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Using fuzzy set to model the stability region on the bicycle derailleur system

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Abstract

In designing the bicycle derailleur system, designers can use the results from the "stability region test" to determine many parameters. Experimentation is the most accurate way to find a stability region because it is very difficult to derive a mathematical expression for the stability region. In this paper, a fuzzy set is used to determine the stability region of the bicycle derailleur system. The database of the fuzzy set model is created from the experimental results. The relationship between the experimental data and fuzzy set model will be described. Many properties of the stability region can be derived from this model. The existence of the stability region, the reduction of experimental noise, the determination of indexed points, and other techniques in designing an indexed derailleur system will also be introduced. This approach can simplify the previous representations of the stability region and the mechanical characteristics. Finally, a case study of the market product is presented. © 2001 Elsevier Science B.V. All rights reserved.

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1. Introduction

In recent years, the fuzzy concept has been given much attention because of its wide applicability in engineering and industry. It provides better and more reasonable solutions for many problems that can or cannot be solved by using a conventional approach. For the most popular human-powered vehicles, bicycles, the fuzzy concept can be used in designing the derailleur system. The derailleur system, which is shown in Fig. 1, is similar to the gear box in a motor vehicle. It consists of four main components: chainwheel and freewheel, front and rear derailleurs, shift levers and

In order to control such systems, many parameters have to be determined, such as indexed points, and values for over-shifting and under-shifting. These mechanical characteristics affect the shifting performance of the derailleur systems and therefore, are very important for the indexed derailleur system. It is believed that many companies have developed their

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cables, and a chain [9]. For different riding conditions, cyclists can choose between speed and labor-saving by moving shift-levers which causes the derailleurs to guide the chain to the desired sprocket. In recent years, "indexed derailleur system" has come to represent the market mainstream. It means that cyclists do not change gears by "feel", they move shift levers to exact "indexed points" on the levers.

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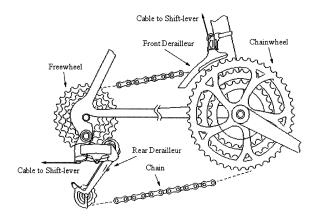


Fig. 1. A bicycle derailleur system.

testing procedures and standards. For business security reasons, few published works can be found by competing companies. In reference papers [3,6,7], a systematic procedure has been proposed to design and test the bicycle derailleur system, and the concept of the "stability region" has also been introduced. Not only many parameters have to be determined from stability region, but also the stability region represents a performance index [7]. In former times, the stability region tests could be accomplished by using three methods: the continuous stepping method, the iterative stepping method [7,8], and the combined method [5].

In this paper, a fuzzy set is used to obtain the stability region of the bicycle derailleur system. The first method obtains a crisp stability region but the experimental results are not very accurate. The other two methods obtain a fuzzy stability region, and the combined method saves much experimental time and effort over the other two methods because a fuzzy logic controller (FLC) is used. Many definitions in fuzzy set theory and bicycle science will be introduced first. The relationship between the experimental data and the mathematical model will be described. Many properties of the stability region can be derived from this model. This approach can simplify the previous representations of the stability region and the mechanical characteristics. Finally, a case study of the market product is presented.

2. Preliminaries

In this section, some concepts related to the fuzzy set theory [1,4] and the bicycle derailleur system will be presented.

2.1. Fuzzy set theory

Let U be a space of objects and x be a generic element of U.

Definition 2.1 ($Fuzzy \ set$). A fuzzy set A in the universe of discourse U is defined as a set of ordered pairs,

$$A = \{ (x, \mu_A(x) \mid x \in U) \}, \tag{1}$$

where $\mu_A(\cdot)$ is called the membership function of A and $\mu_A(x)$ is the membership value of x between 0 and 1. A can also be written as

$$A = \mu_1/x_1 + \mu_2/x_2 + \dots + \mu_i/x_i + \dots + \mu_n/x_n$$

$$= \sum_{i=1}^n \mu_i/x_i.$$
(2)

Definition 2.2 (*Normality*). A fuzzy set is normal if a point $x \in U$ such that $\mu_A(x) = 1$ can always be found.

Definition 2.3 (α -*cut*). The α -cut of a fuzzy set A is a crisp set defined by

$$A_{(\alpha)} = \{ x \mid \mu_A(x) > \alpha \}. \tag{3}$$

Definition 2.4 (*Convexity*). A fuzzy set *A* is convex if and only if for any $\alpha \ge 0$ and any $\lambda \in [0, 1]$,

$$\mu_{A}(\lambda x_{\max} + (1 - \lambda)x_{\min}) \geqslant \mu_{A}(x_{\max})$$
or
$$\mu_{A}(\lambda x_{\max} + (1 - \lambda)x_{\min}) \geqslant \mu_{A}(x_{\min}),$$
(4)

where x_{max} and x_{min} are the maximum and minimum values in $A_{(\alpha)}$.

Definition 2.5 (*Length*). If a fuzzy set A is convex, the length of the α -cut $A_{(\alpha)}$ is

$$LEN(A_{(\alpha)}) = x_{\text{max}} - x_{\text{min}}, \tag{5}$$

where x_{max} and x_{min} is the maximum and minimum values of $A_{(\alpha)}$.

Definition 2.6 (*Intersection*). The intersection of two fuzzy sets A and B is a fuzzy set C represented as $C = A \cap B$ where

$$\mu_C(x) = \min(\mu_A(x), \mu_B(x)). \tag{6}$$

Definition 2.7 (*Union*). The union of two fuzzy sets A and B is a fuzzy set C represented as $C = A \cup B$ where

$$\mu_C(x) = \max(\mu_A(x), \mu_B(x)). \tag{7}$$

Definition 2.8 (*Intensification*). The intensification of a fuzzy set A is INT(A) where

 $\mu_{INT(A)}$

$$= \begin{cases} 2(\mu_A(x))^2 & \text{for } 0 \leq \mu_A(x) \leq 0.5, \\ 1 - 2(1 - \mu_A(x))^2 & \text{for } 0.5 \leq \mu_A(x) \leq 1. \end{cases}$$
 (8)

2.2. Terminology in bicycle science

From reference paper [2], specific terminology used in bicycle science is introduced in the following:

Definition 2.9 (*Down-shifting*). The chain is shifted from a smaller sprocket to a larger sprocket.

Definition 2.10 (*Up-shifting*). The chain is shifted from a larger sprocket to a smaller sprocket.

Definition 2.11 (*Stability region*). The region or interval that is located on the derailleur, in which the chain will not engage a larger sprocket or drop to a smaller sprocket. The chain will remain in the same sprocket and it will be stable while the cable moves around this region.

Definition 2.12 (*Indexed point*). A point in the stability region determines the derailleur position for the current sprocket only and exactly. For example, there

are seven indexed points in a seven-speed rear derailleur system. Such a system is called an indexed derailleur system.

3. Stability region

The stability region is a kind of mechanical characteristic from experimental results. Fig. 2 shows the importance of the stability region. In designing the derailleur system, the control factors: indexed points, over-shifting and under-shifting values, are determined by using the stability region test. The technique of under-shifting elimination is also derived from it. In performance test, many indexes such as stability ratio, width and existence of the stability region are also determined from it. These characteristics will be introduced and simulated in the following sections.

The stability region may be affected by different combinations of derailleur system components and many other factors, such as shown in Fig. 3. Therefore, it is very difficult to get an analytic expression of the stability region. In present studies [8], the stability region can be derived from three experimental methods: the continuous stepping method, the iterative stepping method, and the combined method. In this section, the relationship among the three methods, the crisp set and fuzzy set for the stability region will be described in detail.

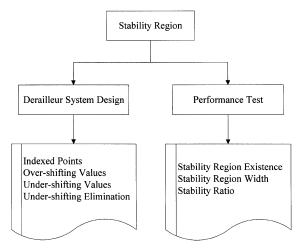


Fig. 2. The importance of the stability region test.

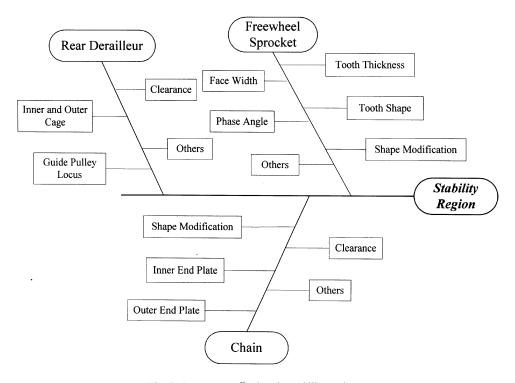


Fig. 3. Parameters affecting the stability region.

3.1. Continuous stepping method: a crisp set of the stability region

Fig. 4a is the rear derailleur system and Fig. 4b is the right-hand-side view of Fig. 4a. When shifting from a smaller sprocket to a larger sprocket, i.e., down-shifting to the nth sprocket, the guide-pulley gradually moves right according to the cable pulled. The chain begins to engage the larger sprocket as the guide-pulley reaches point A. It is said that the points p to p+a do not "belong" to the current sprocket and the points after p+a "belong" to the current sprocket. The crisp set S_{dn} of down-shifting can be defined as

$$S_{\rm dn} = \{ \mu_d(i)_i \mid i = p \ to \ q \},$$
 (9)

where

$$\mu_d(i)_i = \begin{cases} 1 & \text{if "i" belongs to the current} \\ & \text{sprocket,} \\ 0 & \text{if "i" does not belong to the current} \\ & \text{sprocket,} \end{cases}$$

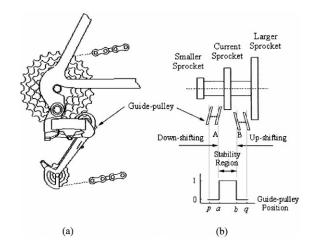


Fig. 4. (a) Rear derailleur system; (b) construction of the crisp set.

and n denotes the nth sprocket. In this example,

$$S_{dn} = \{0_p, 0_{p+1}, 0_{p+2}, \dots, 1_a, 1_{a+1}, \dots, 1_q\}.$$
 (10)

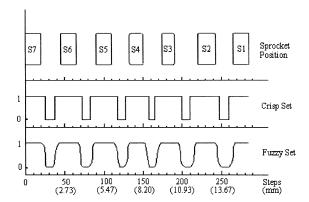


Fig. 5. The stability region of a seven-speed freewheel sprocket.

On the other hand, during up-shifting, the chain begins to leave the larger sprocket as the guide-pulley moves toward point B. In the same way, the crisp set S_{un} of up-shifting is

$$S_{un} = \{1_p, \dots, 1_{b-1}, 1_b, \dots, 0_{q-2}, 0_{q-1}, 0_q\}.$$
 (11)

Therefore, in region AB, the chain does not engage the larger sprocket and drop onto the smaller sprocket; it is thus stable in that region, hence called "stability region". It can be represented as

$$S_n = S_{dn} \cap S_{un}$$

$$= \{0_p, 0_{p+1}, \dots, 1_a, 1_{a+1}, \dots, 1_{b-1}, 1_b, \dots, 0_{q-1}, 0_q\}.$$
(12)

If the cable is pulled from a smaller sprocket to the largest sprocket and pushed in the reverse direction for a seven-speed freewheel, seven stability regions can be formed as shown in Fig. 5 and the expression

$$S = S_1 \cup S_2 \cup \dots \cup S_N, \tag{13}$$

and N denotes the number of the sprocket.

It seems reasonable to determine the stability region by using this method. But some special cases will influence the results. In indexed derailleur systems, there are often auxiliary shifting designs on the sprocket tooth [9]. They guide the chain to engage the sprocket at these points quickly and smoothly. For example, there are four auxiliary shifting designs in a 28T sprocket. If a manufacturing defect or assembly clearance occurs at one point, it will cause the chain to engage at this point more easily than at the other three points. Thus, the chain always shifts at that point

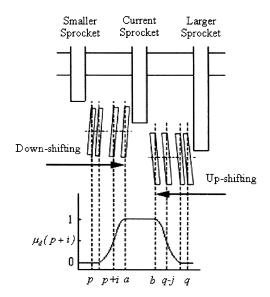


Fig. 6. Construction of the fuzzy set.

with a small cable displacement, but more cable displacement is needed to shift at other designated points. Similarly, if external disturbances occur during the experiment, it will cause the chain to engage or leave the sprocket suddenly. These can be treated as noise factors which affect the results of the stability region experiment.

3.2. Iterative stepping method: a fuzzy set of the stability region

In this method, the statistical concepts are applied to reduce the noise effects. In Fig. 6, point p is the starting point of the experiment. In the first iteration, the cable is pulled from p to p+1 and returned to p several times. For example, if in 10 attempts, no shifting action occurs, the percentage is 0%. In the ith iteration, the cable is pulled from p to p+i and returned to p several times. If the percentage of shifting action is 30%, "0.3" is assigned to the membership value of this position. The process will be continued until the percentage reaches 100%. This point is defined as point A in the down-shifting period. The fuzzy set $S_{\rm dn}$ can be written as

$$S_{\rm dn} = 0/p + 0/(p+1) + \dots + \mu_d(p+i)/(p+i) + \dots + 1/a + 1/(a+1) + \dots + 1/a, \tag{14}$$

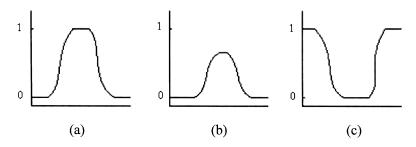


Fig. 7. Curves of the stability region: (a) strict convex, (b) convex and (c) concave.

where $\mu_d(p+i)$ is the membership value of the position (p+i). Similarly, during the up-shifting period, the fuzzy set $S_{\rm un}$ can be written as

$$S_{\text{un}} = 1/p + \dots + 1/(b-1) + 1/b + \dots + \mu_{\nu}(q-j)/(q-j) + \dots + 0/q.$$
 (15)

Therefore, the stability region can be summarized as

$$S_{n} = S_{dn} \cap S_{un} = 0/p + 0(p+1)$$

$$+ \dots + \mu_{d}(p+i)/(p+i) + \dots + 1/a$$

$$+ \dots + 1/b + \dots + \mu_{u}(q-j)/(q-j)$$

$$+ \dots + 0/(q-1) + 0/q.$$
(16)

In a *N*-speed sprocket, the entire stability region can be represented as

$$S = S_1 \cup S_2 \cup \dots \cup S_N. \tag{17}$$

The fuzzy set of the stability region is also shown in Fig. 5.

3.3. Combined method

It is obvious that the results of the continuous stepping method are not very accurate and the iterative stepping method may use a lot of time for the experiment. In reference paper [5], a FLC is used to accelerate the experiment speed. In this method, the continuous stepping method is first applied to pull the cable. The FLC can help to judge whether the guide-pulley is close to the boundary of the stability region or not. If the boundary is reached, the iterative stepping method is then applied to get an accurate stability. Therefore, a fuzzy set of the stability region is obtained and it is the same as that shown in Section 3.2.

4. Properties of the fuzzy stability region

After the stability region is modeled by a fuzzy set, many mechanical characteristics can be derived from the fuzzy theories. In the following, the symbol S_n denotes the fuzzy set of the nth sprocket.

4.1. Existence of a stability region

In previous studies [7], the "linear variation curve" is used to represent and check the existence of the stability region. It is not very convenient because an additional sensor has to be used. In this paper, basic fuzzy concepts can help to avoid this.

Property 4.1. S_n is convex if and only if the stability region exists.

Property 4.2. If S_n is convex and normal, then it is called a "strict stability region".

Fig. 7 shows some representative types of the stability region. In Fig. 7a and b, S_n is convex. Therefore, the stability region exists. Fig. 7a is a strict stability region because the maximum value in the set is 1. The stability region in Fig. 7c does not exist because it is a concave curve. The treatment of this case is described in reference paper [7].

4.2. Noise reduction

Although the iterative stepping method has reduced experimental noise successfully, an intensification operator can emphasize the main effect.

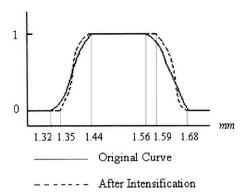


Fig. 8. Original fuzzy set and the set after intensification.

Property 4.3. If the stability region of S_n exists, $INT(S_n)$ emphasizes the boundaries of the stability and reduces the experimental noise.

For example, S_2 , which is shown in Fig. 8 is the fuzzy set of the second sprocket in a seven-speed freewheel.

$$S_2 = 0/13.2 + 0.1/13.5 + 0.4/13.8 + 0.8/14.1$$

$$+ 1/14.4 + 1/14.7 + \dots + 1/15.3 + 1/15.6$$

$$+ 0.9/15.9 + 0.6/16.2 + 0.3/16.5 + 0/16.8,$$
(18)

the unit of the cable displacement is mm. $INT(S_2)$ can be accomplished from Eq. (7) and shown in Fig. 8,

 $INT(S_2)$

$$= 0/13.2 + 0/13.5 + 0.3/13.8 + 0.9/14.1 + 1/14.4 + 1/14.7 + \dots + 1/15.3 + 1/15.6 + 0.8/16.2 + 0.2/16.5 + 0/16.8.$$
(19)

4.3. Determination of indexed points

As mentioned before, there are seven indexed points in a seven-speed derailleur. These points are assigned on the shift-levers and they have to be determined precisely when designing the shift-levers. From reference paper [7], the indexed point of the current sprocket is the middle point of the stability region. There-

fore, it can be easily determined by using the fuzzy set

Property 4.4. The α -cut of S_n , $S_{n(\alpha)}$, denotes the stability region and the choice of α determines the accuracy of the product.

Property 4.5. The length of $S_{n(\alpha)}$, $LEN(S_{n(\alpha)})$, is the width of the stability region.

Property 4.6. The indexed point of the *n*th sprocket is the middle point of the α -cut set.

For example, in the fuzzy set in Eq. (18), if $\alpha = 0.9$ is chosen, a strict stability region is obtained:

$$INT(S_2)_{(0.9)} = \{14.4, 14.7, \dots, 15.6, 15.9\}.$$
 (20)

The length of the stability region is $15.9 - 14.4 = 1.5 \, mm$ and the indexed point is located at the middle point $(14.4 + 15.9)/2 = 15.15 \, mm$. If $\alpha = 0.2$ is chosen, the region becomes

$$INT(S_2)_{(0,2)} = \{13.8, 14.1, \dots, 15.9, 16.2\},$$
 (21)

the length of the stability region is 2.4 mm and the indexed point is 15.0 mm. A larger stability region is assigned in designing a lower quality product because the design tolerance of the components is larger. But in a higher quality product, the accuracy of the components is very good. Therefore, they can be compatible with lower quality products.

4.4. Stability ratio

As mentioned before, a larger stability region is desired in designing the derailleur system. For the sake of convenient comparison between different products, the index called the "stability ratio (SR)" is defined:

Property 4.7. The stability ratio

$$SR = \frac{\sum_{n=1}^{m} LEN(S_{n(\alpha)})}{x_{\text{end}}} \times 100, \tag{22}$$

where m is the number of sprocket and x_{end} is the length of total cable displacement.

The physical meaning of the SR value is the ratio of the total range of stability region to total cable displacement. A larger SR value indicates the derailleur

system has a larger stability region, and the allowable tolerances in manufacturing and assembly of the components can be larger.

4.5. Case study

To demonstrate the above properties, the combinbed method is used to test a seven-speed rear derailleur system. The fuzzy set of the seven sprockets is shown in the following:

$$S_{1} = 0/16.8 + 0.8/17.1 + 1/17.4 + \dots + 1/18.0,$$

$$S_{2} = 0/13.2 + 0.1/13.5 + 0.4/13.8 + 0.8/14.1 + 1/14.4 + \dots + 1/15.6 + 0.9/15.9 + 0.6/16.2 + 0.3/16.5 + 0/16.8,$$

$$S_{3} = 0/10.2 + 0.1/10.5 + 0.9/10.8 + 1/11.3 + \dots + 1/12.3 + 0.2/12.6 + 0/12.9,$$

$$S_{4} = 0/7.5 + 1/7.8 + \dots + 1/9.6 + 0.8/9.9 + 0/10.2,$$

$$S_{5} = 0/4.8 + 0.6/5.1 + 1/5.4 + \dots + 1/6.6 + 0/6.9,$$

$$S_{6} = 0/1.9 + 0.8/2.1 + 1/2.4 + \dots + 1/4.2 + 0.8/4.5 + 0/4.8,$$

$$S_{7} = 1/0.0 + \dots + 1/1.2 + 0.8/1.5 + 0/1.8.$$
(23)

Use the intensification operator to obtain $INT(S_n)$, and then choose $\alpha = 0.8$ for the α -cut operation,

$$INT(S_{1})_{(0.8)} = \{17.1, 17.4, \dots, 18.0\},$$

$$INT(S_{2})_{(0.8)} = \{14.1, 14.4, \dots, 15.9\},$$

$$INT(S_{3})_{(0.8)} = \{10.8, 11.1, \dots, 12.3\},$$

$$INT(S_{4})_{(0.8)} = \{7.8, 8.1, \dots, 9.9\},$$

$$INT(S_{5})_{(0.8)} = \{5.4, 5.7, \dots, 6.6\},$$

$$INT(S_{6})_{(0.8)} = \{2.1, 2.4, \dots, 4.5\},$$

$$INT(S_{7})_{(0.8)} = \{0.0, 0.3, \dots, 1.5\}.$$

The set of calculated indexed points is $\{17.6, 15.0, 11.6, 8.9, 6.0, 3.3, 0.8\}$. And SR = 63.33. From the product itself, the measured indexed points from the shift-lever are $\{17.5, 14.4, 11.4, 8.4, 5.9, 3.3, 0.8\}$. For business security reasons, design strategy of the company cannot be figured out. But, the set of indexed points calculated in Eq. (24) is very close to the set measured on an actual product.

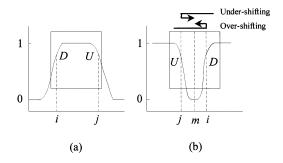


Fig. 9. (a) Stability region exist, and (b) stability region does not exist.

4.6. Other mechanical characteristics

Until present, the discussions are valid when the stability region exists. In Fig. 9a, the block means the position of current sprocket, and $\alpha = 0.8$ is defined. During down-shifting, the chain engages current sprocket at point D. On the other hand, the chain drop on current sprocket at point U during down-shifting. Therefore, the region DU is the stability region. If there are no stability regions in any sprocket, overshifting and under-shifting values have to be taken into consideration such as shown in Fig. 9b. In operation, the chain will not engage the larger sprocket until the cable is pulled to point D and it will not drop on the sprocket until point U. Therefore, during down-shifting, the cable must first be pulled to point D and then pushed back to the middle point of the sprocket. This is called "over-shifting". "Undershifting" can be determined in a similar way. From Eqs. (14) and (15),

Property 4.8 (*Over-shifting*). If the stability of the nth sprocket does not exist, the over-shifting value of the nth sprocket $O'_n = i - m$, where m is the middle point of the sprocket.

Property 4.9 (*Under-shifting*). If the stability of the *n*th sprocket does not exist, the under-shifting value of the *n*th sprocket $U'_n = m - j$.

Property 4.10 (*Under-shifting elimination*). To reduce the under-shifting value, the over-shifting value of the *n*th sprocket $O_n = i - m + \max(U'_n)$, and

 $U_n = 0$. Therefore, new indexed points are equal to old indexed points minus $\max(U'_n)$.

In situations in which only over-shifting is present in shift-lever designs, they can be used for applications by under-shifting elimination and shifting indexed points. Therefore, shift-levers can be designed for only over-shifting with an easier mechanism and lower cost. The detail descriptions can be found in reference paper [7].

5. Conclusions

This paper presents a method to implement the stability region from the experimental data. In addition, the use of the fuzzy concept helps to correct the disadvantages and inconveniences of the traditional modeling in the stability region test. The relationship between the experiment and the fuzzy set model is established. The stability region represented can be determined more reasonably and accurately. From such a fuzzy set, the existence of the stability region can be known easily and effectively. After using the intensification operator, the noise in the experimental process can be reduced and the main effect of the stability region can be emphasized. The indexed points in an indexed derailleur system can also be determined easily. Finally, a case study is used to demonstrate the application of these approaches.

Although bicycle science has developed over a hundred years, some difficulties have been suffered in these years, especially in designing the derailleur system. The problem most recently is to improve the performance of the derailleur system, which directly affects the feelings of the rider. The stability region test and its applications are the most important and the first stage in the performance tests. It is believed

that many companies have developed their own procedures and standards. For business security reasons, few published works or research reports can be found. This paper successfully applies the fuzzy concept in the stability region. It is desired that some progress can be made in subsequent designs by using this approach.

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