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Fuzzy approaches for fault diagnosis of transformers

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Abstract

Dissolved gas analysis has been used as a diagnostic method to determine the conditions of transformers for a long time. The criteria used in dissolved gas analysis are based on crisp value norms. Due to the dichotomous nature of crisp criteria, transformers with similar gas-in-oil conditions may lead to very different conclusions of diagnosis especially when the gas concentrations are around the crisp norms. To deal with this problem, gas-in-oil data of failed transformers were collected and treated in order to obtain the membership functions of fault patterns using a fuzzy clustering method. All crisp norms are fuzzified to linguistic variables and diagnostic rules are transformed into fuzzy rules. A fuzzy system originally proposed by Takagi and Sugeno is used to combine the rules and the fuzzy conditions of transformers to obtain the final diagnostic results. It is shown that the diagnosing results from the combination of several simple fuzzy approaches are much better than traditional methods especially for transformers which have gas-in-oil conditions around the crisp norms.

Keywords: Cluster analysis; Linguistic modeling; Approximate reasoning; Transformer diagnosis; Dissolved gas analysis

1. Traditional methods

Dissolved gas analysis (DGA) [3,4,11] is the most popular method for detecting incipient faults of transformers presently. When there are abnormal phenomena such as overheating or arcing in transformer, degradation of transformer insulating oil result in the formation of many by-products. The ratios of combustible gases, H_2, CO, CH_4, C_2H_6, C_2H_4, and C_2H_2, of these by-products are closely related to the type of abnormality. The pattern and degree of abnormality can be determined by monitoring the concentrations and growth of these combustible gases, and the fault can be prevented from deterioration consequently.

The diagnosing procedure provided by Taiwan Power Company is shown in Fig. 1 and as a typical example to show how a transformer can be diagnosed.

Generally, the condition of a transformer can be determined from the concentration of combustible gases dissolved in the insulating oil, the concentration of total combustible gas (TCG), and the increasing rate of TCG. However, normal transformers can generate combustible gases while running. A means of verifying if

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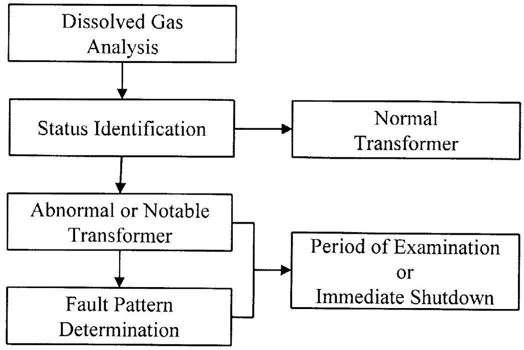

Fig. 1. Diagnostic procedure for transformers.

Table 1
Norms of combustible gases for 69 kV transformers

Status of transformers	Concentration of gases (PPM)							TCG increasing rate
	H_2	CH_4	C_2H_6	C_2H_4	C_2H_2	CO	TCG	
Notable	250	550	450	500	20	300	1500	500 PPM/yr
Abnormal	500	1100	900	1000	40	600	3000	125 PPM/month

a transformer is behaving normally is to compare it with the majority of similar transformers by the gassing characteristics. The judging norms of combustible gases established by Taiwan Power Company for 69 kV transformers are given in Table 1 as an example.

When the concentrations of all the combustible gases, TCG, and TCG increasing rate are below the norms in Table 1, transformers can be considered as "normal" and the period of examination remains unchanged. While if any of these concentrations is beyond its norm of "notable", it means that there might be some incipient fault in the transformer. When any one of the concentrations exceeds its norm of "abnormal", it can be concluded that there must be some fault in the transformer. The examination period of this abnormal transformer must be shortened, or the transformer must be shutdown for repairing at once.

The period of examination depends not only on the status of transformer but also on the pattern of fault. The most commonly used method for identifying fault pattern is the Rogers' method [7]. Four ratios of combustible gases, CH_4/H_2, C_2H_4/CH_4, C_2H_4/C_2H_6, and C_2H_2/C_2H_4, are translated into four independent codes according to their values. Different combinations of these four codes represent different fault patterns, overheating, arcing, and corona.

The final step of diagnosis is to obtain the examination period of suspicious transformers for continuous observation. The current rules developed by Taiwan Power Company for determining examination period of 69 kV transformers are summarized in Table 2.

Basically, both the norms for status identification and the rules for determining examination period are based on crisp value criteria. The boundaries between status of transformer, normal, notable, and abnormal, are fuzzy, and so are the boundaries between fault patterns, overheating, corona, and arcing, due to the possible transition from one fault pattern to another. Even the gas concentrations can also be considered as fuzzy owing to their inevitable measuring error. However, the relationship between fuzzy gas concentrations and the fuzzy conclusion of diagnosis, status and fault pattern, are inferred through crisp criteria. Thus, for gas concentrations that are around one crisp norm misjudging could happen. Through the procedure of diagnosis,

Table 2
Rules for determining examination period of 69 kV transformers

Status of transformers	TCG increasing rate (PPM/month)	Examination period
Notable	45–125	Corona, arcing: 1 month Overheating: 6 months
	$\leqslant 45$	1 yr
Abnormal	$\geqslant 125$	Half a month
	45–125	Corona, arcing: 1 month Overheating: 3 months
	$\leqslant 45$	6 months

Table 3
Gas concentrations of transformers A and B

Transformer	H_2	CH_4	C_2H_4	C_2H_6	C_2H_2	CO	TCG	TCG increasing rate (PPM/month)
A	100	38	80	25	38	0	281	43
B	90	40	78	24	42	0	274	46

Table 4
Diagnostic conclusion of transformers A and B

Transformer	A	B
Status	Notable	Abnormal
Fault pattern (by Rogers' method)	Overheating	Arcing
TCG increasing rate (PPM/month)	$\leqslant 45$	$\geqslant 45$
Examination period	1 yr	One month

accumulating judging error may cause totally different conclusions for transformers that are in similar gas-in-oil conditions.

Suppose that there are two 69 kV transformers A and B with similar gas-in-oil conditions as shown in Table 3. The conclusions of diagnosis by using crisp norms is shown in Table 4.

As can be seen in Table 4, the two transformers with similar gas-in-oil conditions have very different conclusions of diagnosis – examination period of 1 yr for transformer A and one month for transformer B. This is resulted from the inherent dichotomous nature of crisp criteria. Instead of traditional diagnostic method, a new method based on fuzzy cluster analysis and fuzzy inference was proposed for diagnosing transformers.

First of all, gas-in-oil data of failed transformers were collected and analyzed with fuzzy cluster analysis to obtain the membership functions of fault patterns. Then the crisp criteria for status identification and examination period determination were replaced by linguistic variables and transformed into fuzzy rules. Finally, a fuzzy inference method was introduced to combine the status and fault pattern of transformers, which are represented by memberships, together to obtain the examination periods of transformers. The transformer A and B, which have similar gas-in-oil conditions, were reexamined, and the examination periods of these two transformers by using fuzzy approaches are 7.8 months for transformer A and 7.6 months for transformer B, respectively.

Table 5
Principal component analysis of gas-in-oil data

Principal component	Eigenvalues of the covariance matrix			
	Eigenvalue	Difference	Proportion	Cumulative
P_1	933·202	780·690	0·796	0·796
P_2	152·512	99·530	0·130	0·926
P_3	52·982	26·829	0·045	0·971
P_4	26·153	18·111	0·022	0·993
P_5	8·043	8·041	0·007	1·000
P_6	0·002	0	0	1·000

Gas	Eigenvectors P_1	P_2	P_3	P_4	P_5	P_6
H_2	0·584	0·533	0·048	0·442	− 0·105	0·407
CH_4	− 0·498	0·164	− 0·688	0·091	− 0·277	0·409
C_2H_6	− 0·120	− 0·055	− 0·038	0·067	0·900	0·408
C_2H_4	− 0·494	− 0·020	− 0·711	0·169	− 0·234	0·408
C_2H_2	0·180	0·186	0·075	− 0·868	− 0·082	0·408
CO	0·347	− 0·806	0·107	0·101	− 0·200	0·409

2. Membership functions of fault patterns

The membership functions of fault patterns of transformers were obtained with fuzzy cluster analysis from gas-in-oil data of failed transformers. The data of combustible gases were collected from 16 failed transformers which had been proven to have incipient fault from 1983 to 1994. The data collection of every transformer was started when the transformer was found to have symptom of fault and ended when the transformer had been repaired.

According to the key gases method for identifying fault pattern [3], it is the percentages of combustible gases in TCG rather than the magnitude of concentrations that are related to the fault patterns of transformers. Thus, the original data of combustible gas concentrations were transformed in terms of percentages in TCG before further analyzing.

The data were then treated with Ward's method [10] to reject outliers. According to Punj and Stewart [6], all hierarchical methods of clustering are sensitively influenced by outliers, and Ward's method is better than others in the existence of outliers. They also found that k-means method [5] is less influenced by outliers. The data of combustible gas concentrations after removing outliers are shown in Appendix A. The fuzzy clustering method adapted for analyzing gas-in-oil data, the fuzzy c-means algorithm [1], resembles the k-means method. This is the reason why the algorithm was selected to obtain the membership functions of fault patterns.

From the concept of key gases and the observation of data in Appendix A, it can be found that there are strong correlation among combustible gases of the same fault pattern. It means that fewer variables will be enough in identifying the fault pattern of transformers. The results of principal component analysis (Table 5) reveal that two principal components will be enough to account for 93% of the total variance. Fewer variables not only favor the subsequent task of computation but also the representation and realization of diagnostic results.

As shown in Fig. 2, it is obvious that the data approximately belong to two clusters which can be clearly divided by the x-axis. Above the x-axis, the arcing cluster is found, and below the overheating one. It is important for the fuzzy c-means algorithm to predefine the number of clusters before the iteration of the algorithm. According to Fig. 2, one can easily define the number of clusters as 2.

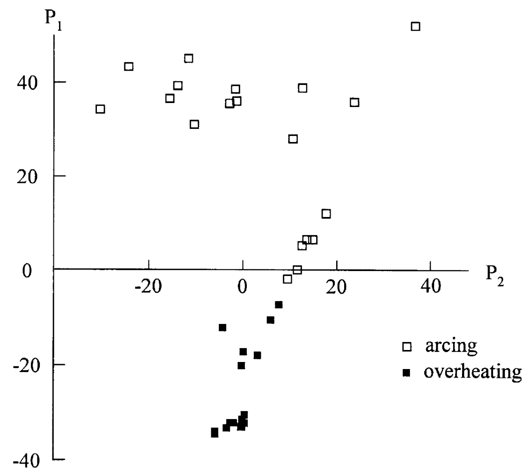

Fig. 2. Results of principal component analysis: P_1 vs. P_2.

For a fuzzy clustering problem of n objects and c clusters, let the data set $X = \{x_1, \ldots, x_n\}$, and the vector of all cluster centers $V = \{v_1, \ldots, v_c\}$. $\tilde{U} = [\mu_{ik}]$ is the membership matrix where μ_{ik} denote the degree of membership of object x_k to cluster $\tilde{s}_i$, that is, $\mu_{ik} = \mu_{\tilde{s}_t}(x_k)$. According to [1],

$$v_i = \frac{\sum_{k=1}^{n} (\mu_{ik})^m x_k}{\sum_{k=1}^{n} (\mu_{ik})^m}, \quad i = 1, \ldots, c,$$

$$\mu_{ik} = \frac{(1/\|x_k - v_i\|_G^2)^{1/(m-1)}}{\sum_{j=1}^{c} (1/\|x_k - v_j\|_G^2)^{1/(m-1)}}, \quad i = 1, \ldots, c, \ k = 1, \ldots, n.$$

For the gas-in-oil data, let $c = 2$ according to the observation from Fig. 2 and the exponential weight $m = 2$. A simplified version of fuzzy c means algorithm with Euclidean distance, $G = I$, comprises the following steps:

Step 1: Initialized $\tilde{U}^{(0)}$ with all $\mu_{ik} = 0.5$ and set $l = 0$.

Step 2: Calculate the c fuzzy cluster centers $\{V_i^{(l)}\}$ by using $\tilde{U}^{(l)}$ from

$$v_i = \frac{\sum_{k=1}^{n} (\mu_{ik})^2 x_k}{\sum_{k=1}^{n} (\mu_{ik})^2}, \quad i = 1, \ldots, c.$$

Step 3: Calculate the new membership matrix $\tilde{U}^{(l+1)}$ by using $\{v_i^{(l)}\}$ from

$$\mu_{ik} = \frac{1/(\|x_k - v_i\|^2)}{\sum_{j=1}^{c} 1/(\|x_k - v_j\|^2)}, \quad i = 1, \ldots, c, \ k = 1, \ldots, n,$$

where $\|x_k - v_j\|^2 = (x_k - v_j)^{\mathrm{T}} (x_k - v_j)$.

Step 4: Calculate $\Delta = \|\tilde{U}^{(l+1)} - \tilde{U}^{(l)}\|$. If $\Delta > \varepsilon = 0.01$ set $l = l + 1$ and go to step 2.

If $\Delta \leqslant \varepsilon$ then stop.

After iterating, the $\mu_{\tilde{s}_i}(x_k)$ for $\tilde{s}_1 =$ overheating and $\tilde{s}_2 =$ arcing are as in Appendix B. The centers of clusters $v_i = (p_{1i}, p_{2i})$ are

$$v_{\text{overheating}} = (-23.75, -0.15), \qquad v_{\text{arcing}} = (34.97, -2.05),$$

and the corresponding membership functions of fault patterns are

$$\mu_{\text{overheating}}(x_k) = \frac{(p_1 - 34.97)^2 + (p_2 + 2.05)^2}{(p_1 + 23.75)^2 + (p_2 + 0.15)^2 + (p_1 - 34.97)^2 + (p_2 + 2.05)^2},$$

$$\mu_{\text{arcing}}(x_k) = \frac{(p_1 + 23.75)^2 + (p_2 + 0.15)^2}{(p_1 + 23.75)^2 + (p_2 + 0.15)^2 + (p_1 - 34.97)^2 + (p_2 + 2.05)^2}.$$

Before calculating the membership to each fault pattern, the data of combustible gases must be transformed into principal components using eigenvectors 1 and 2 in Table 5.

3. Linguistic variables and fuzzy rules

By taking C_2H_2 for example, the rules for identifying the status of transformers which use crisp norms of Table 1 are as below

IF concentration of $C_2H_2 < 20$ THEN status is normal.

IF concentration of $C_2H_2 \geqslant 20$ AND concentration of $C_2H_2 < 40$ THEN status is notable.

IF concentration of $C_2H_2 \geqslant 40$ THEN status is abnormal.

After replacing the crisp norms with linguistic variables, the rules will become:

IF concentration of C_2H_2 is low THEN status is normal.

IF concentration of C_2H_2 is medium THEN status is notable.

IF concentration of C_2H_2 is high THEN status is abnormal.

The terms “low”, “medium”, and “high” are linguistic variables of combustible gas concentration. Every linguistic variable corresponds to a fuzzy set. For different gases the domain of discourse and fuzzy sets are different according to the types of gases and the value of crisp norms. However, the linguistic variables of status, “normal”, “notable”, and “abnormal”, are not fuzzy sets at all because there does not exist any corresponding domain of discourse for status of transformers. The inferential results of fuzzy rules are not memberships to any fuzzy set but only represent degrees of notable or abnormal. The rules can still work well considering that the linguistic variables of transformer status are fuzzy sets belonging to some pseudo domain of discourse.

Unlike the crisp value rules which will be fired individually, more than one fuzzy rules may be fired simultaneously. The results of inference by fuzzy rules for a transformer will be fuzzy status such as 0.3 notable and 0.7 abnormal. To avoid generating triple conclusion like 0.1 normal, 0.9 notable, and 0.1 abnormal, the definition of linguistic variables had better followed the requirement of fuzzy partition. Triple conclusion would not do anything good but only to complicate the problem.

According to the definition of Butnariu [2], if the union and intersection operations of two fuzzy sets in a same discourse of domain are

$$\mu_{\tilde{A}\cap\tilde{B}} = \max(\mu_{\tilde{A}}(x) + \mu_{\tilde{B}}(x) - 1, 0), \tag{1}$$

$$\mu_{\tilde{A}\cup\tilde{B}} = \min(\mu_{\tilde{A}}(x) + \mu_{\tilde{B}}(x), 1). \tag{2}$$

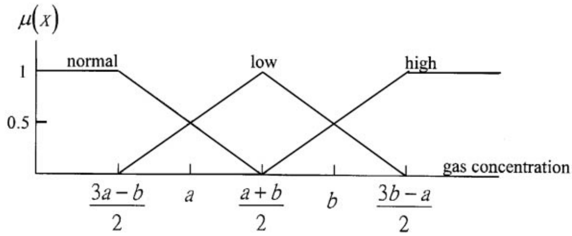

Fig. 3. Linguistic variables of gas concentration.

If fuzzy sets $\tilde{A}_i$, $i=1,\dots,n$, $n\geqslant 2$ in domain U have

$$\tilde{A}_i \cap \tilde{A}_j = \emptyset, \quad i,j=1,\dots,n,\ \ i\neq j, \tag{3}$$

and

$$\bigcup_{i=1}^{n} \tilde{A}_i = U, \tag{4}$$

then $\tilde{A}_i$, $i=1,\dots,n$ is a fuzzy partition of U. Thus, for every $x\in U$ there must be

$$\mu_{\cup_{i=1}^{a}\tilde{A}_i}(x) = \min\left(\sum_{i=1}^{n} \mu_{\tilde{A}_i}(x), 1\right) = 1,$$

that is,

$$\sum_{i=1}^{n} \mu_{\tilde{A}_i} = 1. \tag{5}$$

By following the requirements of (1)–(4), the linguistic variables of every combustible gas with notable norm a and abnormal norm b can be defined as in Fig. 3.

The membership functions are as (6)–(8).

$$\mu_{\text{low}}(x) = \begin{cases} 1 & \text{for } x < \dfrac{3a-b}{2}, \\ 1 - \dfrac{x-(3a-b)/2}{b-a} & \text{for } \dfrac{3a-b}{2} \leqslant x < \dfrac{a+b}{2}, \\ 0 & \text{for } x \geqslant \dfrac{a+b}{2}, \end{cases} \tag{6}$$

$$\mu_{\text{medium}}(x) = \begin{cases} 0 & \text{for } x < \dfrac{3a-b}{2}, \\ \dfrac{x-(3a-b)/2}{b-a} & \text{for } \dfrac{3a-b}{2} \leqslant x < \dfrac{a+b}{2}, \\ 0 - \dfrac{x-(a+b)/2}{b-a} & \text{for } \dfrac{a+b}{2} \leqslant x < \dfrac{3b-a}{2}, \\ 1 & \text{for } x \geqslant \dfrac{3b-a}{2}, \end{cases} \tag{7}$$

$$\mu_{\text{high}}(x) = \begin{cases} 0 & \text{for } x < \dfrac{a+b}{2}, \\ \dfrac{x-(a+b)/2}{b-a} & \text{for } \dfrac{a+b}{2} \leqslant x < \dfrac{3b-a}{2}, \\ 1 & \text{for } x \geqslant \dfrac{3b-a}{2}. \end{cases} \tag{8}$$

Different gases will have the same structure of fuzzy rules but different norms in rules. Different rules and different gas concentrations may cause different conclusions of status such as 0.5 abnormal by CH_4 and 0.7 abnormal by C_2H_2. Under this situation, the stronger evidence C_2H_2 should be taken into account, and the weaker ones should be neglected.

The fuzzy rules for examination period determination also have linguistic variables. But, it should be noted that the linguistic variables of TCG increasing rate for identifying fault pattern are different to that for examination period determination in spite of having the same name. However, the definition of membership functions are still by the same way as (6)–(8). The rules for examination period determination (Table 2) after introducing linguistic variables will be of the following form:

IF status is notable and TCG increasing rate is low
AND fault pattern is overheating THEN examination period is 6 months.

The above rule is not a pure fuzzy rule because the right-hand side of the rule is a crisp value. The inference method of these "semifuzzy rules" are different with the pure fuzzy logic systems and will be discussed hereafter.

4. Inference of fuzzy rules

The configuration of a pure fuzzy logic system [9] is shown in Fig. 4 where the fuzzy rule base consists of a collection of fuzzy rules of the following form:

$$R^{(l)}:\quad IF\ x_1\ is\ F_1^l\ and \cdots and\ x_n\ is\ F_n^l\ THEN\ y\ is\ G^l. \tag{9}$$

For using the pure fuzzy logic system in engineering systems where inputs and outputs are real-valued variables, the most straightforward way is to add a fuzzifier to the input and a defuzzifier to the output of the pure fuzzy system. The configuration of fuzzy logic system with fuzzifier and defuzzifier is shown in Fig. 5.

Instead of considering the fuzzy rules in the form of (9), Takagi and Sugeno [8] proposed the following fuzzy rules:

$$L^{(l)}:\quad IF\ x_1\ is\ F_1^l\ and \cdots and\ x_n\ is\ F_n^l\ THEN\ y^l = c_0^l + c_1^l x_1 + \cdots + c_n^l x_n,$$

where F_i^l are fuzzy sets, c_i are real-valued parameters, y^l is the system output due to rule $L^{(l)}$, and $l = 1, 2, \ldots, M$. The left-hand side of the rules are fuzzy but the right-hand sides are crisp. The output is a linear combination of input variables. For a real-valued input vector $\underline{x} = (x_1, \ldots, x_n)^{\mathrm{T}}$, the output $y(\underline{x})$ is a weighted average of the y^l's:

$$y(\underline{x}) = \frac{\sum_{l=1}^{M} \omega^l y^l}{\sum_{l=1}^{M} \omega^l},$$

where the weight ω^l is calculated as

$$\omega^l = \prod_{i=1}^{n} \mu_{F_i^l}(x_i).$$

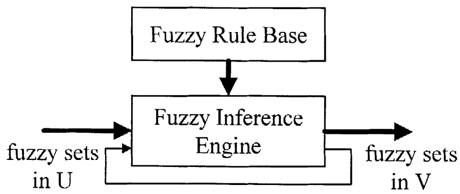

Fig. 4. Configuration of pure fuzzy logic system.

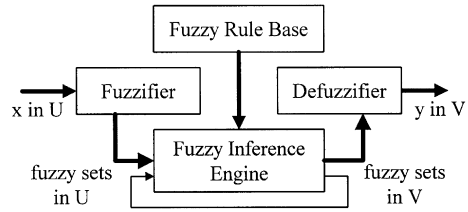

Fig. 5. Configuration of pure fuzzy logic system with fuzzifier and defuzzifier.

The most attractive feature of fuzzy logic systems is that they provide a framework to incorporate fuzzy IF–THEN rules from human experts of which both the IF part and the THEN part are usually fuzzy. However, this is not the case for transformer diagnosis since the THEN part of fuzzy rules derived from Table 2 are already crisp. The defuzzifier will be unnecessary only if a weight representing the overall truth value of the premise can be obtained. This can be done by Takagi and Sugeno's fuzzy logic system. It means that this fuzzy logic system is more suitable for transformer diagnosis than that with a fuzzifier and a defuzzifier.

For the problem here, y^l is not necessary to be a linear combination of input variables, so that the fuzzy rules become

$$L^{(l)}: \quad IF\ x_1\ is\ F_1^l\ and \cdots and\ x_n\ is\ F_n^l\ THEN\ y^l = c^l.$$

Besides, the linguistic of status, TCG increasing rate, and fault pattern are all fuzzy partition of each domain. For every domain the property of (5) will be sustained, and it can be easily proved that

$$\sum_{l=1}^{M} \omega^l = 1.$$

The output $y(\underline{x})$ will then be calculated as

$$y(\underline{x}) = \sum_{l=1}^{M} \omega^l c^l.$$

The transformer A and B mentioned before are diagnosed again with fuzzy approaches. The results are shown in Table 6. It can be seen that the summation of ω^l from every rule is equal to 1, and that the examination period of transformers A and B are 7.80 and 7.64, respectively. For the two transformers, the examination periods are closer to each other and are more reasonable than that of traditional diagnostic method.

5. Conclusions

It is obvious from the results of Table 6 that the combination of several simple fuzzy approaches is better than traditional methods for diagnosing transformers especially for those that have gas-in-oil conditions around the crisp norms. By the fuzzy approaches, transformers of similar gas-in-oil condition will have diagnostic results that are close to each other.

Table 6
Diagnosis of transformers A and B by fuzzy approaches

Transformer	Status	TCG increasing rate (PPM/month)	Membership to fault patterns	ω^l	Contribution of rule (month)	Examination period (month)
A	Notable 0·48	Normal 0·52		0·25	3·00	7·80
		Low 0·48	Arcing 0·55	0·13	0·13	
			Overheating 0·45	0·10	0·62	
	Abnormal 0·52	Normal 0·52		0·27	3·24	
		Low 0·48	Arcing 0·55	0·14	0·14	
			Overheating 0·45	0·11	0·67	
B	Notable 0·4	Normal 0·49		0·20	2·35	7·64
		Low 0·51	Arcing 0·51	0·10	0·10	
			Overheating 0·49	0·10	0·60	
	Abnormal 0·6	Normal 0·49		0·29	3·53	
		Low 0·51	Arcing 0·51	0·16	0·16	
			Overheating 0·49	0·15	0·90	

The validity of diagnostic results still depends on the fundamental diagnostic knowledge, that is, norms for transformer status, rules for examination period, and membership functions of fault patterns. Basically, the former two are derived from diagnostic experience or with statistical methods and have been used for a long period of time. Yet the identification of fault patterns by using membership functions is a new approach, and the representativeness of membership functions is directly related to the gas-in-oil data that have been collected. The amount of data is crucial to the representativeness of membership functions. Since that the occurrence of failed transformer is rare, the membership functions must be updated whenever new data are available. Besides, for transformers of different voltages the membership functions of the same fault pattern may be different.

It should also be noted that the membership functions of fault patterns have no validity on singular points not only because that singular points had been expelled from the data in the beginning of analysis but also because of the limitation of fuzzy c-means algorithm. By fuzzy c-means algorithm, the summation of memberships of every gas-in-oil data to all fault patterns equals to 1 even the point is far from any center of cluster. Thus, more researches are needed to obtain knowledge for pretesting singularity.

Another factor that influences the validity of diagnosis is the design of linguistic variables representing diagnostic norms. The design and definition of these linguistic variables are usually subjective. Trial and error will be unavoidable if there is not any objective approach.

Appendix A. Gas-in-oil data of failed transformers (% of TCG)

No.	Pattern	CO	H_2	CH_4	C_2H_6	C_2H_4	C_2H_2
1	HH	0.3	2.17	45.59	7.97	43.94	0.04
2	HH	0	2.78	41.03	7.82	48.36	0
3	HH	9.3	14.08	30.14	7.32	39.15	0
4	HH	2.16	12.51	31.68	8.19	45.18	0.27
5	HH	2.87	14.23	32.05	8.16	42.62	0.07
6	HH	0.67	15.64	35.35	9.74	38.48	0.12
7	HH	1.22	22.06	37.58	10.36	28.78	0
8	HH	0.92	25.56	34.44	9.19	29.9	0
9	HH	25.21	6.78	21.82	11.44	33.47	1.27
10	HH	0	4.02	55.39	11.2	29.39	0
11	HH	0	2.21	52.71	10.54	34.54	0
12	HH	0	4.89	52.06	9.69	33.53	0
13	HH	0	3.69	48.44	11.13	36.73	0
14	HH	0	4.16	49.74	11.2	34.9	0
15	HH	0	4.63	49.77	11.2	34.41	0
16	HH	0	2.31	38.24	14.91	44.54	0
17	HH	0	2.76	39.84	14.43	42.97	0
18	HH	0	1.22	46.16	12.98	39.64	0
19	HH	0	1.35	46.38	12.9	39.38	0
20	MH	0	0	31.33	17.59	51.08	0
21	MH	0	0	33.88	15.57	50.55	0
22	AR	0	24.79	27.22	3.15	25.85	18.86
23	AR	0	21.29	27.99	3.56	27.32	19.85
24	AR	0	34.72	31.75	7.7	17.06	8.77
25	AR	0	39.42	30.37	7.92	14.54	7.75
26	AR	0	33.55	30.5	11.11	16.12	8.71
27	AR	0	29.96	31.43	12.03	13.71	12.87
28	SD	35.44	35.02	8.86	8.4	2.53	9.7
29	SD	34.4	36.22	7.52	6.38	2.51	12.98
30	AR	46.58	25.23	9.93	7.11	5.1	6.04
31	AR	45.95	35.14	5.41	2.7	1.35	9.46
32	SD	35.29	41.18	4.41	1.47	1.47	16.18
33	AR	25.88	39.05	11.47	3.16	11.93	8.51
34	AR	26.18	45.08	10.86	2.55	9.58	5.75
35	AR	25.89	46.03	10.47	2.07	8.86	6.67
36	AR	25.6	48.38	10.03	1.88	8.05	6.06
37	AR	2.33	56.82	9.86	2.16	11.59	17.97
38	SD	10.36	46.63	6.22	1.55	7.25	27.98
39	AR	20.18	35.88	9.3	1.59	15.4	17.66
40	SD	3.57	75	2.98	0.6	3.57	14.29

HH: High overheating; MH: Medium overheating; SD: Small discharging; AR: Arcing.

Appendix B. Results of fuzzy clustering

No.	Pattern	P_1	P_2	$\mu_{\text{overheating}}(x_k)$	$\mu_{\text{arcing}}(x_k)$
1	HH	−33.62	−2.04	0.98	0.02
2	HH	−33.27	−2.31	0.98	0.02
3	HH	−12.42	−5.35	0.95	0.05
4	HH	−20.61	−0.30	1.00	0.00
5	HH	−18.31	0.12	0.99	0.01
6	HH	−18.03	3.20	0.98	0.02
7	HH	−10.51	6.68	0.91	0.09
8	HH	− 7.42	8.32	0.85	0.15
9	HH	− 5.47	−23.32	0.71	0.29
10	HH	−30.73	0.91	0.99	0.01
11	HH	−32.92	−0.56	0.98	0.02
12	HH	−30.34	0.83	0.99	0.01
13	HH	−31.08	−0.55	0.99	0.01
14	HH	−30.56	−0.05	0.99	0.01
15	HH	−30.06	0.21	0.99	0.01
16	HH	−31.12	−3.33	0.99	0.01
17	HH	−30.82	−2.77	0.99	0.01
18	HH	−33.05	−2.41	0.98	0.02
19	HH	−32.94	−2.29	0.98	0.02
20	MH	−32.58	−5.98	0.98	0.02
21	MH	−33.35	−5.44	0.98	0.02
22	AR	1.53	11.39	0.71	0.29
23	AR	−1.49	9.79	0.71	0.29
24	AR	7.05	15.48	0.48	0.52
25	AR	11.52	17.61	0.38	0.62
26	AR	7.04	14.47	0.48	0.52
27	AR	6.31	13.48	0.49	0.51
28	SD	38.18	−16.27	0.05	0.95
29	SD	40.03	−14.29	0.04	0.96
30	AR	34.03	−30.96	0.16	0.84
31	SD	44.84	−24.95	0.10	0.90
32	SD	46.47	−11.98	0.04	0.96
33	AR	31.69	−6.10	0.01	0.99
34	AR	36.35	−3.66	0.00	1.00
35	AR	37.58	−2.77	0.00	1.00
36	AR	39.38	−1.45	0.00	1.00
37	AR	36.65	24.01	0.14	0.86
38	SD	39.36	13.41	0.06	0.94
39	AR	29.06	−1.83	0.01	0.99
40	SD	54.64	31.06	0.17	0.83

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