# Analytical Model for Design Criteria of Passively Q-Switched Lasers

Y. F. Chen, Y. P. Lan, and H. L. Chang

*Abstract—***A general and straightforward model was developed** for the design of passively *Q*-switched lasers. With the second**threshold criterion and using a numerically fitting procedure, the output pulse energy was expressed as an analytical function of the initial transmission of the saturable absorber and the reflectivity of the output coupler. An analytical expression for the optimal output reflectivity was also obtained for maximizing the output pulse en**ergy of a passively Q-switched laser with a given initial transmis**sion of the saturable absorber. Excellent agreement was studied between the present results and detailed theoretical computations.** A Nd:YAG laser with  $Cr^{4+}$ :YAG as a saturable absorber was per**formed to illustrate the use of the present model.**

*Index Terms*—Passively  $Q$ -switched laser, saturable absorber, **solid-state laser, Cr**4+**:YAG.**

## I. INTRODUCTION

**P** ASSIVELY Q-switched solid-state lasers [1]–[5] are still playing an important role in many applications, such as range finders, pollution detection, lidars, and medical systems. Several recent theoretical investigations [6]–[8] were proposed to optimize the performance of  $Q$ -switched lasers. Degnan [6] derived the key parameters of an energy-maximized passively -switched laser as functions of two variables and generated several design curves. More recently, Xiao and Bass [7] and Zhang *et al.* [8] followed Degnan's approach to include the effect of excited state absorption (ESA) in the saturable absorber into the analysis. A saturable absorber exhibiting ESA is basically called a reverse saturable absorber. The properties of ESA have important applications in optical limiting devices involving reverse saturable absorbers [9]. In addition, Harter *et al.* theoretically demonstrated the possibility of mode-locking a laser containing both a saturable absorber and a reverse saturable absorber [10].

By analogy with the analysis of the actively  $Q$ -switched case, Degnan [6] used the variable  $z = 2\sigma n_i l/L$  in the analysis of the passively  $Q$ -switched laser for a convenient comparison. Here,  $n_i$  is the initial population density in the gain medium,  $\sigma$  is the stimulated emission cross section of the gain medium,  $l$  is the length of the gain medium, and  $L$  is the nonsaturable intracavity round-trip dissipative optical loss. In the actively  $Q$ -switched laser,  $n_i$  is normally proportional to the pump rate. As a re-

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sult, it is practically useful to model the output pulse energy of the actively  $Q$ -switched laser with the variable  $z$ . However, for the passively  $Q$ -switched laser,  $n_i$  does not depend on the pump rate; it is determined by the initial transmission of the saturable absorber  $(T<sub>o</sub>)$  and the reflectivity of the output mirror  $(R)$ . Therefore, it should be more practical to model the output pulse energy of the passively  $Q$ -switched laser with the parameters  $T<sub>o</sub>$  and R than with the variable z. Similarly, it is of great practical interest to determine the optimum output reflectivity as a function of  $T<sub>o</sub>$  for a given gain medium in a passively Q-switched laser.

In this work, we first derive a general formula for the second threshold criterion by including the influence of intracavity focusing and the ESA effect into the rate-equation analysis [11]. With the derived formula, we defined two parameters. One parameter is related to the upper bound of  $T<sub>o</sub>$ , which can result in normal  $Q$ -switching behavior for a given  $R$ , a given gain medium and a given saturable absorber. With this parameter, the output pulse energy was explicitly fitted as an analytical function of  $T<sub>o</sub>$  and R. The other parameter is related to the lower bound of  $R$ , which can result in normal  $Q$ -switching behavior for a given  $T_o$ , a given gain medium and a given saturable absorber. With this parameter, the optimum output reflectivity for maximizing the output pulse energy was successfully fitted as an analytical function of  $T<sub>o</sub>$ . Finally, a Nd:YAG laser with  $Cr<sup>4+</sup>:YAG$  as a saturable absorber is examined to illustrate the use of the present model.

### II. SECOND THRESHOLD OF PASSIVELY  $Q$ -SWITCHED LASERS

The physical meaning of the second threshold is whether the saturable absorber will saturate first, thereby allowing the photon density to turn upward and produce a giant pulse. Alternatively, the gain might saturate first, so that the photon density never turns upward to develop a giant pulse. To model the operation of a passively  $Q$ -switched laser, we shall assume uniform pumping of the gain medium, assume the intracavity optical intensity as axially uniform, and assume complete recovery of the saturable absorber, i.e., for a single-shot or low pulse rate. The coupled rate equations have been used to model a passively  $Q$ -switched laser in many investigations [6]–[8]. Here, we extend previous results by including the influence of intracavity focusing and the ESA effect. The coupled equations for three or four level gain media are modified as

$$
\frac{d\phi}{dt} = \frac{\phi}{t_r} \left[ 2\sigma n l - 2\sigma_{\rm gs} n_{\rm gs} l_s - 2\sigma_{\rm es} n_{\rm es} l_s - \left( \ln \left( \frac{1}{R} \right) + L \right) \right]
$$
(1)

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$$
\frac{dn}{dt} = -\gamma c \sigma \phi n \tag{2}
$$

$$
\frac{dn_{\rm gs}}{dt} = -\frac{A}{A_s}c\sigma_{\rm gs}\phi n_{\rm gs}
$$
\n
$$
n_{\rm gs} + n_{\rm es} = n_{\rm so} \tag{3}
$$

where



- $\cal R$ reflectivity of the output mirror;
- inversion reduction factor ( $\gamma = 1$  and  $\gamma = 2$  $\gamma$ correspond to, respectively, four-level and threelevel systems; see [6]);

 $t_r = 2l'/c$  round-trip transit time of light in the cavity optical length  $l'$ , where c is the speed of light.

Note that a four-level saturable absorber such as that considered by Hercher [12] is used in the present analyses.

Dividing (2) by (3) and integrating gives

$$
n_{\rm gs} = n_{\rm so} \left(\frac{n}{n_i}\right)^{\alpha} \tag{5}
$$

where

$$
\alpha = \frac{1}{\gamma} \frac{\sigma_{\rm gs}}{\sigma} \frac{A}{A_s} \tag{6}
$$

and  $n_i$  is the initial population inversion density in the gain medium.  $n_i$  is determined from the condition that the round-trip gain is exactly equal to the round-trip losses just before the  $Q$ -switch opens. Thus

$$
2\sigma n_i l - 2\sigma_{\rm gs} n_{\rm so} l_s - \left(\ln\left(\frac{1}{R}\right) + L\right) = 0. \tag{7}
$$

To keep the parallels between the present analysis and previous works [6]–[8], we use the following expression:

$$
T_o = \exp(-\sigma_{\rm gs} n_{\rm so} l_s),\tag{8}
$$

where  $T<sub>o</sub>$  is the initial transmission of the saturable absorber. In terms of  $T<sub>o</sub>$  in (7) can be expressed as

$$
n_i = \frac{\ln\left(\frac{1}{T_o^2}\right) + \ln\left(\frac{1}{R}\right) + L}{2\sigma l}.\tag{9}
$$

Dividing (1) by (2) and substituting (5) into the result gives

$$
\frac{d\phi}{dn} = -\frac{l}{\gamma l'} \left[ 1 - \frac{(1-\beta)}{2\sigma ln_i} \ln\left(\frac{1}{T_o^2}\right) \left(\frac{n}{n_i}\right)^{\alpha - 1} - \frac{\beta \ln\left(\frac{1}{T_o^2}\right) + \ln\left(\frac{1}{R}\right) + L}{2\sigma ln} \right]
$$
\n(10)

where

$$
\beta = \frac{\sigma_{\rm es}}{\sigma_{\rm gs}}\tag{11}
$$

Since the first derivative of  $\phi$  with respective to n at  $n = n_i$  is equal to zero, the criterion for  $Q$ -switching behavior is whether the second derivative of  $\phi$  with respective to n at  $n = n_i$  has a positive or a negative sign. If positive, the growth curve for the photon intensity will turn increasingly upward. With (9), we obtain

$$
\frac{d^2\phi}{dn^2} = -\frac{l}{\gamma l'} \left[ -\frac{(1-\beta)(\alpha-1)}{2\sigma ln_i^2} \ln\left(\frac{1}{T_o^2}\right) \left(\frac{n}{n_i}\right)^{\alpha-2} + \frac{\beta \ln\left(\frac{1}{T_o^2}\right) + \ln\left(\frac{1}{R}\right) + L}{2\sigma ln^2} \right].
$$
\n(12)

Substituting  $n = n_i$  into (12), the criterion for a giant pulse to occur is then given by

$$
[\alpha(1-\beta) - 1] \ln \left(\frac{1}{T_0^2}\right) - \ln \left(\frac{1}{R}\right) - L > 0. \tag{13}
$$

Substituting (6) into (13), the criterion for the second threshold becomes

$$
\frac{\ln\left(\frac{1}{T_o^2}\right)}{\ln\left(\frac{1}{T_o^2}\right) + \ln\left(\frac{1}{R}\right) + L} \frac{\sigma_{gs}}{\sigma} \frac{A}{A_s} > \frac{\gamma}{1 - \beta.}
$$
(14)

The main difference between our derivation and the previous result [11] is that we take account of the influence of excitedstate absorption and intracavity focusing. Therefore, in the case of  $\beta = 0$  and  $A/A_s = 1$ , (14) can be reduced to the previous result [11]. In addition, (14) is practically useful because the parameters used in the present derivation are directly related to the design of a passively  $Q$ -switched laser.

Note that the parameters  $\alpha$  and  $\beta$  are determined from the cavity configuration and the physical properties of the gain medium and the saturable absorber. From (13), it can be found that the initial transmission of the saturable absorber has an upper bound  $(T<sub>o</sub>)<sub>upper</sub>$  for producing a giant pulse for a given  $\ln(I/R) + L$ , a given  $\alpha$ , and a given value of  $\beta$ , i.e.,

$$
(T_o)_{\text{upper}} = \exp\left[-\frac{\ln\left(\frac{1}{R}\right) + L}{2(\alpha(1-\beta)-1)}\right].\tag{15}
$$

On the other hand, the reflectivity of the output coupler has a lower bound  $(R)_{\text{upper}}$  to build up a giant pulse for a given L, a given  $T_o$ , a given  $\alpha$ , and a given  $\beta$ , i.e.,

$$
(R)_{\text{lower}} = \exp\left[-(\alpha(1-\beta)-1)\ln\left(\frac{1}{T_o^2}\right)+L\right].
$$
 (16)

As described later, the parameters  $(T_o)_{\text{upper}}$  and  $(R)_{\text{lower}}$  can be conveniently used to express the output pulse energy and optimum output reflectivity as an analytical function, respectively.

# III. ANALYTICAL MODEL FOR OUTPUT PULSE ENERGY AND OPTIMUM OUTPUT COUPLING

In [6]–[8] and in many other referenes, it is assumed that the transverse-mode profile is a plane-wave distribution.



Fig. 1. Calculation results for the output pulse energy as a function of  $T<sub>o</sub>$  for several values of  $R$ ,  $\alpha$  and  $\beta$ . Dashed lines: results from (26)–(28). Solid lines: results through numerically solving (23) and substituting the solution into (21).

However, an aperture is often used to generate the  $TEM_{00}$ mode in a stable cavity. Therefore, it is more practical to derive the output energy for the Gaussian beam. Here, we shall follow specifically the approach developed in [13] to derive the output pulse energy for the Gaussian beam distribution with the ESA effect.

The energy  $(E_g)$  extracting from the gain medium of a passively  $Q$ -switched laser includes three parts. Some of the energy  $(E_s)$  is lost in bleaching of the saturable absorber, some  $(E_i)$  is lost due to intracavity losses or ESA, and another part  $(E)$  leaves the cavity as the output energy. The equation for energy balance is given by [13]

$$
E_g = E_s + E_i + E \tag{17}
$$

Although general expressions for the parameters in (17) were derived in [13] for the Gaussian beam profile, the ESA effect was not considered. To modify the expressions of [13] by including the ESA effect, we obtain

$$
E_g = h\nu n_i A F(x) \tag{18}
$$

$$
E_s = \frac{h\nu A_s}{2\sigma_s} (1 - \beta) \ln\left(\frac{1}{T_o^2}\right) F(\alpha x)
$$
 (19)

$$
E_i = \frac{h\nu A}{2\sigma\gamma} \left[ L + \beta \ln\left(\frac{1}{T_o^2}\right) \right] x \tag{20}
$$

$$
E = \frac{h\nu A}{2\sigma\gamma} \ln\left(\frac{1}{R}\right)x\tag{21}
$$

and

$$
F(x) = \int_0^x \frac{1 - \exp(-y)}{y} \, dy \tag{22}
$$

where  $h\nu$  is the laser photon energy. The parameter x is the energy density at the maximum of the transverse distribution of the laser mode in the gain medium, normalized to the saturation energy density. The physical meaning of the parameter  $x$  represents the extraction efficiency of the energy stored in the gain medium through the lasing process [13].

From (17)–(21) using (9), the equation for x can be given by

$$
F(x) - \frac{(1-\beta)\ln\left(\frac{1}{T_c^2}\right)}{\ln\left(\frac{1}{T_c^2}\right) + \ln\left(\frac{1}{R}\right) + L} \frac{F(\alpha x)}{\alpha}
$$

$$
- \frac{\beta\ln\left(\frac{1}{T_c^2}\right) + \ln\left(\frac{1}{R}\right) + L}{\ln\left(\frac{1}{T_c^2}\right) + \ln\left(\frac{1}{R}\right) + L} x = 0.
$$
(23)

The output energy can be found from (21) if (23) is solved for  $x$ . Equation (23) indicates that the parameter  $x$  can be expressed as a function of the parameters  $\alpha$ ,  $\beta$ ,  $T_o$ , and  $\ln(1/R) + L$ . It is worth while noting that the parameter  $x$  is independent of the pump level  $n_i$ , as mentioned early. In other words, above threshold the output pulse energy of a passively  $Q$ -switched laser should not depend on the pump level; it is mainly determined by the properties of the gain medium and the saturable absorber. This is why above threshold, increasing the pump energy only increases the number of output pulses but does not lead to an obvious increase for the output energy per pulse, as shown in [1] and [2] and in our experimental results described later.

With some transformation, (23) can be reduced to the previous formula derived from the plane-wave approximation. If the transverse-mode profile is assumed to be the plane wave, we have

$$
F(x) = 1 - \exp(-x). \tag{24}
$$

In this case, if  $A/A_s = 1$ , then (23) becomes

$$
1 - \exp(-x) - \frac{(1-\beta)\ln\left(\frac{1}{T_c^2}\right)}{\ln\left(\frac{1}{T_0^2}\right) + \ln\left(\frac{1}{R}\right) + L} \frac{1 - \exp(-\alpha x)}{\alpha}
$$

$$
- \frac{\beta \ln\left(\frac{1}{T_0^2}\right) + \ln\left(\frac{1}{R}\right) + L}{\ln\left(\frac{1}{T_c^2}\right) + \ln\left(\frac{1}{R}\right) + L} x = 0. \tag{25}
$$

With the substitution of  $x = \ln(n_i/n_f)$ , (25) becomes the formula obtained in previous analysis [7], [8].

Since the output pulse energy is a function of the parameters  $\alpha, \beta, T_o$ , and  $\ln(1/R) + L$ , it is of great practical interest to express the output pulse energy as an analytical function of the parameters  $\alpha$ ,  $\beta$ ,  $T_o$ , and  $\ln(1/R) + L$  for the design of passively  $Q$ -switched lasers. Through the numerical calculation, we found that the pulse energy can be satisfactorily fitted as in (26), shown at the bottom of the next page, where

$$
\eta = \left[\frac{1 + 3\exp(-50\alpha^{-3})}{\beta + 0.08}\right]
$$

$$
\times (1 - \exp(1 - \alpha + \alpha\beta)) \bigg] / \ln\left(\frac{1}{R}\right) + L \quad (27)
$$

and

$$
f(\alpha, \beta) = 1.15 - 0.2 \exp(-5\beta) - 0.9\beta^2 - \frac{\exp(1 - \alpha)}{\sqrt{\alpha - 1}} + \frac{0.15 + 0.9\beta}{\exp(150\alpha^{-3})}
$$
(28)

The functional form of (26) was based on the fact that the pulse energy is proportional to the modulation losses of the saturable absorber  $(1-\beta) \ln(1/T_0^2)$  and inversely proportional to the total cavity losses in the situation that the saturable absorber saturates  $\beta \ln(1/T_o^2) + \ln(1/R) + L$ . The other term  $1 - (T_o/(T_o)_{\text{upper}})^{\eta}$ in (26) was used to satisfy the condition of the second threshold. The parameters  $\eta$  and  $f(\alpha, \beta)$  were used to obtain a good fitting result. Since the present model can cover the typical extent of  $\alpha > 2$  and  $\beta < 0.7$ , it is suitable for most passively Q-switched lasers.

Fig. 1 shows the output pulse energy as a function of  $T<sub>o</sub>$  for several values of  $R, \alpha$  and  $\beta$ . Note that one can use a single parameter  $\ln(1/R) + L$  to omit the dependence of E on L. Here, we just used  $L = 0.05$  in the calculation for convenience. In this figure, we compare the results obtained directly from (26)–(28) with the numerical data obtained through solving (23) and substituting the solution into (21) to show the accuracy of the analytical expression for the output energy. Good agreement is found for all cases.

To illustrate the utility of the present model, a Nd:YAG miniature laser with a  $Cr^{4+}$ :YAG crystal as a saturable absorber is considered and performed experimentally. The plane-plane resonator of the length 8 cm includes the rear mirror, whose reflectivity is  $>99/8\%$ , the Cr<sup>4+</sup>:YAG crystal is near the rear mirror, the Nd:YAG medium with a 3-mm in diameter and 50-mm in length pumped by a xenon flashlamp, and the output mirror with  $R = 40\%$ . We used several Cr<sup>4+</sup>:YAG crystals with different initial transmission to test the laser. The input energy of the xenon flashlamp can be adjusted by adjusting the voltage



Fig. 2. Plot of the experimental and theoretical results for the output pulse energy as a function of  $T_o$  in the Nd:YAG/Cr<sup>4+</sup>:YAG  $Q$ -switched laser. Symbols: experimental data. Solid line: numerical data from solving (21) and (23). Dashed line: calculation results from (26)–(28)



Fig. 3. Plot of the experimental output energy as a function of the input voltage of the xenon flashlamp with  $T_o = 0.3$ .



Fig. 4. Plot of the calculation results (solid lines) from (26)–(28) for the output pulse energy as a function of the output reflectivity for several initial transmissions  $T_o$  with the values of  $\alpha$  and  $\beta$  in Nd:YAG/Cr<sup>4+</sup>:YAG Q-switched laser. Dashed line indicates the position of the optimal output reflectivity.

of the xenon flashlamp to ensure a single laser pulse to be obtained. For a plane–plane cavity,  $A/A_s$  is nearly equal to unity. To demonstrate the influence of  $A/A<sub>s</sub>$ , the rear mirror was replaced by a concave mirror with a radius of curvature of 10-m, where  $A/A_s$  is about 0.86.

The parameters of the  $Cr^{4+}$ :YAG crystal as given in [1] are: cm<sup>2</sup> and  $\sigma_{\rm es} = 2.2 \times 10^{-19}$  cm<sup>2</sup>. For the Nd:YAG crystal,  $\gamma = 1$  and  $\sigma = 2.8 \times^{-19}$  cm<sup>2</sup> [14]. Thus, the parameters  $\alpha$  and  $\beta$  from (6) and (11) are 3.11 and 0.25, respectively. With a beam radius of 1.5 mm, the value of  $(h\nu A/2\sigma\gamma)$ is 24 mJ.  $L$  is estimated as 0.03 from the free-running experiment. Substituting the values of  $\alpha$ ,  $\beta$ ,  $R$ ,  $L$ , and  $(h\nu A/2\sigma\gamma)$  into (26)–(28), the output pulse energy can be predicted as a function of initial transmission  $T<sub>o</sub>$ .

Fig. 2 shows the experimental and theoretical results for the dependence of the on  $T<sub>o</sub>$ . For comparison, the theoretical results calculated from (21) with the numerical solution of (23) are also shown in Fig. 2. It can be seen that the prediction of the analytical model agrees well with the experimental data and the theoretical calculation. In addition, from (15),  $(T<sub>o</sub>)<sub>upper</sub>$  was found to be about 0.701 for  $A/A_s = 1$  and 0.615 for  $A/A_s = 0.86$  that

$$
E = \begin{cases} \frac{h\nu A}{2\sigma\gamma} \ln\left(\frac{1}{R}\right) \frac{(1-\beta)\ln\left(\frac{1}{T_o^2}\right)}{\beta \ln\left(\frac{1}{T_o^2}\right) + \ln\left(\frac{1}{R}\right) + L} \left[1 - \left(\frac{T_o}{(T_o)_{\text{upper}}}\right)^{\eta}\right] f(\alpha, \beta), & \text{for } T_o < (T_o)_{\text{upper}} \\ 0, & \text{for } T_o \ge (T_o)_{\text{upper}} \end{cases}
$$
(26)



Fig. 5. The calculation results for  $R_{\rm opt}$  as a function of  $T_o$  for several typical values of  $\alpha$  and  $\beta$ . Solid lines: results obtained from solving (23) and substituting the solution into (21) and finding the optimal output reflectivity. Dashed lines: results calculated from (29) and (30).

is quite close to the experimental results, as shown in Fig. 2. It is worthwhile to mention that the quoted values for  $\sigma_{gs}$  and  $\sigma_{es}$ from [1] are small by a factor of 2 when compared with more recent measurements [15]. Nevertheless, the analytical model shown in (26)–(28) indicates clearly that the output energy is seen to be quite insensitive to the absolute values of  $\sigma_{\rm gs}$  and  $\sigma_{\rm es}$ when  $\alpha$  is much greater than 1. This is the case of the present experiment and analysis.

Fig. 3 depicts the experimental output energy as a function of the input voltage of the xenon flashlamp with  $T<sub>o</sub> = 0.3$ . The curve reveals three steep steps, each reflecting the appearance of an additional pulse. The output energy per pulse is shown to be nearly unchanged as increasing the pump energy. As mentioned early, the output pulse energy in the passively  $Q$ -switched laser is mainly determined by  $T<sub>o</sub>$  and  $R$ , not by the pump energy. The step-like input-output characteristic is a common feature in the passively Q-switched laser [1], [2]. With the values of  $\alpha$  and  $\beta$ for the Nd:YAG/Cr<sup>4+</sup>:YAG  $Q$ -switched laser and (26)–(28), the output pulse energy was calculated as a function of the output reflectivity for several valuse of the initial transmission  $T<sub>o</sub>$ . Fig. 4 shows the calculated results. The dashed line in this figure shows that there is an optimal output reflectivity for maximizing the output pulse energy for a given  $T<sub>o</sub>$ . Therefore, it is practically useful to obtain the optimal output reflectivity as a function of  $T_o$ ,  $\alpha$ , and  $\beta$ .

As shown in (16), the output reflectivity should be greater than  $(R)_{\text{lower}}$  for normal  $Q$ -switching behavior. Consequently, the optimal output reflectivity for maximizing pulse energy should be larger than  $(R)_{\text{lower}}$ . Therefore, we use the following function form to express optimal output reflectivity:

where the factor  $m(\alpha, \beta)$  is smaller than unity. Through numerical analysis, we find that  $m(\alpha, \beta)$  can be satisfactorily fitted to

$$
m(\alpha, \beta) = \frac{(2.85 - \beta)(\alpha - 1.1)}{\alpha(\alpha + 1)} \left[ 1 - \frac{2(\alpha - 2)^{0.5}}{\exp(0.83\alpha)} + 5\exp\left(-\frac{1.5}{\alpha} - \frac{1.3}{\beta}\right) \right].
$$
 (30)

Fig. 5 shows the calculated results for  $R_{\rm opt}$  as a function of  $T<sub>o</sub>$  for several typical values of  $\alpha$  and  $\beta$ . Once again, we just used  $L = 0.05$  in the calculation for convenience. To reveal the accuracy of the analytical function, we compare the results calculated from (29) and (30) with the numerical data obtained from solving (23) and substituting the solution into (21) and finding the optimal output reflectivity. Good agreement is also found for all cases. In general, the optimal output reflectivity is an increasing function of  $T<sub>o</sub>$  for a given  $\alpha$  and a given  $\beta$ .

# IV. CONCLUSION

We have included the effects of the intracavity focusing and ESA in the coupled rate equation to derive the condition of the second threshold for a passively  $Q$ -switched laser. From the criterion of the second threshold, we obtain two parameters  $(T<sub>o</sub>)<sub>upper</sub>$  and  $(R)<sub>lower</sub>$ . In terms of  $(T<sub>o</sub>)<sub>upper</sub>$ , the output pulse energy was explicitly fitted to an analytical function of the variables  $\alpha$ ,  $\beta$ ,  $T_o$ , and  $\ln(1/R) + L$ . A passively Q-switched Nd:YAG laser with  $Cr^{4+}$ :YAG as a saturable absorber was used to verify the validity of the analytical function. Furthermore, we used the parameter  $(R)_{\text{lower}}$  to find an analytical function for the optimal output reflectivity. The present model provides a straightforward procedure for the design of passively Q-switched lasers.

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