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A fuzzy reasoning based diagnosis system for $\bar{X}$ control charts

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This paper describes a new diagnosis system, which is based on fuzzy reasoning to monitor the performance of a discrete manufacturing process and to justify the possible causes. The diagnosis system consists chiefly of a knowledge bank and a reasoning mechanism. The knowledge bank provides knowledge of the membership functions of unnatural symptoms that are described by Nelson's rules on $\bar{X}$ control charts and knowledge of cause-symptom relations. We develop an approach called maximal similarity method (MSM) for knowledge acquisition to construct the fuzzy cause-symptom relation matrix. Through the knowledge bank, the diagnosis system can first determine the degrees of an observation fitting each unnatural symptom. Then, using the fuzzy cause-symptom relation matrix, we can diagnose the causes of process instability. In conclusion we provide a numerical example to illustrate the system.

Keywords: Fuzzy reasoning, knowledge acquisition, diagnosis system, process control, $\bar{X}$ control chart

1. Introduction

Rapid diagnosis of the manufacturing process in a plant plays an important role in maintaining product quality. A good diagnosis system should have the capability to monitor the status of a process and to justify the possible causes when a process is becoming unstable.

Traditionally, statistical process control (SPC) is widely used to detect a discrete manufacturing process behavior. When instability is indicated on an SPC chart, the operators are under stress to justify the possible causes quickly. By removing the root cause of instability, the process can be improved.

However, in general the operators lack the knowledge or the experience needed to interpret the signals indicated on the SPC chart and to relate the signals with the possible causes. In this paper, we develop an on-line diagnosis system that is an operational support system to assist operators to quickly find the correct causes.

Our diagnosis system is a fuzzy reasoning based system. $\bar{X}$ control charts are applied to monitor whether or not a process is under control and if not, to find the cause of variation. Parallel to human thinking, the diagnosis system needs to detect the unnatural patterns of symptoms on $\bar{X}$ control charts and construct the knowledge base for diagnosis.

At present, several pattern recognition algorithms have been developed to detect unnatural patterns of symptoms on $\bar{X}$ control charts. Al-Ghanim (1995) adopted statistical correlation analysis to generate a set of optimal matched filters for identifying three specific unnatural patterns (trends, cycles, and systematic variables). Unfortunately his method gets poor performance on diagnosing trend patterns. Gwee (1996) applied fuzzy logic control in recognition of unnatural pattern for $\bar{X}$ and R charts, However, he did not mention how to define suitable sets of membership functions and rules. Guo and Dooley (1992) proposed the back-propagation algorithm and Bayesian statistical classification procedure to identify a structure change in the process behavior. Moreover, Hwarng and Hubele (1991, 1993) developed an $\bar{X}$ control chart pattern recognizer based on back-propagation neural networks paradigm. As a whole, these pattern recognition algorithms need much more sophisticated skill than the rules-test method.

A rules-test method was firstly proposed by West Electric Company (1985) and developed by Nelson (1984, 1985) for detecting the unnatural patterns of symptoms on $\bar{X}$ control charts. This method is simple and most popular in factories, however, the method lacks the ability to distinguish the degree of an observation fitting symptoms. For example, if an observation point on an $\bar{X}$ control chart is very near but not beyond the control limit, then by Nelson's rule the state of the process is judged as being in-control as the state that an observation is near the centerline. In fact, the two unlike phenomena have reflected the different essential in a process. The former indicates that the process has a higher possibility to be unstable in the near future than those in the latter case.

In this research, the concept of fuzzy set is applied to modify Nelson's rules. The unnatural patterns of symptoms on $\bar{X}$ control charts are indicated by the modified Nelson's rules. In addition, we represent the cause-symptom relation as a fuzzy relation matrix form. Then we develop a new method to acquisition knowledge from data, called the maximal similarity method, which is used to establish the fuzzy relation matrix. Finally by approximate reasoning our diagnosis system can justify the possible causes for unstable processes.

This paper is organized as follows. In Section 2, we depict the framework of the fuzzy diagnosis system. In Section 3, we interpret the development of the system, including the construction of the membership function of unnatural symptoms and the cause-symptom relation matrix. In Section 4, a numerical example is shown. Conclusions are made in the final section.

2. Framework of fuzzy diagnosis system

In this section, we will depict the concept of the proposed fuzzy reasoning based diagnosis system for $\bar{X}$ control charts.

As shown in Fig. 1, the system consists of a knowledge bank and an inference mechanism module. The knowledge bank comprises both the membership functions for the unnatural symptoms and the knowledge of cause-symptom relations that supports the inference mechanism to detect the occurrence of

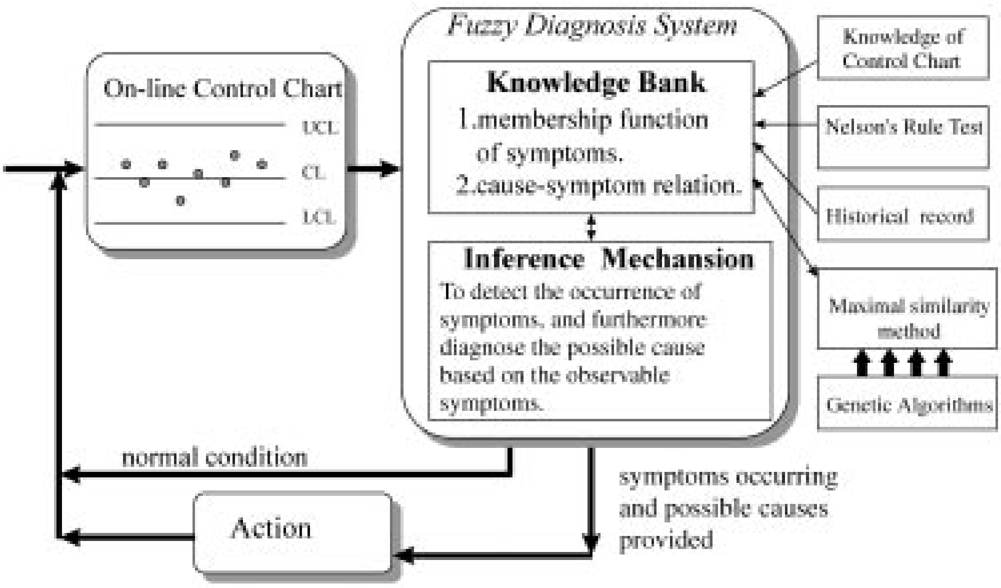

Fig. 1. Framework of diagnosis system.

symptoms and furthermore diagnose the possible cause based on the observable symptoms.

The diagnosis of the inference mechanism is based on fuzzy reasoning and the process is stated as follows:

$$[a_{i1} \;\cdots\; a_{ij} \;\cdots\; a_{im}] \circ \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1c} \\ r_{21} & \ddots & & \\ \vdots & & r_{jk} & \\ r_{m1} & & \cdots & r_{mc} \end{bmatrix} = [b_{i1} \;\cdots\; b_{ik} \;\cdots\; b_{ic}] \qquad (1)$$

abbreviated as $\mathbf{a_i} \circ \mathbf{R} = \mathbf{b_i}$, where $0 \leq a_{ij}, r_{jk}, b_{ik} \leq 1$, $i = 1, \ldots, n$; $j = 1, \ldots, m$; $k = 1, \ldots, c$. Element a_{ij} of vector $\mathbf{a_i}$ means the membership grade of ith input observation satisfying the jth symptom. There are m symptoms considered. Element r_{jk} of matrix R denotes the relational grade of the jth symptom and the kth cause. Vector $\mathbf{b_i}$ represents the diagnostic results and element b_{ik} means the possibility of the kth cause occurring. ''$\circ$'' is the maxmin composition operator. In terms of the structure of fuzzy reasoning, it is apparent that the diagnostic result depends intensely on the construction of the system's knowledge bank.

In the next section, we will introduce the approach to establish the knowledge bank which contains the constructions of membership functions of symptoms and the cause-symptom relation matrix.

3. Establishment of fuzzy diagnosis

In this section, we explain how to develop the knowledge bank of the system that includes knowledge about detecting symptoms on $\bar{X}$ control charts and the relationship between assignable causes and symptoms.

3.1. *Detection of symptoms*

Since rules-test provides a simple and useful tool in identifying unnatural symptoms, we adopt Nelson's rules (Nelson, 1985) (Table 1) to detect the unnatural symptoms on $\bar{X}$ control charts. In applying Nelson's rule tests, one half of the control band at a time is considered, that is, the area between the central line and one of the control limits. This area can be partitioned into three equal zones, labeled as c, b, a, as shown in Fig. 2. Each symptom is defined as the fitness of observed points to each rule.

Table 1. Nelson's rules

Rule 1	One point beyond zone a
Rule 2	Nine points in a row in zone c or beyond
Rule 3	Six points in a row steadily increasing or decreasing
Rule 4	Fourteen points in a row altering up and down
Rule 5	Two out of three points in a row in zone a or beyond
Rule 6	Four out of five points in zone b or beyond
Rule 7	Fifteen points in a row in zones c, above and below the centerline
Rule 8	Eight points in a row on both sides of the centerline with none in zone c

In order to capture the most information contained in the observed points, we extend the crisp threshold of Nelson's rules to fuzzified results. The extension by softening the threshold is according to Beliakov's technique (Beliakov, 1996). For example, we soften the threshold of Nelson's first rule as ''fuzzily one point beyond zone a'', which is denoted as $\tilde{A}$. In this case, we assume the process is monitored by an $\bar{X}$ control chart with centerline $u_0 = 0$ and control limits $u_0 \pm 3\sigma$. Then applying Beliakov's technique we can define the corresponding membership function, $\mu_{\tilde{A}(x)}$ of set $\tilde{A}$:

$$\mu_{\tilde{A}(x)} = \begin{cases} 0 & x \leq 2.33\sigma \\ \int_{2.33\sigma}^{x} \rho(t)dt \Big/ \int_{2.33\sigma}^{3\sigma} \rho(t)\,dt & 2.33\sigma \leq x \leq 3\sigma \\ 1 & 3\sigma \leq x \end{cases} \qquad (2)$$

The membership function (as shown in Fig. 3) is defined as the normalized distance from the observed point to the control limit, by a specific metric, $d(\cdot,\cdot)$, $d(x, y) = |\int_{x}^{y} \rho(t)\,dt|$, where $\rho(t)$ (as shown in Fig. 3) means the probability density of $\bar{X}$ in the in-control state. The subscript number, 2.33σ, of integral signs in the above formula is chosen as the critical point to maintain the probability of type I error under 0.01, hence the membership grades of points located within

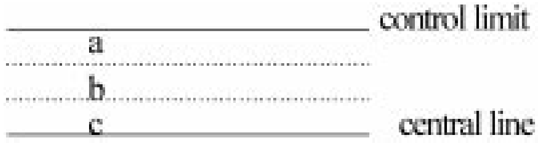

Fig. 2. One half of the control band.

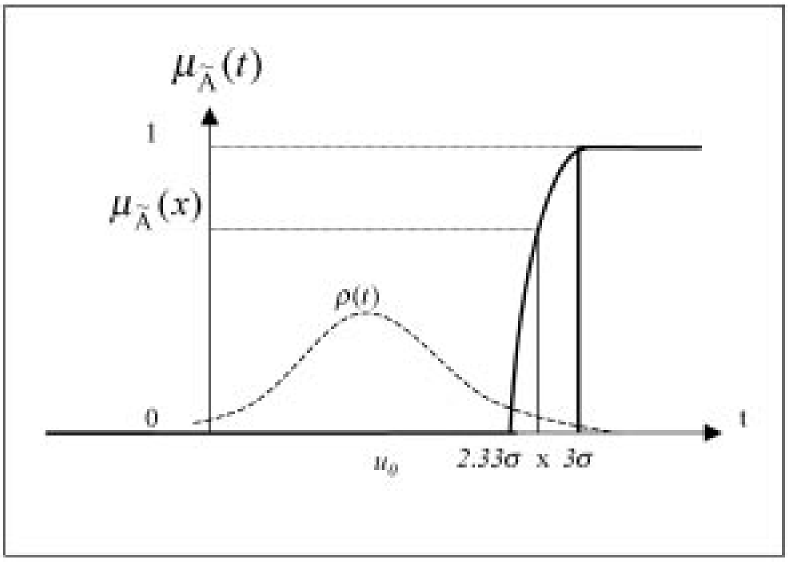

Fig. 3. Fuzzification of the crisp threshold curve.

2.33σ is set to zero. By the definition, it is apparent that the smaller distance has the larger membership grade of the set ''fuzzily one point beyond zone a''. Therefore, through the extension we can distinguish the phenomenon of a point near but not beyond control limit from the phenomenon of a point near the centerline.

Similarly, the membership functions for softening other Nelson's rules (described in Table 1) are constructed and shown in the Appendix. In the next section, we will describe the other important part of the knowledge bank, the knowledge of cause-symptom relations.

3.2. *Construction of cause-symptom relation matrix*

The detection of unnatural symptoms on $\bar{X}$ control chart tells us when to look for trouble, but it can not tell us the cause of the trouble. Cause-symptom relation is the important tool connecting symptoms and causes. Therefore, it is necessary to construct the relationship knowledge in the diagnosis system. We use the fuzzy relation matrix R to express the knowledge of cause-symptom relations. In other words, the task of constructing knowledge of cause-symptom relations involves finding the fuzzy relation matrix from expert's knowledge or a historical data set. In the following section, we will show how to construct it with historical data set.

From the maintenance record and its corresponding $\bar{X}$ control charts, we can define the membership grades of symptoms and the true assignable cause of unstable process in each maintenance record. The maintenance record is treated as the historical data set and is represented as the form (A, B). A is a fuzzy input matrix and B is a crisp output matrices. Each *i*th row vector of matrix A and B represents the membership grades of symptoms and the corresponding assignable cause for a maintenance data respectively. Hence, finding the cause-symptom relation matrix in a fuzzy reasoning based diagnosis system can be described as solving the following fuzzy relation equation given known matrices A and B,

$$\mathrm{A} \circ \mathrm{R} = \mathrm{B} \tag{3}$$

However, the exact solution of equation (3) in most case does not exist since the element of given matrix B is a crisp value, which is either 0 or 1. Therefore, we hope to find a matrix $\hat{\mathrm{R}}$ such that matrix $\hat{\mathrm{B}}$, obtained from $\mathrm{A} \circ \hat{\mathrm{R}} = \hat{\mathrm{B}}$, is as close to matrix B as possible, that is, approximately to solve the above equations.

Attempts to obtain the approximate solution of fuzzy relation equations were initiated by Pedrycz (1983), in which the Quasi-Newton method was used to find the approximate solution in order to minimize the variance of $\hat{\mathrm{B}}$ and B. But this method is unsuited to our problem. The prime reason is that the maximal deviations of elements between the optimized $\hat{\mathrm{B}}$ and B may be equal to 1 (Wang, 1993), which means the diagnostic result is completely opposite to the truth, the diagnosis ability will be doubtful. Therefore, we develop another approach, namely a maximal similarity method, to construct the cause-symptom relation matrix.

3.2.1. *Maximal similarity method (MSM)*

The maximal similarity method (denoted as MSM) essentially desires to find an ideal matrix $\hat{R}$ such that the composite matrix $A \circ \hat{R}$ is as similar to matrix B as possible. Here, two matrices with the same size is said to be similar, meaning that all the corresponding elements located in the same position of these two matrices are close. Therefore, MSM attempts to make each element $\hat{b}_{i,j}(=\max_k \min(a_{ik}, \hat{r}_{kj}))$ of the composite matrix as possibly approaching the corresponding element $b_{i,j}$ of matrix B. Since each element $b_{i,j}$ of matrix B is either 0 or 1, we would like $\hat{b}_{i,j}$ to approach 1 as possible when $b_{i,j} = 1$, that is, preferring the term $\{\max_k \min(a_{ik}, \hat{r}_{kj})\}$ larger. Similarly, we would like $\hat{b}_{i,j}$ to approach 0 as possible when $b_{i,j} = 0$, that is, preferring the term $\{1 - \max_k \min(a_{ik}, \hat{r}_{kj})\}$ larger. Integrating the above analysis, the general term can be shown below. Then we can make $\hat{b}_{i,j}$ possibly approach $b_{i,j}$ regardless whether $b_{i,j} = 1$ or 0.

$$\text{Maximize } \left\{\max_k \min(a_{ik}, \hat{r}_{kj})\right\}^{b_{ij}} \cdot \left\{1 - \max_k \min(a_{ik}, \hat{r}_{kj})\right\}^{1-b_{ij}} \tag{4}$$

Simultaneously consider overall elements of matrix B and composite matrix $\hat{B}$, the aggregated equation is shown in Equation 5. The reason for using the multiplier to aggregate the general terms is to avoid the contrast between $b_{i,j}$ and $\hat{b}_{i,j}$ for any i and j.

Problem A:

$$\begin{aligned} \text{Max } S(\hat{R}) = \prod_{i=1}^{n}\prod_{j=1}^{c} & \left\{\max_k \min(a_{ik}, \hat{r}_{kj})\right\}^{b_{ij}} \\ & \cdot \left\{1 - \max_k \min(a_{ik}, \hat{r}_{kj})\right\}^{1-b_{ij}} \\ \text{s.t. } & 0 \le a_{ik},\ \hat{r}_{kj} \le 1; b_{ij} = 0 \text{ or } 1; \\ & i = 1, \ldots, n; j = 1, \ldots, c;\ k = 1, \ldots, m \end{aligned} \tag{5}$$

where the function $S(\hat{R})$, named similarity function, is used to measure the similarity between the evaluated matrix $A \circ \hat{R}$ and matrix B. According to this method we can completely avoid the contrast condition. When the contrast condition occurs, the value of function $S(\hat{R})$ is equal to zero. In solving the process, the contrast condition can be avoided.

Since the optimization of problem A is very cumbersome, we solve the optimal solutions using genetic algorithms instead of the conventional search method and briefly introduce genetic algorithms in the next section.

3.2.2. *Genetic algorithms applied to MSM*

Genetic algorithms (referred to as GAs hereafter) (Davis, 1991; Goldberg, 1989) are global search and optimization techniques motivated by the process of natural selection in biological system. GAs are different from other search procedures in the following ways (Karr, 1993): (1) GAs consider many points in the search space simultaneously, rather than a single point; (2) GAs work directly with strings of characters representing the parameter set, not the parameters themselves; (3) GAs use probabilistic rules to guide their search, not deterministic rules. Because GAs consider many points in the search space simultaneously there is a reduced chance of converging to local optima. In a conventional search, based on a decision rule, a single point is considered and that is unreliable in multimodal space.

The primary distinguishing features of GAs are an encoding, a fitness function, a selection mechanism, a crossover mechanism, a mutation mechanism, and a culling mechanism.

GAs can be formulated as the following steps:

(1) Randomly generate an initial solution set (population) of N strings and evaluate each solution by fitness function.

(2) If the termination condition was not met, do
Repeat {Select parents for crossover.
Generate offspring.
Mutate some of the numbers.
Merge mutants and offspring into population.
Cull some members of the population.}

(3) Stop and return the best fitted solution.

In order to apply GAs to problem A, we define the similarity function $S(\hat{R})$ as the fitness function and the search space of size L. Each point on the space with size L represents a solution (a set of parameters) of Problem A. The length L is determined by the product of the number of parameters and the bits required representing each parameter. For example, we assume the size of matrix $\hat{R}$ is m^*c, and each element of matrix $\hat{R}$ is represented by r bits. Therefore, the length L of a string is equal to $(m^*c)r$.

In GAs work, the binary strings $(s_{kj})_2$ of r bits within each search point are used to describe the value of the elements $\hat{r}_{kj}$ of $\hat{R}$ as shown as follows. These binary strings can be transferred to decimal integers $(s'_{kj})_{10}$, which range from 0 to $2^r - 1$. In order to evaluate each search point, we must decode these strings to elements of $\hat{R}$. Since each element $\hat{r}_{kj}$ in $\hat{R}$ belongs to $[0, 1]$, we decode the strings to $\hat{r}_{kj}$ by $\hat{r}_{kj} = (s'_{kj})_{10}/(2^r - 1)$. After decoding we will put these parameters to the fitness function and evaluate it.

The termination condition is achieved when the number of generations is large enough or a satisfied fitness value is obtained. In the next subsection, we will illustrate the diagnosis system with a numerical example.

4. Numerical example

Suppose only three of Nelson's rules are taken into account as symptoms s_1, s_2, and s_3 on $\bar{X}$ control chart, and six underlying causes c_1, c_2, c_3, c_4, c_5, and c_6 are considered in a manufacturing process. We simulate 100 records of historical data (A, B) via the true cause-symptom relation matrix R;

$$\mathrm{R} = \begin{bmatrix} 0.7 & 0.6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.7 & 0.8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & 0.5 \end{bmatrix}_{3^*6}$$

where r_{kj} represents the grade of fuzzy relation between the kth symptom and the jth cause for $i = 1, 2, 3$; $j = 1, 2, \ldots, 6$. Matrix A in historical data (A, B) is a fuzzy matrix of size 100^*3, and the maximal elements of each row are set greater than or equal to 0.5 to represent each data having some unnatural pattern. Matrix B in historical data (A, B) is a crisp matrix of size 100^*6, where the element $b_{ij} = 1$ represents c_j is one of the causes to make the ith record having unnatural symptom.

In the real world, the true cause-symptom relation matrix R is unknown. Hence, we can only find the matrix $\hat{R}$ according to the simulated historical data (A, B). As mentioned above, MSM applied to the example can be illustrated as

$$\text{Max } \mathrm{S}(\hat{R})$$

$$\mathrm{S}(\hat{\mathrm{R}}) = \prod_{i=1}^{100}\prod_{j=1}^{6}\left\{\max_k \min(a_{ik}, r_{kj})\right\}^{b_{ij}} \cdot \left\{1 - \max_n \min(a_{ik}, r_{kj})\right\}^{1-b_{ij}}$$

$$0 \le a_{ik},\ r_{kj} \le 1,\ b_{ij} = 0 \text{ or } 1.\ i = 1, \ldots, 100; \quad j = 1, \ldots, 6; k = 1, 2, 3$$

Applying GAs to this problem, we can obtain the evaluated matrix $\hat{R}$ as follows.

$$\hat{\mathrm{R}} = \begin{bmatrix} 0.73 & 0.68 & 0.03 & 0.08 & 0.05 & 0 \\ 0 & 0.02 & 0.71 & 0.89 & 0.05 & 0 \\ 0.02 & 0.03 & 0.02 & 0.05 & 0.54 & 0.67 \end{bmatrix}$$

In order to measure the similarity between matrix R and $\hat{R}$, we select two distance indices, which are defined as follows:

$$d_1(\mathrm{R}, \hat{\mathrm{R}}) = \left(\frac{1}{mc}\sum_{j,k}^{m,c}|r_{jk} - \hat{r}_{jk}|\right) \quad \text{and} \quad d_\infty(\mathrm{R}, \hat{\mathrm{R}}) = \sup_{j,k}|r_{jk} - \hat{r}_{jk}| \tag{6}$$

The first index is used to measure the mean deviation of all corresponding elements in R and $\hat{R}$, and the second index is used to measure the maximal deviation of that. In the above example, $d_1 = 0.043$ and $d_\infty = 0.17$.

For a new input observed on $\bar{X}$ control chart, using the method explained in Section 3.1, we can transfer the observation into the degrees of satisfying the three symptoms with membership grades 0.1, 0.1, and 0.9 respectively. The output of approximate reasoning will be obtained as follows.

$$[0.1 \quad 0.1 \quad 0.9] \circ \begin{bmatrix} 0.73 & 0.68 & 0.03 & 0.08 & 0.05 & 0 \\ 0 & 0.02 & 0.71 & 0.89 & 0.05 & 0 \\ 0.02 & 0.03 & 0.02 & 0.05 & 0.54 & 0.67 \end{bmatrix}$$

$$= [0.1 \quad 0.1 \quad 0.1 \quad 0.1 \quad 0.54 \quad 0.67]$$

According to the output, an action controlled by the adopted decision rule may be taken or not. The following two decision rules based on a threshold level α are considered in a discrete case. For the first rule, if the greatest membership grades in the output exceed α, the diagnosis system will give an alarm, and suggest engineers to check the cause with the greatest membership grade. Otherwise, no action is taken. For the second rule, if any membership grade exceeds α, the diagnosis system will give an alarm, and suggest engineers to check all the causes with the membership grades exceeding α. In the above example, if α is set as 0.5, then the diagnosis system will give an alarm, in addition, c_6 will be considered by the first decision rule while c_5 and c_6 will be considered by the second one.

5. Conclusions

A new on-line diagnosis system based on fuzzy reasoning to monitor and diagnose the process has been described. This diagnosis system will support the operators to quickly identify the possible causes when a process is going unstable.

In this research we apply the concept of fuzzy sets and membership functions for softening Nelson's rules to detect unnatural patterns of symptoms. With these improvements on Nelson's rules, we can represent the status of a process accurately. Moreover, in knowledge acquisition aspects, we also present a new methodology, named MSM, to acquire the knowledge about the relationship between causes and symptoms from data. MSM method has good performances to justify the possible causes. In a future study, we plan to add Range control charts to monitor and diagnosis the process in our diagnosis system.

Acknowledgments

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Appendix
Membership grade for rule set

Rule 2. Nine points in a row in zone c or beyond

X(Points)	*Probability*	*Membership grade*
9	0.001	1
8	0.002	0.9320
7	0.0039	0.7959
6	0.0078	0.5306
5	0.0156(> 0.01)	0

Rule 3. Six points in a row steadily increasing or decreasing

X(Points)	*Probability*	*Membership grade*
6	0.0050	1
5	0.0208(> 0.01)	0

Rule 4. Fourteen points in a row altering up and down

X(Points)	*Probability*	*Membership grade*
14	0.0018	1
13	0.0029	0.8947
12	0.0045	0.7251
11	0.0079	0.4620
10	0.011(> 0.01)	0

Rule 5. Two out of three points in a row in zone a or beyond

X(Points)	*Probability*	*Membership grade*
3	0.00001	1
2	0.0015	1
1	0.0653(> 0.01)	0

Rule 6. Four out of five points in zone b or beyond

X(Points)	*Probability*	*Membership grade*
4	0.0027	1
3	0.0283(> 0.01)	0

Rule 7. Fifteen points in a row in zones c, above and below the centerline

X(Points)	*Probability*	*Membership grade*
15	0.001	1
14	0.015	0.949
13	0.0022	0.8731
12	0.0032	0.7614
11	0.0048	0.5990
10	0.0070	0.3553
9	0.0102(> 0.01)	0

Rule 8. Eight points in a row on both sides of the centerline with none in zone c

X(Points)	*Probability*	*Membership grade*
8	0.00007	1
7	0.00032	0.993
6	0.0007	0.9793
5	0.0022	0.9019
4	0.0069	0.6838
3	0.0218(> 0.01)	0

References

Al-Ghanim, A. M. and Kamat, S. J. (1995) Unnatural pattern recognition on control charts using correlation analysis techniques. *Computers and Industrial Engineering*, **29**(1–4), 43–47.

Beliakov, G. (1996) Fuzzy sets and membership functions based on probabilities. *Information Science*, **91**, 95–111.

Davis, L. (1991) *Handbook of Genetic Algorithms*, Ed. Van Nostrand Reinhold, New York.

Goldberg, D. E. (1989) *Genetic Algorithms in Search, Optimization, and Machine Learning*, Addison-Wesley, Reading, MA.

Guo, Y. and Dooley, K. J. (1992) Identification of change structure in statistical process control. *International Journal of Production Research*, **30**(7), 1655–1669.

Gwee, B. H., Lim, M. H. and Soong, B. H. (1996) Self-adjusting diagnostic system for the manufacture of crystal resonators. *IEEE Transactions on Industry Application*, **32**(1), 73–79.

Hwarng, H. B. and Hubele, N. F. (1991) X-bar chart pattern recognition using neural nets. *Annual Quality Congress Transactions*, **45**, 884–889.

Hwarng, H. B. and Hubele, N. F. (1993) Back-propagation pattern recognition for $\bar{X}$ control charts: methodology and performance. *Computers and Industrial Engineering*, **24**(2), 219–235.

Karr, C. L. and Gentry, E. J. (1993) Fuzzy control of pH using genetic algorithms. *IEEE Transactions on Fuzzy Systems*, **1**(1), 46–53.

Nelson, L. S. (1984) The Shewhart control chart—test for special causes. *Journal of Quality Technology*, **16**, 237–239.

Nelson, L. S. (1985) Interpreting Shewhart $\bar{X}$ control charts. *Journal of Quality Technology*, **17**(2), 114–116.

Pedryrcz, W. (1983) Numerical and applicational aspects of fuzzy relational equations. *Fuzzy Sets and Systems*, **11**, 1–18.

Wang, H. F. (1993) Numerical analysis on fuzzy relation equations with various operators. *Fuzzy Sets and Systems*, **53**, 155–166.

Western Electric Co. (1985) *Statistical Quality Control Handbook*, Western Electric Co. Inc, Indianapolis, Indiana.