R}{2} - (1 - w)\mu L_2\right] + \lambda\left[B(L_1) + B(L_2) - \frac{(100 - \psi)\mu R}{100}\right],$$

and setting the derivatives with respect to S, w and λ equal to zero, we can obtain

$$\frac{L_2}{R} = \frac{P(k_2)}{P(k_1) + P(k_2)}, \tag{12}$$

$$B(L_1) + B(L_2) = \frac{(100 - \psi)\mu R}{100}, \tag{13}$$

where $0 < w < 1$ and $P(\cdot)$ is the complement of the cumulative distribution function for the standard normal variable. It is evident from (12) and (13) that the optimal S and w (given a certain R) do not depend on the values of O, I, and h. To find the optimal combination of R, S, and w, we use (12) and (13) to find the optimal S and w for a given R. Then we tabulate the total cost as a function of R to determine the optimal R.

4. Computational results

We investigate the effect of the inter-arrival time L_2 on the performance of the two-delivery model. We also examine the effect of cost parameters, demand variability, and service level on the performance of the two-delivery model relative to the single-delivery model.

4.1. Effect of the inter-arrival time

We first investigate the effect of L_2 on the performance of the two-delivery model. It appears from Table 1 that the total cost is at a minimum when L_2 is approximately equal to $R/2$. Note that this result is also obtained under other levels of L_1 and

Table 2
Single-delivery model versus two-delivery model under different levels of O. Data: $\mu = 10$ units/day, $\sigma = 2$ units, 1 year = 250 days, $L_1 = 10$ days, $L_2 = R/2$, $I = \$1$, $h = \$0.5$/unit/year, $\psi = 99.90$

O	Single-delivery model			Two-delivery model				% savings
	R	S	$C(R,S)$	R	S	w	$C(R,S,w)$	
\$0.0	10	217	58.5	20	321	0.4818	59.6	− 1.88
0.125	11	227	61.6	20	321	0.4818	61.2	0.65
0.25	11	227	64.4	20	321	0.4818	62.7	2.64
0.5	12	237	69.8	22	342	0.4780	65.7	5.87
1.0	14	258	79.7	24	362	0.4822	71.2	10.67
2.0	17	288	95.6	28	403	0.4818	80.9	15.38

Table 3
Single-delivery model versus two-delivery model under different levels of I. Data: $\mu = 10$ units/day, $\sigma = 2$ units, 1 year = 250 days, $L_1 = 10$ days, $L_2 = R/2$, $O = \$1$, $h = \$0.5$/unit/year, $\psi = 99.90$

I	Single-delivery model			Two-delivery model				% savings
	R	S	$C(R,S)$	R	S	w	$C(R,S,w)$	
\$0.25	11	227	64.4	17	291	0.4744	52.7	18.17
0.5	12	237	69.8	20	321	0.4818	59.6	14.61
1.0	14	258	79.7	24	362	0.4822	71.2	10.67
2.0	17	288	95.6	31	433	0.4862	89.5	6.38
4.0	22	339	121.3	41	535	0.4861	117.2	3.38
8.0	30	420	160.0	51	697	0.4895	157.8	1.37

R, although the computations are not shown here. We are unable to prove this result due to the partial expectation functions involved. However, we could explain as follows. As mentioned above, we can let the arrival of the first shipment of an order initiate a cycle. Then if $L_2 = R/2$, the arrival of second shipment is exactly halfway through a cycle. It is as though we place an order (in a single shipment) every period of length $R/2$ and the arrival epochs would be the same. For computational purposes, it is much easier to fix L_2 at $R/2$ and find the optimal combination of R, S and w, instead of treating L_2 also as a variable. Moreover, it is easy for both the buyer and the supplier to implement the two-delivery contract when L_2 is simply fixed at $R/2$. Consequently, we set $L_2 = R/2$ in the computations thereafter.

4.2. *Effect of cost parameters*

We next consider the effect of cost parameters. Note that the cost structure is characterized by the ratio of A/h or $(O + I)/h$. Thus, we fix the value of h to investigate the effect of the amount of A (relative to h) on the performance of the two-delivery model. Since $A = O + I$, we vary the value of O and I, respectively, to examine their effect. As we see from Tables 2 and 3, the two-delivery model becomes more attractive as O increases and I decreases, respectively (other things being equal). These results came with no surprise. Since the ordering cost equals $O + 2I$ in the two-delivery model, a smaller I results in only a slight increase in the ordering cost and thus the two-delivery approach becomes more effective by reducing the average cycle stock.

Table 4
Single-delivery model versus two-delivery model under different levels of the ratio $O/(O+I)$. Data: $\mu = 10$ units/day, $\sigma = 2$ units, 1 year = 250 days, $L_1 = 10$ days, $L_2 = R/2$, $A = O + I = \$2$, $h = \$0.5$/unit/year, $\psi = 99.90$

O	Single-delivery model			Two-delivery model				% savings
	R	S	$C(R,S)$	R	S	w	$C(R,S,w)$	
\$2.0	14	258	79.7	19	311	0.4801	59.6	25.22
1.75	14	258	79.7	21	331	0.4851	62.8	21.20
1.5	14	258	79.7	22	342	0.4780	65.7	17.56
1.25	14	258	79.7	23	352	0.4802	68.5	14.05
1.0	14	258	79.7	24	362	0.4822	71.2	10.67
0.75	14	258	79.7	25	372	0.4841	73.8	7.40
0.5	14	258	79.7	26	382	0.4860	76.2	4.39
0.25	14	258	79.7	27	393	0.4801	78.6	1.38
0	14	258	79.7	28	403	0.4818	80.9	− 1.51

On the contrary, a smaller O saves the two-delivery model very little of the ordering cost and thus makes it less effective. This has a very important implication. As we mentioned before, the use of EDI in inventory systems will lower O and thus the optimal period length R is shortened, as can be seen from Table 2. If we assume that the firm and its supplier(s) have made an investment in EDI, the use of order splitting does not appear particularly attractive as O is decreased to a smaller level in the long run. Nevertheless, the firm benefits from the use of the two-delivery approach, since the two-delivery approach obtains a lower total cost than the traditional single-delivery approach as long as O is not decreased to a level of near zero. This result also is apparent from Table 4. If we fix the value of A but change the proportion of O in A, we see that the two-delivery approach yields a smaller percentage cost savings as the ratio of O/A decreases to zero. In an extreme case of $O/A \approx 0$, splitting an order into two deliveries during each cycle will increase the total cost.

As a note, the two-delivery approach yields a larger R than the single-delivery model, although the inter-arrival time L_2 between the two deliveries is shorter than the optimal R of the single-delivery model. This implies that the buyer will order a larger quantity and thus is more likely to obtain quantity discounts under the two-delivery model.

4.3. *Effect of demand variability and service level*

Next, we examine the effect of demand variability and service level on the performance of the single-delivery model versus the two-delivery model. It appears from Tables 5 and 6 that the two-delivery model performs better under lower levels of σ or ψ (other things being equal). This is because there are two stockout possibilities (one more than the single-delivery model) during each order cycle in the two-delivery model. Thus, the two-delivery model is less vulnerable to stockouts if demand variability or service level is low. This agrees with the finding of Chiang and Chiang [6].

5. A multiple-delivery model

We consider the possibility of the arrangement of n shipments during each order cycle. Let $L_i = R/n$, $i = 2, \ldots, n$, be the inter-arrival time between the $(i-1)$th and ith shipments. We suppose that the supplier agrees to deliver the ith shipment after $\sum_{j=1}^{i} L_j$, $i = 1, \ldots, n$, and the first shipment has size $Y_1 - (1 - w_1)\mu R$ (note again that the average size of the first shipment is $w_1 \mu R$), the second shipment has size $w_2 \mu R, \ldots$, the $(n-1)$th shipment has size $w_{n-1}\mu R$, and the nth shipment has size $(1 - \sum_{j=1}^{n-1} w_j)\mu R$. Let $B(L_i)$, $i = 2, \ldots, n$ be the average backorder that might build up before the

Table 5
Single-delivery model vs. two-delivery model under different levels of σ. Data:$\mu = 10$ units/day, 1 year = 250 days, $L_1 = 10$ days, $L_2 = R/2$, $A = \$2$, $(O = I = \$1)$, $\psi = 99.90$, $h = \$0.5$/unit/year

σ	Single-delivery model			Two-delivery model				% savings
	R	S	$C(R, S)$	R	S	w	$C(R, S, w)$	
0.5	14	243	72.2	24	344	0.4953	63.0	12.74
1.0	14	247	74.2	24	349	0.4958	65.5	11.73
2.0	14	258	79.7	24	362	0.4822	71.2	10.67
4.0	14	280	90.7	24	390	0.4595	83.8	7.61

Table 6
Single-delivery model vs. two-delivery model under different levels of ψ. Data: $\mu = 10$ units/day, $\sigma = 2$ units, 1 year = 250 days, $L_1 = 10$ days, $L_2 = R/2$, $A = \$2 (O = I = \$1)$, $h = \$0.5$/unit/year

ψ	Single-delivery model			Two-delivery model				% savings
	R	S	$C(R, S)$	R	S	w	$C(R, S, w)$	
95.00	14	235	68.2	24	336	0.5066	59.6	12.60
99.00	14	247	74.2	24	350	0.4895	65.6	11.59
99.90	14	258	79.7	24	362	0.4822	71.2	10.67
99.99	14	266	83.7	24	371	0.4765	75.3	10.03

receipt of the ith shipment. Then,

$$B(L_i) = \sigma \sqrt{\sum_{j=1}^{i} L_j}\; G(k_i), \quad i = 2, \dots, n, \tag{14}$$

where

$$k_i = \left[S - \left(1 - \sum_{j=1}^{i-1} w_j\right)\mu R - \mu \sum_{j=1}^{i} L_j \right] \Big/ \sigma \sqrt{\sum_{j=1}^{i} L_j}. \tag{15}$$

We assume that the ordering cost is $O + nI$ when n shipments during each cycle are arrenged with the supplier. Noticing that the average cycle stock is reduced by $(1 - w_1)\mu L_2 + (1 - w_1 - w_2)\mu L_3 + \cdots + (1 - \sum_{j=1}^{n-1} w_j)\mu L_n$, we can express the decision problem as

$$\begin{aligned} \text{Min } C(R, S, w_1, \dots, w_{n-1}) &= [D(O + nI)/\mu R] \\ &+ h\Bigg\{ S - \mu L_1 - \frac{\mu R}{2} - (1 - w_1)\mu L_2 \\ &- (1 - w_1 - w_2)\mu L_3 - \cdots - \left(1 - \sum_{j=1}^{n-1} w_j\right)\mu L_n \Bigg\} \end{aligned} \tag{16}$$

s.t.

$$\sum_{i=1}^{n} B(L_i) - \frac{(100 - \psi)\mu R}{100} = 0, \tag{17}$$

$$0 < w_j < 1, \quad j = 1, \dots, n-1, \tag{18}$$

$$\sum_{j=1}^{n-1} w_j < 1. \tag{19}$$

To find the optimal combination of S and w_j, $j = 1, \dots, n-1$, for a given R, we formulate the Lagrangian of this multiple-delivery model and set the derivatives with respect to S and w_j, $j = 1, \dots, n-1$, equal to 0, yielding

$$\frac{L_2}{R} = \frac{P(k_2)}{P(k_1) + P(k_2) + \cdots + P(k_n)},$$

$$\frac{L_3}{R} = \frac{P(k_3)}{P(k_1) + P(k_2) + \cdots + P(k_n)}, \dots,$$

$$\frac{L_n}{R} = \frac{P(k_n)}{P(k_1) + P(k_2) + \cdots + P(k_n)}, \tag{20}$$

$$\sum_{i=1}^{n} B(L_i) = \frac{(100 - \psi)\mu R}{100} \tag{21}$$

Table 7
Two-delivery model versus three-delivery model under different levels of the ratio $O/(O+I)$. Data: $\mu = 10$ units/day, $\sigma = 2$ units, 1 year = 250 days, $L_1 = 10$ days, $O + I = \$2$, $h = \$0.5$/unit year, $\psi = 99.90$

O	Two-delivery model				Three-delivery model					% savings
	R	S	w	$C(R,S,w)$	R	S	w_1	w_2	$C(R,S,w_1,w_2)$	
\$2.0	19	311	0.4801	59.6	24	364	0.3050	0.3463	51.1	14.26
1.5	22	342	0.4780	65.7	29	415	0.3076	0.3458	60.6	7.76
1.0	24	362	0.4822	71.2	33	456	0.3086	0.3446	68.7	3.51
0.5	26	382	0.4860	76.2	37	496	0.3120	0.3462	75.8	0.52
0	28	403	0.4818	80.9	41	537	0.3124	0.3448	82.2	− 1.61

Table 8
Multiple-delivery models. Data: $\mu = 10$ units/day, $\sigma = 2$ units, 1 year = 250 days, $L_1 = 10$ days, $O = \$1.5$, $I = \$0.5$, $h = \$0.5$/unit/year, $\psi = 99.90$

n	R	S	$C(R,S,w_1,\dots,w_{n-1})$	n	R	S	$C(R,S,w_1,\dots,w_{n-1})$
2	22	342	65.7	3	29	415	60.6
4	36	487	58.2	5	42	550	56.9
6	48	612	56.1	7	55	684	55.7
8	61	746	55.5	9	67	808	55.4
10	74	880	55.5	11	80	942	55.7

(see the appendix for details). Since $L_i = R/n$, $i = 2, \dots, n$, it follows from (20) that $k_1 = k_2 = \cdots = k_n$. Hence, we can use the following procedure to obtain the optimal S and w_j, $j = 1, \dots, n-1$.

Step 1. Substitute $k_2 = k_1, \dots, k_n = k_1$ into (21) to obtain k_1 and thus S.

Step 2. Use $k_i = k_1$, $i = 2, \dots, n$, to obtain w_j by using (15), $j = 1, \dots, n-1$, respectively.

We then tabulate the total cost as a function of R to determine the best R. Table 7 gives the computational results for the relative performance of the two-delivery model versus the three-delivery model. As we see, the total cost may be further reduced if we split an order into three deliveries during each cycle.

A question arises at this point: does there exist an *optimal number* of deliveries per cycle that results in minimum total cost (as in [6]). To investigate this, we carry out the computation further. As we see, for example, the optimal number of deliveries per cycle is 9 in Table 8. This also illustrates the frequent-delivery approach that Hotai Motor Co. Ltd. (as mentioned in the introduction) employs to reduce the inventory carrying cost. As Hotai Motor Co. Ltd. works with its major suppliers on a long-term relationship, the ordering cost of an item is small. Often, delivery of a split procurement for an item is part of a joint shipment which includes hundreds of items, and there is no inspection after the procurement arrives. The suppliers also absorb some of the transportation cost.

In summary, if the ordering cost structure agrees with what we assume, the buyer should consider negotiating an optimal number of deliveries for each cycle with the supplier.

6. Conclusion

In this paper, we investigate the possibility of the multiple-delivery arrangement during each order cycle for periodic review systems. We show that splitting an order into multiple deliveries can reduce the average cycle stock and thus the total cost, especially if the cost of despatching an order (which includes the review cost) is not small. Although the use of information technology such as EDI decreases the ordering cost and thus shortens the period length, order splitting remains a cost-effective approach as long as the cost of despatching an order is not close to zero. Moreover, we show that there exists an optimal number of deliveries per cycle such that the lowest total cost is obtained. As very few assumptions are made in this research, firms can apply the approach of order splitting in practice immediately, as long as multiple shipments of an order can be arranged with suppliers. Finally, we should note that this research also provides a rationale for the JIT frequent-delivery approach.

Appendix A

In this appendix, we derive expression (20) of Section 5. The Lagrangian including (16) and (17) with a multiplier λ is

$$[D(O+nI)/\mu R] + h\left\{S - \mu L_1 - \frac{\mu R}{2} - (1-w_1)\mu L_2 - (1-w_1-w_2)\mu L_3 - \cdots - \left(1 - \sum_{j=1}^{n-1} w_j\right)\mu L_n\right\} + \lambda\left\{\sum_{j=1}^{n} B(L_j) - \frac{(100-\psi)\mu R}{100}\right\}.$$

Differentiating it with respect to S and setting the derivative equal to zero, we obtain

$$\lambda = h/(P(k_1) + P(k_2) + \cdots + P(k_n)). \quad \text{(A.1)}$$

Next, we differentiate the Lagrangian with respect to w_{n-1}, set it to zero, and substitute (A.1) into the expression to give

$$L_n/R = P(k_n)/(P(k_1) + P(k_2) + \cdots + P(k_n)). \quad \text{(A.2)}$$

Then, we differentiate the Lagrangian with respect w_{n-2}, set it to zero, and substitute (A.1) into the expression to give

$$(L_{n-1} + L_n)/R = (P(k_{n-1}) + P(k_n))/(P(k_1) + P(k_2) + \cdots + P(k_n)),$$

and substitute (A.2) into the above expression to yield

$$L_{n-1}/R = P(k_{n-1})/(P(k_1) + P(k_2) + \cdots + P(k_n)). \quad \text{(A.3)}$$

Continue this way and differentiate with respect to $w_{n-3}, \ldots,$ and w_1 to obtain respectively

$$L_{n-2}/R = P(k_{n-2})/(P(k_1) + P(k_2) + \cdots + P(k_n)),$$
$$\vdots$$
$$L_2/R = P(k_2)/(P(k_1) + P(k_2) + \cdots + P(k_n)),$$

The above expressions together with (A.2) and (A.3) are expression (20) in Section 5.

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