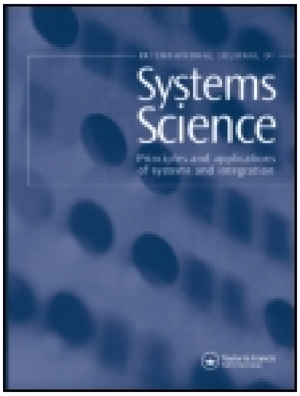
This article was downloaded by: [National Chiao Tung University 國立交通大學]
On: 27 April 2014, At: 23:32
Publisher: Taylor & Francis
Informa Ltd Registered in England and Wales Registered Number: 1072954 Registered office: Mortimer House, 37-41 Mortimer Street, London W1T 3JH, UK



International Journal of Systems Science
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/tsys20

Variable structure control scheme for landing on a celestial object

Der-Cherng Liaw [a] & Chiz-Chung Cheng [a]
[a] Department of Electrical and Control Engineering , National Chiao Tung University , Hsinchu, Taiwan , 30039

Published online: 26 Nov 2010.

To cite this article: Der-Cherng Liaw & Chiz-Chung Cheng (2001) Variable structure control scheme for landing on a celestial object, International Journal of Systems Science, 32:3, 295-301, DOI: 10.1080/002077201300029584

To link to this article: http://dx.doi.org/10.1080/002077201300029584

PLEASE SCROLL DOWN FOR ARTICLE

Taylor & Francis makes every effort to ensure the accuracy of all the information (the “Content”) contained in the publications on our platform. However, Taylor & Francis, our agents, and our licensors make no representations or warranties whatsoever as to the accuracy, completeness, or suitability for any purpose of the Content. Any opinions and views expressed in this publication are the opinions and views of the authors, and are not the views of or endorsed by Taylor & Francis. The accuracy of the Content should not be relied upon and should be independently verified with primary sources of information. Taylor and Francis shall not be liable for any losses, actions, claims, proceedings, demands, costs, expenses, damages, and other liabilities whatsoever or howsoever caused arising directly or indirectly in connection with, in relation to or arising out of the use of the Content.

This article may be used for research, teaching, and private study purposes. Any substantial or systematic reproduction, redistribution, reselling, loan, sub-licensing, systematic supply, or distribution in any form to anyone is expressly forbidden. Terms & Conditions of access and use can be found at http://www.tandfonline.com/page/terms-and-conditions

International Journal of Systems Science, 2001, volume 32, number 3, pages 295–301

Variable structure control scheme for landing on a celestial object

DER-CHERNG LIAW† and CHIZ-CHUNG CHENG‡

This paper considers the landing of a space vehicle on a celestial object. By assuming the air drag in two specific forms and the uncertainty in matching type, their effects are studied. Through the construction of a time-varying boundary layer, a new guidance control law for landing on a celestial object is proposed via the variable structure control (VSC) technique to guarantee that tracking performance is achieved at an exponential convergence rate. The proposed guidance law is continuous and alleviates chattering drawback by classic VSC design. Finally, simulation results are presented to illustrate the use of the main design.

Downloaded by [National Chiao Tung University] at 23:32 27 April 2014

1. Introduction

In recent years, the study of the rendezvous of a space vehicle with a space station or a celestial object has attracted considerable attention (see for example, Steffan (1961), Niemi (1963), Jensen (1984), Guelman (1991), Yuan and Hsu (1993), Guelman and Harel (1994)). The interest and importance of the rendezvous problem arise from both its theoretical considerations and the variety of possible applications. Among the studies, for instance, Guelman and Harel (1994) studied a power-limited soft landing on an asteroid under the gravitational effect while neglecting the drag. Jensen (1984) dealt with the kinematics of rendezvous manoeuvres based on proportional navigation techniques. Yuan and Hsu (1993) investigated the problem of spacecraft rendezvous in the exo-atmospheric flight via a modified proportional navigation scheme. However, the existing studies tend to neglect the combinatorial impact of atmosphere and gravity. For this reason, the main goal of this work is to extend the study of Guelman and Harel (1994) to derive a new rendezvous guidance law for two-dimensional landing with the effects of the drag and gravity. Guelman (1991) pointed out that the rendezvous with a celestial object has three phases: firstly, the cruise or transfer to the vicinity of the celestial body, secondly, the approach, and thirdly, the manoeuvres near the celestial body. This paper will focus on the study of the manoeuvres near the celestial body. Wertz and Larson (1991) quantified the environmental disturbance of spacecraft control such as the gravity gradient, solar radiation, non-spherical asteroid and aerodynamics. Using a by Monte Carlo simulation analysis, Lafontaine (1992) studied autonomous spacecraft navigation and control for comet landing with environmental disturbances.

The main goals of this paper is to employ the variable structural control (VSC) scheme to study the landing problem. In the study, solar radiation and non-spherical asteroids are unmodelled and treated as system perturbation. It is known that VSC possesses the advantages of fast response and less sensitivity to uncertainties or disturbances (see for example DeCarlo *et al.* (1988)). However, traditional VSC techniques often result in a chattering behaviour because of a discontinuous switching control law. The chattering behaviour has some drawbacks including damage to the mechanisms and excitation of unmodelled dynamics. Moreover, although the traditional boundary layer method with fixed boundary layer in VSC can attenuate the degree of high-frequency behaviour, its stability is guaranteed only outside the boundary layer and asymptotic tracking usually cannot be achieved if the boundary layer is not sufficiently small (see for example Slotine and Li (1991)). Furthermore, a fixed switching control gain for reaching the sliding surface often costs too much energy for the purpose of trajectory tracking. Owing to these disadvantages of traditional VSC (i.e. with fixed switching gain and fixed boundary layer), in this paper we synthesize a continuous-type control law

Received 14 April 1999. Revised 17 November 1999. Accepted 12 January 2000.

† Department of Electrical and Control Engineering, National Chiao Tung University, Hsinchu 30039, Taiwan. Fax: 886-3-5715998; e-mail: dcliaw@cc.nctu.edu.tw.

‡ Department of Electrical and Control Engineering, National Chiao Tung University, Hsinchu 30039, Taiwan.

International Journal of Systems Science ISSN 0020–7721 print/ISSN 1464–5319 online © 2001 Taylor & Francis Ltd
http://www.tandf.co.uk/journals

Downloaded by [National Chiao Tung University] at 23:32 27 April 2014

with time-varying boundary layer and varying switching control gain to reduce the control efforts in both magnitude and frequency while ensuring asymptotically vanishing tracking error. Related research using VSC was found only in the paper by Brierley and Longchamp (1990) for an air–air interception problem but not a landing problem. The objective of the interception problem is to match the position vector only, while for the landing problem the objective is to match both the position and the velocity vectors simultaneously.

The organization of the paper is as follows. In § 2, we describe the landing problem together with two kinds of air drag. In § 3 the design of a discontinuous control law that fulfils the desired performance requirements is presented. To alleviate the classic chattering behaviour drawback, in § 4 the control law is modified to be continuous by the construction of a time-varying boundary layer. It is followed by numerical simulations for two kinds of drag to illustrate the use of the primary result. Finally, § 6 gives the conclusions.

2. Problem Formulation

It is known that, when a space vehicle docks on a celestial body, the relative velocity must be driven to zero. This means that the commanded acceleration of the active vehicle in both the direction normal to the line of sight (LOS) and the direction along the LOS must reduce to zero as the space vehicle lands on a celestial object. The two-dimensional system equations under air drag, gravitational effects and disturbances can be derived by the construction of system Lagrangian as given by

$$\ddot{r} - r\dot{\theta}^2 = a_{Cr} - \frac{\mu}{r^2} + F + d_1, \tag{1}$$

$$r\ddot{\theta} + 2\dot{r}\dot{\theta} = a_{C\theta} + G + d_2, \tag{2}$$

where the landing geometry is given in figure 1 while the parameters in (1) and (2) are described as follows: r is the spacecraft position, θ is the LOS angle with respect to the celestial object, μ is the gravitational constant multiplied by the mass of the celestial object, a_{Cr} and $a_{C\theta}$ are the commanded accelerations in the $\boldsymbol{e}_r$ and $\boldsymbol{e}_\theta$ directions respectively, F and G are the components of the air drag in the $\boldsymbol{e}_r$ and $\boldsymbol{e}_\theta$ direction, respectively and d_1 and d_2 are unmodelled perturbation forces due to the environment. Here, similar to the work of Guelman and Harel (1994), for simplicity, the spacecraft is assumed to be of unit mass. Myint-U (1968) pointed out that a close Earth satellite is subjected to various perturbation forces such as asteroid oblateness and solar radiation pressure, which are small and have different orders of magnitude. In addition, Wertz and Larson (1991) pointed out the perturbation forces from non-spherical asteroids and

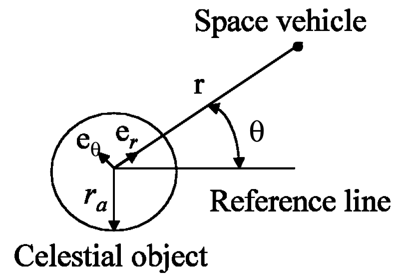


Figure 1. The geometry for the landing problem.

solar radiation might cause periodic variations. Thus, these effects can be formulated as periodic functions for analysis. We then impose the following assumption on the perturbation forces.

Assumption 1: *For uncertainty* $\boldsymbol{d} = (d_1, d_2)^{\mathrm{T}}$*, there exists a continuous function* $w(\boldsymbol{x})$ *such that*

$$\|\boldsymbol{d}\| \leqslant w(\boldsymbol{x}) \tag{3}$$

for all $\boldsymbol{x} = (r, \theta, \dot{r}, \dot{\theta})$ *and* $t > 0$*. Here,* $\|\cdot\|$ *denotes the Euclidean norm and will be in effect throughout the paper.*

It is, at present, not easy to construct a reliable expression for the air drag resulting from the motion of landing. However, if we suppose that the drag is caused by some sort of gas, it is reasonable to assume that the drag is proportional to some power of the space vehicle's velocity as well as the increase in the density of the gas as the vehicle approaches the celestial body. Two types of frictional force are adopted from Myint-U (1968) and Persen (1958) in the study as given below.

Type 1 is given by

$$F = -\beta \frac{[\dot{r}^2 + (r\dot{\theta})^2]^{1/2}}{e^{kr}} \dot{r}, \tag{4}$$

$$G = -\beta \frac{[\dot{r}^2 + (r\dot{\theta})^2]}{e^{kr}} r\dot{\theta}, \tag{5}$$

where β denotes the air drag coefficient. Note that this type of air drag assumes that the frictional force is proportional to the second power of space vehicle velocity relative to the celestial body.

Type 2 is given by

$$F = -\beta \frac{\dot{r}}{r^2}, \tag{6}$$

$$G = -\beta \frac{r\dot{\theta}}{r^2}. \tag{7}$$

This case assumes that the motion of the vehicle occurs in a space with a very low gas density. Note that, in this case, the frictional force is directly proportional to the space vehicle's velocity. Moreover, the variation in density is also taken into account by assuming that F and G are decreasing with respect to the second power of r.

Downloaded by [National Chiao Tung University] at 23:32 27 April 2014

Assuming the air drag to be in either of the above forms, we proceed to develop a guidance law that ensures that the vehicle land on the celestial body and satisfies the landing constraint. That is, the relative velocity must decrease to zero as the space vehicle meets the celestial object (i.e. the distance between the vehicle and celestial object $r = r_a$, where r_a is the radius of a celestial body).

3. Design of the guidance control law

The VSC design, in general, consists of three steps. The first step is to select sliding surface vector, which is a function of system states. It is followed by the design of the so-called 'equivalent control' to govern the motion on the sliding surface. Indeed, for any initial state lying on the sliding surface, the equivalent control is equivalent to forcing the system state to remain on the slide surface thereafter and sliding toward the origin. The final step is to design an extra control effort to guarantee the reaching condition. That is, the second control guarantees that the system state will reach the sliding surface in a finite time. The VSC technique is now applied to the landing problem of the system

To achieve the main goals of the paper, we denote

$$\boldsymbol{x} = (\boldsymbol{x}_1^{\mathrm{T}} \quad \boldsymbol{x}_2^{\mathrm{T}})^{\mathrm{T}}, \tag{8}$$

where

$$\boldsymbol{x}_1 = (x_1 \quad x_2)^{\mathrm{T}} = (r \quad \theta)^{\mathrm{T}}, \tag{9}$$

$$\boldsymbol{x}_2 = (x_3 \quad x_4)^{\mathrm{T}} = (\dot{r} \quad \dot{\theta})^{\mathrm{T}}. \tag{10}$$

Equations (1) and (2) can then be rewritten as

$$\dot{\boldsymbol{x}}_1 = \boldsymbol{x}_2, \tag{11}$$

$$\dot{\boldsymbol{x}}_2 = \boldsymbol{A}(\boldsymbol{x}) + B(\boldsymbol{x})\boldsymbol{u} + B(\boldsymbol{x})\boldsymbol{d}. \tag{12}$$

Here,

$$\boldsymbol{A}(\boldsymbol{x}) = \begin{pmatrix} x_1 x_4^2 - \dfrac{\mu}{x_1^2} + F \\ -\dfrac{2x_3x_4}{x_1} + \dfrac{G}{x_1} \end{pmatrix}, \tag{13}$$

$$B(\boldsymbol{x}) = \begin{pmatrix} 1 & 0 \\ 0 & \dfrac{1}{x_1} \end{pmatrix}, \tag{14}$$

$$\boldsymbol{d} = (d_1 \quad d_2)^{\mathrm{T}}, \tag{15}$$

$$\boldsymbol{u} = (a_{Cr} \quad a_{C\theta})^{\mathrm{T}}. \tag{16}$$

Note that the dynamical system (11) and (12) is in a regular form (for definition, see for example DeCarlo *et al.* (1988)). In addition, since $x_1 \to r_a$, the matrix $B(\boldsymbol{x})$ is never singular during the landing process. After rearranging the system equations into the form of (11) and (12) above, the design procedure proceeds via the variable structure control technique.

Let $\boldsymbol{x}_d(t) = (x_{d1}(t) \quad x_{d2}(t))^{\mathrm{T}}$ be the desired trajectory of $\boldsymbol{x}_1$, where $x_{d1}(t)$ and $x_{d2}(t)$ are both twice differentiable functions of t. It is noted that $x_{d1}(t)$ must be chosen so that $x_{d1}(t) \to r_a$, the radius of the celestial body. However, to guarantee that the landing process can be accomplished in a finite time, instead, we choose $x_{d1}(t) \to r_a - \delta$ for some $\delta > 0$. Indeed, the landing process for $x_1 \to r_a$ can be realized within a finite time if the system state x_1 approaches x_{d1} at an exponential convergence rate.

Define $\boldsymbol{e} = (\boldsymbol{e}_1^{\mathrm{T}}, \boldsymbol{e}_2^{\mathrm{T}})^{\mathrm{T}}$ as the error function, where $\boldsymbol{e}_1 = \boldsymbol{x}_1 - \boldsymbol{x}_d(t)$ and $\boldsymbol{e}_2 = \dot{\boldsymbol{e}}_1$. Let $\boldsymbol{y}_d(t) = [\boldsymbol{x}_d^{\mathrm{T}}(t), \dot{\boldsymbol{x}}_d^{\mathrm{T}}(t)]^{\mathrm{T}}$. The system equations (11) and (12) can be rewritten as the following error model:

$$\dot{\boldsymbol{e}}_1 = \boldsymbol{e}_2, \tag{17}$$

$$\dot{\boldsymbol{e}}_2 = -\ddot{\boldsymbol{x}}_d(t) + \boldsymbol{A}(\boldsymbol{e}, t) + B(\boldsymbol{e}, t)(\boldsymbol{u} + \boldsymbol{d}), \tag{18}$$

where $\boldsymbol{A}(\boldsymbol{e}, t) = \boldsymbol{A}(\boldsymbol{e} + \mathbf{y}_d(t))$ and $B(\boldsymbol{e}, t) = B(\boldsymbol{e} + \mathbf{y}_d(t))$. The landing process is then transformed into a tracking problem such that the closed-loop system having an error vector $\boldsymbol{e}$ approaches zero. Follow the VSC design process, we choose a time-varying sliding surface vector as

$$\boldsymbol{S}(\boldsymbol{e}, t) = \boldsymbol{e}_2 + M\boldsymbol{e}_1 = \boldsymbol{0}, \tag{19}$$

where $M \in \mathbb{R}^{2\times 2}$ is a positive definite matrix. It is known (see for example DeCarlo *et al.* (1988)) that the VSC control for the system (17) and (18) is in the form of

$$\boldsymbol{u} = \boldsymbol{u}_{eq} + \boldsymbol{u}_N, \tag{20}$$

where $\boldsymbol{u}_{eq}$ is the equivalent control for the nominal system (i.e. (17) and (18) with $\boldsymbol{d} = \boldsymbol{0}$) and $\boldsymbol{u}_N$ is to be designated to compensate non-zero uncertainties. Now, we proceed the design of the equivalent control $\boldsymbol{u}_{eq}$ for the nominal system.

It is noted that the input-related matrix $B(\boldsymbol{e}, t) = B(\boldsymbol{x})$ as given in (14) is non-singular and, for (19),

$$\begin{aligned} \dot{\boldsymbol{S}} &= \dot{\boldsymbol{e}}_2 + M\dot{\boldsymbol{e}}_1 \\ &= -\ddot{\boldsymbol{x}}_d + \boldsymbol{A}(\boldsymbol{e}, t) + B(\boldsymbol{e}, t)\boldsymbol{u} + M\boldsymbol{e}_2. \end{aligned} \tag{21}$$

To provide $\dot{\boldsymbol{S}} = \boldsymbol{0}$ for any $\boldsymbol{e}$ such that $\boldsymbol{S}(\boldsymbol{e}, t) = \boldsymbol{0}$, the equivalent control can then be chosen as

$$\boldsymbol{u}_{eq} = -B^{-1}(\boldsymbol{e}, t)[-\ddot{\boldsymbol{x}}_d + A(\boldsymbol{e}, t) + M\boldsymbol{e}_2]. \tag{22}$$

From (17) and (19), the reduced model for the system (17) and (18) with $\boldsymbol{S} = \boldsymbol{0}$ becomes

$$\dot{\boldsymbol{e}}_1 = -M\boldsymbol{e}_1. \tag{23}$$

Since M is a positive definite matrix, we then have $\boldsymbol{e}_1(t) \to \boldsymbol{0}$ as $t \to \infty$ (i.e. $\boldsymbol{x}_1 \to \boldsymbol{x}_d(t)$). That is, for any initial $\boldsymbol{e}$ such that $\boldsymbol{S}(\boldsymbol{e}, t) = \boldsymbol{0}$, the tracking error vector $\boldsymbol{e}$ approaches zero as the time t increases.

Next, we design $\boldsymbol{u}_\mathrm{N}$ to compensate disturbances. Suppose that the disturbance $\boldsymbol{d}$ satisfies assumption 1. Let

$$\boldsymbol{u}_\mathrm{N} = -[w(\boldsymbol{e}) + \eta] B^{-1}(\boldsymbol{e}, t)\,\mathrm{sgn}\,(\boldsymbol{S}), \tag{24}$$

where $\eta > 0$, $\mathrm{sgn}\,(\boldsymbol{S}) = [\mathrm{sgn}\,(s_1), \mathrm{sgn}\,(s_2)]^\mathrm{T}$ with

$$\mathrm{sgn}\,(s) = \begin{cases} 1 & \text{if } s > 0, \\ 0 & \text{if } s = 0, \\ -1 & \text{if } s < 0. \end{cases} \tag{25}$$

For any $\boldsymbol{e}$ such that $\boldsymbol{S}(\boldsymbol{e}, t) \neq \boldsymbol{0}$, from (17), (18) and (22) we have

$$\begin{aligned} \frac{1}{2}\frac{\mathrm{d}||\boldsymbol{S}||^2}{\mathrm{d}t} &= \boldsymbol{S}^\mathrm{T}\dot{\boldsymbol{S}} = \boldsymbol{S}^\mathrm{T}[-\ddot{\boldsymbol{x}}_\mathrm{d} + \boldsymbol{A}(\boldsymbol{e}, t) + B(\boldsymbol{e}, t)\boldsymbol{u} + M\boldsymbol{e}_2] \\ &= \boldsymbol{S}^\mathrm{T} B(\boldsymbol{e}, t)(\boldsymbol{u}_\mathrm{N} + \boldsymbol{d}) \\ &\leqslant -[w(\boldsymbol{e}) + \eta]\sum_{i=1}^{2}|s_i| + ||\boldsymbol{S}||\,||B(\boldsymbol{e}, t)||\,||\boldsymbol{d}|| \\ &\leqslant -[w(\boldsymbol{e}) + \eta]||\boldsymbol{S}|| + w(\boldsymbol{e})||\boldsymbol{S}|| \\ &\leqslant -\eta||\boldsymbol{S}||. \end{aligned} \tag{26}$$

Here, we use the fact that $||B(\boldsymbol{e}, t)|| = ||B(\boldsymbol{x})|| = ||\mathrm{diag}\,(1, 1/x_1)|| = 1$ since, in general, $x_1 > 1$ throughout the landing. Moreover, we have $\mathrm{d}||\boldsymbol{S}||^2/\mathrm{d}t = 2||\boldsymbol{S}||\,\mathrm{d}||\boldsymbol{S}||/\mathrm{d}t = 2\boldsymbol{S}^\mathrm{T}\dot{\boldsymbol{S}}$. From (26), we then have $\mathrm{d}||\boldsymbol{S}||/\mathrm{d}t \leqslant -\eta/2$. This implies that the state will reach the sliding surface in a finite time. Indeed, the first time t_reach required for the state to contact the sliding surface satisfies $t_\mathrm{reach} - t_0 \leqslant 2(||\boldsymbol{S}(\boldsymbol{e}_0, t_0)||/\eta)$, where $\boldsymbol{e}_0$ and t_0 denote the initial state of the system (17) and (18) and the initial time respectively.

It is clear that $\dot{\boldsymbol{S}} \neq \boldsymbol{0}$ when the state is driven by $\boldsymbol{u}_\mathrm{N}$ to touch the sliding surface. This will induce the so-called 'chattering' problem. Slotine and Li (1991) pointed out that the chattering must be eliminated by a proper control action. In fact, it can be achieved by smoothing out the discontinuity of control law with a thin boundary layer near the switching surface. In the next section, a continuous type of control law is proposed to alleviate chattering behaviour.

4. Smoothing the control law

It is known that the sign function as given in (24) and (25) results in discontinuity of the control law and induce chattering of system dynamics. However, in practical applications, chattering is generally undesirable since it involves extremely high control activity and further may excite high-frequency dynamics neglected in the course of modelling. In the following, a continuous type of control law is proposed to alleviate the chattering behaviour while retaining exponential tracking performance.

Let the function

$$\boldsymbol{g}(\boldsymbol{S}) = \frac{2\boldsymbol{S}}{||\boldsymbol{S}|| + \epsilon\,\mathrm{e}^{-\gamma t}}, \tag{27}$$

replace the sign function in (24), where $\epsilon > 0$ and $\gamma > 0$ can be selected by the designer to satisfy the condition which will become clear later. The control law $\boldsymbol{u}_\mathrm{N}$ as in (24) can be modified as follows:

$$\boldsymbol{u}_\mathrm{N} = -[\alpha w(\boldsymbol{e}) + \eta] B^{-1}(\boldsymbol{e}, t)\boldsymbol{g}(\boldsymbol{S}), \tag{28}$$

where $\alpha > 1$ is a constant. It is noted that the modified control law $\boldsymbol{u}_\mathrm{N}$ is well defined and continuous everywhere with $\boldsymbol{u}_\mathrm{N} = \boldsymbol{0}$ on the sliding surface. The concept behind the modified control law is to construct a time-varying boundary layer $\Gamma(\boldsymbol{e}, t)$ as given in (30) below.

First, consider the case in which the tracking error $\boldsymbol{e}$ lies outside $\Gamma(\boldsymbol{e}, t)$. That is, $||\boldsymbol{S}(\boldsymbol{e}, t)|| \geqslant \epsilon\,\mathrm{e}^{-t}$. By taking the control input $\boldsymbol{u} = \boldsymbol{u}_\mathrm{eq} + \boldsymbol{u}_\mathrm{N}$, from $||\boldsymbol{d}|| \leqslant w(\boldsymbol{e})$ and $||B(\boldsymbol{e}, t)|| = 1$, we have

$$\begin{aligned} \boldsymbol{S}^\mathrm{T}\dot{\boldsymbol{S}} &= \boldsymbol{S}^\mathrm{T} B(\boldsymbol{e}, t)(\boldsymbol{u}_\mathrm{N} + \boldsymbol{d}) \\ &\leqslant -2[\alpha w(\boldsymbol{e}) + \eta]\frac{||\boldsymbol{S}||^2}{||\boldsymbol{S}|| + \epsilon\,\mathrm{e}^{-\gamma t}} + ||\boldsymbol{S}||\,||B(\boldsymbol{e}, t)||\,||\boldsymbol{d}|| \\ &\leqslant -[\alpha w(\boldsymbol{e}) + \eta]||\boldsymbol{S}|| + w(\boldsymbol{e})||\boldsymbol{S}|| \\ &\leqslant -\eta||\boldsymbol{S}||. \end{aligned} \tag{29}$$

Similarly, from the discussions in § 3 and (23) above, for any $\boldsymbol{e}$ such that $||\boldsymbol{S}(\boldsymbol{e}, t)|| \geqslant \epsilon\,\mathrm{e}^{-\gamma t}$, the error vector $\boldsymbol{e}$ will reach the time-varying boundary layer $\Gamma(\boldsymbol{e}, t)$ in a finite time with the reaching time less than $2||\boldsymbol{S}(\boldsymbol{e}_0, t_0)||/\eta$, where

$$\Gamma(\boldsymbol{e}, t) = \{\boldsymbol{e}|\ ||S(\boldsymbol{e}, t)|| \leqslant \epsilon\,\mathrm{e}^{-\gamma t}\}. \tag{30}$$

In addition to reaching the boundary layer in a finite time, we claim here that, once the state $\boldsymbol{e}$ enters the boundary layer, it will stay within the boundary layer hereafter. To see this, we note that

$$\begin{aligned} ||\boldsymbol{S}||\frac{\mathrm{d}}{\mathrm{d}t}||\boldsymbol{S}|| &= \boldsymbol{S}^\mathrm{T}\dot{\boldsymbol{S}} \\ &= \boldsymbol{S}^\mathrm{T} B(\boldsymbol{e}, t)(\boldsymbol{u}_\mathrm{N} + \boldsymbol{d}) \\ &\leqslant -2[\alpha w(\boldsymbol{e}) + \eta]\frac{||\boldsymbol{S}||^2}{||\boldsymbol{S}|| + \epsilon\,\mathrm{e}^{-\gamma t}} \\ &\quad + ||\boldsymbol{S}||\,||B(\boldsymbol{e}, t)||\,||\boldsymbol{d}|| \\ &\leqslant -2[\alpha w(\boldsymbol{e}) + \eta]\frac{||\boldsymbol{S}||^2}{||\boldsymbol{S}|| + \epsilon\,\mathrm{e}^{-\gamma t}} + w(\boldsymbol{e})||\boldsymbol{S}|| \\ &\leqslant \frac{||\boldsymbol{S}||}{||\boldsymbol{S}|| + \epsilon\,\mathrm{e}^{-\gamma t}}\{-2[\alpha w(\boldsymbol{e}) + \eta]||\boldsymbol{S}|| \end{aligned}$$

Downloaded by [National Chiao Tung University] at 23:32 27 April 2014

$$
\begin{aligned}
&+ w(\boldsymbol{e})(\|\boldsymbol{S}\| + \epsilon\,\mathrm{e}^{-\gamma t})\} \\
&\leqslant \frac{\|\boldsymbol{S}\|}{\|\boldsymbol{S}\| + \epsilon\,\mathrm{e}^{-\gamma t}}[-2\eta\|\boldsymbol{S}\| + w(\boldsymbol{e}) \\
&\times (\|\boldsymbol{S}\| + \epsilon\,\mathrm{e}^{-\gamma t} - 2\alpha\|\boldsymbol{S}\|)]. \qquad (31)
\end{aligned}
$$

From (31), we have

$$
\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}\|\boldsymbol{S}\| \leqslant \frac{-2\eta\|\boldsymbol{S}\|}{\|\boldsymbol{S}\| + \epsilon\,\mathrm{e}^{-\gamma t}} + \frac{w(\boldsymbol{e})}{\|\boldsymbol{S}\| + \epsilon\,\mathrm{e}^{-\gamma t}} \\
\times [(1-2\alpha)\|\boldsymbol{S}\| + \epsilon\,\mathrm{e}^{-\gamma t}]. \qquad (32)
\end{aligned}
$$

Since $w(\boldsymbol{e}) \geqslant 0$, we then have $\mathrm{d}\|\boldsymbol{S}(\boldsymbol{e},t)\|/\mathrm{d}t < 0$ for any $\boldsymbol{e} \in \Gamma(\boldsymbol{e},t)$ with $\|\boldsymbol{S}(\boldsymbol{e},t)\| \geqslant [1/(2\alpha - 1)]\epsilon\,\mathrm{e}^{-\gamma t}$. This implies that the error vector $\boldsymbol{e}$ will never go outside the boundary layer once it enters the boundary layer $\Gamma(\boldsymbol{e},t)$.

From the discussions above, we know that the error state $\boldsymbol{e}$ will reach the boundary layer in a finite time and remains inside there. Moreover, the state $\boldsymbol{x}_1$ will track the desired trajectory $\boldsymbol{x}_\mathrm{d}$ exponentially as given in the next theorem.

Downloaded by [National Chiao Tung University] at 23:32 27 April 2014

Theorem 1: *Suppose that the disturbance* $\boldsymbol{d}$ *satisfies assumption 1. Then, the landing performance of the system* (11) *and* (12) *can be achieved at an exponential convergence rate, by the control law* $\boldsymbol{u} = \boldsymbol{u}_\mathrm{eq} + \boldsymbol{u}_\mathrm{N}$, *where* $\boldsymbol{u}_\mathrm{eq}$ *and* $\boldsymbol{u}_\mathrm{N}$ *are given in* (22) *and* (28), *if* M *is a positive definite matrix and* $\gamma > 0$ *satisfies* $\lambda_{\max}(M) - \lambda_{\min}(M) < \gamma$. *Here,* $\lambda_{\min}(\cdot)$ *and* $\lambda_{\max}(\cdot)$ denote the smallest and the largest eigenvalues respectively. Moreover, the control law is continuous and alleviates the chattering behaviour.

Proof: From the discussions above, we know that the system state $\boldsymbol{e}$ will enter the boundary layer $\Gamma(\boldsymbol{e},t)$ in a finite time and remain inside there hereafter. To complete the proof of the theorem, we shall only show that $\boldsymbol{e}_1(t) \to \boldsymbol{0}$ exponentially as $t \to \infty$ for any $\boldsymbol{e} \in \Gamma(\boldsymbol{e},t)$. Let $\boldsymbol{z}(t) = \dot{\boldsymbol{e}}_1 + M\boldsymbol{e}_1$ for any $\boldsymbol{e} \in \Gamma(\boldsymbol{e},t)$. It follows that

$$
\boldsymbol{e}_1(t) = \mathrm{e}^{-Mt}\boldsymbol{e}_1(0) + \int_0^t \mathrm{e}^{-M(t-\tau)}\boldsymbol{z}(\tau)\,\mathrm{d}\tau. \qquad (33)
$$

Since M is a positive definite matrix, we have $\mathrm{e}^{-Mt}\boldsymbol{e}_1(0) \to \boldsymbol{0}$ exponentially as $t \to \infty$. Moreover, for the integral part of (33), we have

$$
\begin{aligned}
&\left\|\int_0^t \mathrm{e}^{-M(t-\tau)}\boldsymbol{z}(\tau)\,\mathrm{d}\tau\right\| \\
&\leqslant \|\mathrm{e}^{-Mt}\| \int_0^t \|\mathrm{e}^{M\tau}\|\,\|\boldsymbol{z}(\tau)\|\,\mathrm{d}\tau \\
&\leqslant \mathrm{e}^{-\lambda_{\min}(M)t}\int_0^t \mathrm{e}^{\lambda_{\max}(M)\tau}\epsilon\,\mathrm{e}^{-\gamma\tau}\,\mathrm{d}\tau \\
&\leqslant \begin{cases}
\mathrm{e}^{-\lambda_{\min}(M)t}\,\epsilon t & \text{if } \gamma = \lambda_{\max}(M), \\
\mathrm{e}^{-\lambda_{\min}(M)t}\left(\dfrac{\epsilon}{\lambda_{\max}(M) - \gamma}(\mathrm{e}^{[\lambda_{\max}(M)-\gamma]t} - 1)\right) & \text{if } \gamma \neq \lambda_{\max}(M).
\end{cases}
\end{aligned}
$$

By L'Hôpital's rule (see for example Buck (1978)), it follows that $\int_0^t \mathrm{e}^{-M(t-\tau)}\boldsymbol{z}(\tau)\,\mathrm{d}\tau \to \boldsymbol{0}$ exponentially as $t \to \infty$ when $\gamma > \lambda_{\max}(M) - \lambda_{\min}(M)$. Thus, from (33), we can claim that $\boldsymbol{e}_1(t) \to \boldsymbol{0}$ exponentially as $t \to \infty$ for any $\boldsymbol{e} \in \Gamma(\boldsymbol{e},t)$. This implies that $\boldsymbol{x}_1 \to \boldsymbol{x}_\mathrm{d}(t)$ as $t \to \infty$. Since the tracking error $\boldsymbol{e}$ goes to zero, the conclusion of the theorem is hence provided. □

5. Simulation results

In this section, we present two examples to illustrate the use of the main result. Let the desired trajectory $\boldsymbol{x}_\mathrm{d}(t)$ be given by

$$
\boldsymbol{x}_\mathrm{d}(t) = \begin{pmatrix} r_\mathrm{a} + (r_0 - r_\mathrm{a})\,\mathrm{e}^{-t} - \delta \\ \theta_\mathrm{f} \end{pmatrix},
$$

where θ_f denotes the desired final angle θ. The initial conditions for the numerical study are adopted from Guelman and Harel (1994) while the coefficients for the drag are from Persen (1958). Other system parameters are chosen to satisfy the landing requirements. The parameters and initial states are then given as follows: $r_\mathrm{a} = 10\,\mathrm{km}$, $r_0 = 200\,\mathrm{km}$, $\theta_0 = 0.2\,\mathrm{rad}$, $\dot{r}_0 = -5\,\mathrm{km\,h^{-1}}$, $\dot{\theta}_0 = 0.2\,\mathrm{rad\,hour^{-1}}$, $\epsilon = 5$, $\gamma = 0.01$, $\theta_\mathrm{f} = 0.5\,\mathrm{rad}$, $k = -0.1$, $\eta = 2$, $\delta = 0.1\,\mathrm{km}$, $\beta = 0.02$ and M is given by $2.414I_2$, where I_2 denotes the identity matrix of dimension two. The disturbances are assumed to be $d_1 = 0.1\sin(x_2)$ and $d_2 = 0.5\sin(x_2)$. The matrix $w(\boldsymbol{x})$ and scalar α can then be selected as $w(\boldsymbol{x}) = 0.5$ for all $\boldsymbol{x}$ and $\alpha = 1.1$. The following two examples are presented with respect to two types of air drag to demonstrate the use of the proposed control scheme as listed in theorem 1.

Example 1: This example considers the frictional force in the form of (4) and (5) with $\beta = 0.02$ from Person (1958). Numerical results are given in figures 2–5. Figure 2 shows the norm of the error function $\boldsymbol{e}$ while figure 3 exhibits the norm of the control force $\boldsymbol{u}$ with respect to different gravitational constants. In addition, figure 4 gives the effects of air drag and gravity and figure 5 displays the timing responses for the four state variables r, θ, $\dot{r}$ and $\dot{\theta}$. It is observed from figure 2 that the tracking performance is achieved at an exponential rate, which agrees with the main conclusion of this study. From figure 3, we observe that a larger energy consumption is required for landing as the gravitational effect becomes heavier. The reason is that the commanded

Downloaded by [National Chiao Tung University] at 23:32 27 April 2014

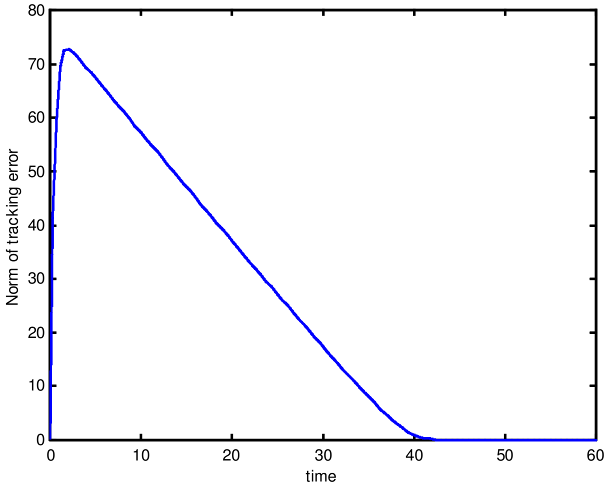


Figure 2. Norm of the error *e*.

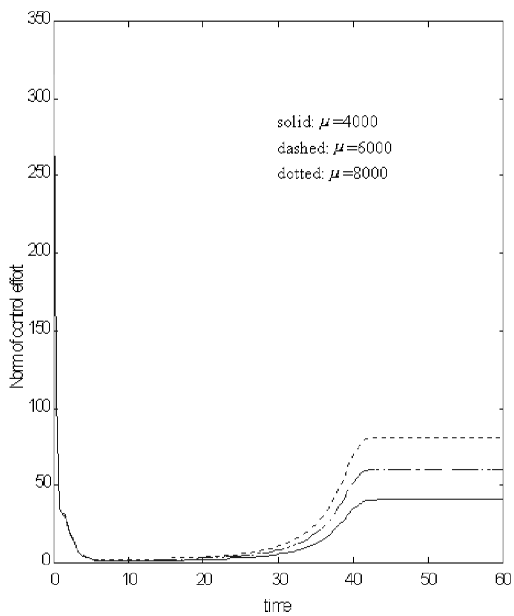


Figure 3. Norm of the control *u* with different kinds of gravitational constant.

acceleration is required to resist the gravity so that the vehicle's velocity approaches zero at the end of landing. From figure 4, it should be noted that the effect of gravity is more significant than that of air drag through the landing process. Finally, it is observed from figure 5 that the four state variables r, $\dot{r}$, θ and $\dot{\theta}$ are all convergent. Moreover, the facts that $\dot{r}$ and $\dot{\theta}$ approach zero implies that the relative velocity decreases to zero as the space vehicle docks on the celestial object. However, from figure 5(*c*), $|\ddot{r}|$ at around 1 s is very large. This would produce large a g force for touching the boundary layer. □

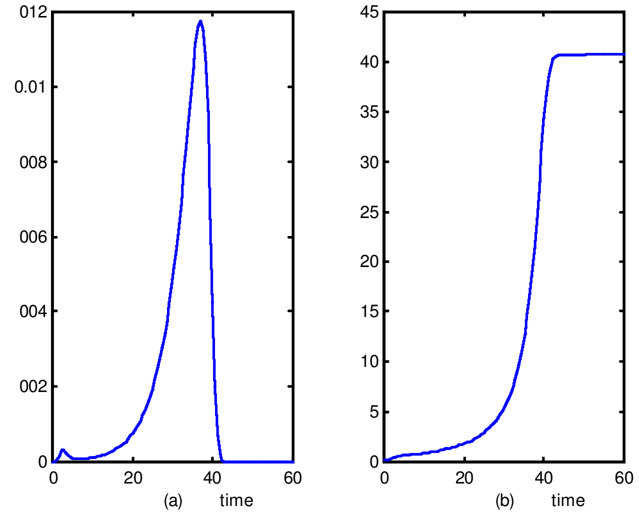


Figure 4. (*a*) Norm of the air drag; (*b*) the gravitational effect μ/r^2.

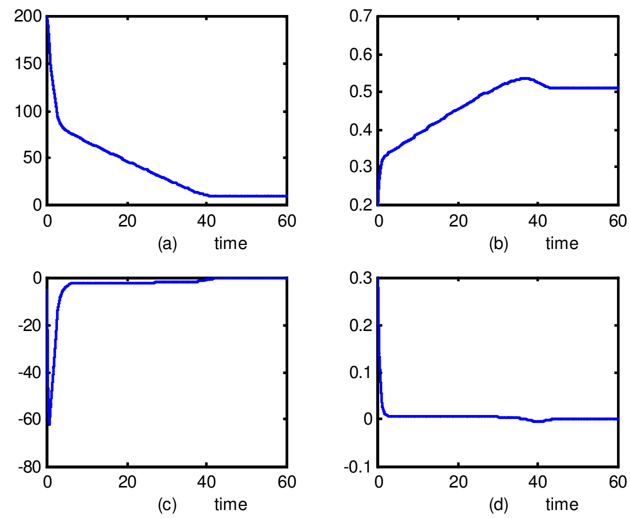


Figure 5. Timing responses of (*a*) r, (*b*) θ, (*c*) $\dot{r}$ and (*d*) $\dot{\theta}$.

Example 2: We consider the frictional force in the form of (6) and (7) with $\beta = 0.02$ from Person (1958). It is observed from figure 6 that the effect of air drag is very small in this case. Numerical results for the control signal and system states resemble those of example 1. Details are not given. □

6. Conclusions

In this paper, we have considered the rendezvous of a space vehicle with a celestial object. The study included the effects of air drag and disturbance. By use of the VSC scheme, a continuous type of guidance control law was proposed to guarantee the tracking performance and to alleviate the classic chattering drawback of discontinuous controls. The tracking performance

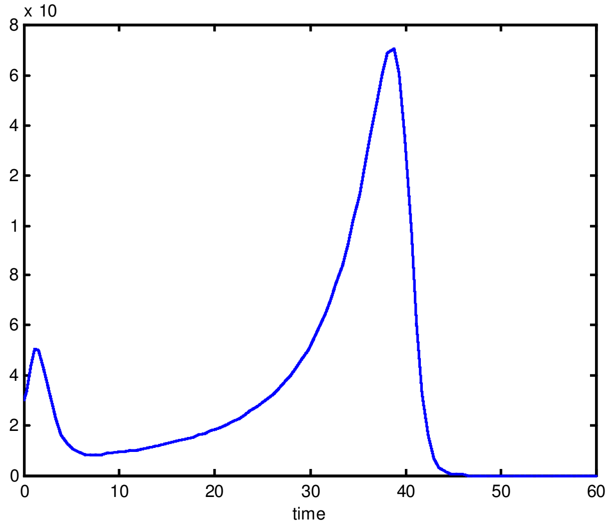


Figure 6. Norm of the air drag.

features an exponential convergence rate, which can be assigned by the designer. Two numerical examples were also presented to demonstrate the use of the main result under different kinds of air drag.

Acknowledgements

The authors are grateful to the reviewers for their comments and suggestions. This research was supported by the National Science Council, Taiwan, R.O.C. under Grants NSC 84-2212-E009-002 and NSC 89-CS-D-009-013.

References

Brierley, S. D., and Longchamp, R., 1990, Application of sliding-mode control to air–air interception problem. *IEEE Transactions on Aerospace and Electronic System*, **26,** 306–325.

Buck, R. C., 1978, *Advanced Calculus*, third edition (New York: McGraw-Hill).

DeCarlo, R. A., Zak, S. H., and Matthews, G. P., 1988, Variable structure control of nonlinear multivariable systems: a tutorial. *Proceedings of the IEEE*, **76,** 212–232.

Guelman, M., 1991, Guidance for asteroid rendezvous. *Journal of Guidance, Control, and Dynamics*, **14,** 1080–1083.

Guelman, M., and Harel, D., 1994, Power limited soft landing on an asteroid. *Journal of Guidance, Control, and Dynamics*, **17,** 15–20.

Jensen, D. L., 1984, Kinematics of rendezvous manoeuvres. *Journal of Guidance, Control, and Dynamics*, **7,** 307–314.

Lafontaine, J. D., 1992, Autonomous spacecraft navigation and control for comet landing. *Journal of Guidance, Control, and Dynamics*, **15,** 567–576.

Myint-U, T., 1968, Effect of oblateness and drag on equatorial orbits with small eccentricities. *Jet Spacecraft*, **5,** 123–125.

Niemi, N. J., 1963, Investigation of a terminal guidance system for a satellite rendezvous. *AIAA Journal*, **1,** 405–411.

Persen, L. N., 1958, Motion of a satellite with friction. *Jet Propulsion*, **28,** 750–752.

Slotine, J. J., and Li, W., 1991, *Applied Nonlinear Control* (Englewood Cliffs, New Jersey: Prentice-Hall).

Steffan, K. F., 1961, Satellite rendezvous terminal guidance system. *American Rocket Society Journal*, **31,** 1516–1521.

Wertz, J. R., and Larson, W. J., 1991, *Space Mission Analysis and Design* (Deventer: Kluwer).

Yuan, P. J., and Hsu, S. C., 1993, Rendezvous guidance with proportional navigation. *Journal of Guidance, Control, and Dynamics*, **17,** 409–411.

Downloaded by [National Chiao Tung University] at 23:32 27 April 2014