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Transmission Coefficients in the Electron-Atom Scattering in Intense Laser Field

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A non-perturbative method is used to investigate the one-dimensional problem of the electron-atom scattering in the presence of intense field by solving the time-dependent Schrodinger equation in momentum space. The exact results of the transmission and reflection coefficients for the system are derived. Some errors in the previous heuristic results are found.

Most of the theoretical studies for the scattering of electron by atoms in the intense radiation field are based on the perturbative theory [1-3]. Starting with the well-known Kroll and Watson [4] work on the soft photon approximation, there exists only a few non-perturbative approaches for this problem. Shakeshaft [5] has formulated a non-perturbative method of coupled integral equations for calculating the scattering cross section by assuming the potential is seperable. Rosenberg [6] applied the variational method for Coulomb scattering in the laser field using low-frequency approximation. Bhatt et al [7] and Gavrila and Kaminski [8] used the Kramers-Henneberger transformation to study the problem and the one-dimensional scattering by a polarized potential is considered. Recently, a new and efficient non-perturbative method of solving the time dependent Schrödinger equation in the momentum space for a system undergoing multiphoton processes has been introduced [9-10]. This theory has been applied to study the above-threshold ionization of atoms [11] and the electron scattering by atoms in the intense laser field [12]. Successful results are obtained. In the case of one-dimensional scattering problem, the most important quantities are the transmission and reflection coefficients during the scattering. These coefficients have been discussed briefly in the previous paper [10] for the case without radiation field, and only ' heuristic results are guessed when the field is included but no detailed derivation is given. In this report, we present a detailed analysis for this problem and the exact results for the transmission and reflection coefficeints are derived. Some errors in the previous heuristic expression are found.

The time-dependent Schödinger equation for an electron travelling in the radiation field of a vector potential $A(t)$ and scattering by the atomic potential V is

$$i\hbar\frac{d}{dt}\left|\psi(t)\right> = \left[\frac{p^2}{2m} + Ve^{-\epsilon|t|} + H_I(t)\right]\left|\psi(t)\right> \tag{1}$$

where H_I is the interaction between the electron and the laser field

$$H_I = \frac{eA(t)}{mc}p = -\frac{f(t)p}{m} \tag{2}$$

with $f(t) = \frac{eA(t)}{C}$. The $A^2(t)$ term in H_I has been removed by a trivial contact transformation [4]. Initially, at time $t \to -\infty$, the electron is far from atom, i.e., free of the potential; thus V can be replaced by $Ve^{-\epsilon|t|}$, in which ϵ is positive and infinitesimal. When the potential V is turned off completely, the problem becomes the free electron scattering in the radiation field. The solution to the time-dependent Schrödinger equation was originally derived by Volkov [13] and can be written in the following form

$$|\chi_k(t)> = \exp\{-i[E(K)t/\hbar + \theta_k(t)]\}|k> \tag{3}$$

where $E(k) = \hbar^2k^2/2m, |k>$ is the eigenvector of p with momentum eigenvalue $\hbar k$ normalized as

$$<x|k> = (2\pi)^{-1/2}e^{ikx} \tag{4}$$

and the real phase $\theta_k(t)$ is given by

$$\begin{aligned}\theta_k(t) &= \frac{1}{\hbar}\int_0^t dt'\left[E(k) - E(K) - \frac{p}{m}f(t')\right] \\ &= [E(k) - E(K)]t/\hbar - \frac{k}{m}\int_0^t f(t')dt'\end{aligned} \tag{5}$$

where K is the initial momentum of the electron.

For our purpose, we take the Volkov states as forming a complete set. The solution of Eq. (1) is expressed as

$$|\psi(t)> = |\chi_K(t)> + |\phi(t)> \tag{6}$$

in which the electron is initially in the state $|\chi_K(t)>$ and $|\phi(t)>$ is expanded as

$$|\phi(t)> = \int dk a_k(t)|\chi_k(t)> \tag{7}$$

with the boundary condition $|\psi(t)> \to |\chi_K(t)>$ as $t \to -\infty$, i.e.: $|\phi(-\infty)> = 0$. Substituting Eqs. (6) and (7) into Eq. (1), we obtain the inhomogeneous integro-differential equation for the coefficient $a_k(t)$:

$$i\hbar\frac{d}{dt}a_k(t) = e^{i\theta_k(t)}[e^{-i\theta_K(t)} < k|Ve^{-\epsilon|t|}|K > +b_k(t)] \tag{8}$$

$$\begin{aligned} b_k(t) &= e^{iE(K)t/\hbar} < k|Ve^{-\epsilon|t|}|\phi(t) > \\ &= \int dk' e^{-i\theta_{k'}(t)} a_{k'}(t) < k|Ve^{-\epsilon|t|}|k' > \end{aligned} \tag{9}$$

with the boundary condition $a_k(-\infty) = b_k(-\infty) = 0$. This set of equations have been used in the previous paper [12] to describe successfully the multiphoton process in the electron-atom scattering in the intense laser field.

In order to obtain the transmission and reflection coefficients, introducing the Green's function defined by

$$\Big(\frac{p^2}{2m} - \frac{f(t)p}{m} - i\hbar\frac{\partial}{\partial t}\Big)G(t, t', E) = \delta(t - t') \tag{10}$$

For positive $(t - t')$, the Green's function in coordinate representation is given as [4]

$$< x|G|x' > = \frac{i}{2\pi\hbar}\int_{-\infty}^{\infty} \exp[ig_k(t') - ig_k(t)]e^{ik(x-x')}dk \tag{11}$$

where

$$g_k(t) = \frac{E(K)t}{\hbar} + \theta_k(t) \tag{12}$$

Therefore, the scattering state $|\phi(t)>$ corresponding to an incoming wave with momentum $\hbar K$ in coordinate representation is

$$\begin{aligned} < x|\phi(t) > &= -\int_{-\infty}^{t} dt' \int_{-\infty}^{\infty} dx' < x|G|x' >< x'|Ve^{-\epsilon|t'|}|\psi_K(t') > \\ &= -\frac{i}{2\pi\hbar}\int_{-\infty}^{t} dt' \int_{-\infty}^{\infty} dx' \int_{-\infty}^{\infty} dk \exp[ig_k(t') - ig_k(t)]e^{ik(x-x')} \\ &\quad \times < x'|Ve^{-\epsilon|t'|}|\psi_K(t') > \end{aligned} \tag{13}$$

For a linearly polarized field $A(t) = a \cos \omega t$, we have

$$\theta_k(t) = [E(k) - E(K)]t/\hbar - \frac{epa}{mcw}\sin\omega t \tag{14}$$

It may be shown, by letting $t \to t + \frac{2\pi}{w}$ and $t' \to t' + \frac{2\pi}{w}$ in Eqs. (6) and (13), that $\exp\{i[\frac{E(K)t}{\hbar} - \frac{k}{m}\int_0^t f(t')dt']\}|\psi_K(t) >$ is periodic in t with period $2\pi/w$ and therefore can be Fourier expanded

$$e^{i[E(K)t/\hbar - \frac{k}{m}\int_0^t f(t')dt']}|\psi_K(t) > = \sum_n \varphi_n(K, k)e^{-in\omega t} \tag{15}$$

Substituting this expansion into Eq. (13), we obtain

$$
\begin{aligned}
&< x|\phi(t) > \\
&= -\frac{i}{2\pi\hbar}\sum_n \int_{-\infty}^{\infty} dx' \int_{-\infty}^{\infty} dk \int_{-\infty}^{t} dt' \exp\Big\{i\Big[\frac{E(k)-E(K)}{\hbar}t' - nwt'\Big]\Big\} \\
&\times \exp\Big\{-i\Big[\frac{E(k)t}{\hbar} - \frac{k}{m}\int_0^t f(t')dt'\Big]\Big\} e^{ik(x-x')} < x'|Ve^{-\epsilon|t|}|\varphi_n(K,k) > \\
&= \sum_n \Big(-\frac{mi}{\hbar^2 k_n}\Big)\int_{-\infty}^{\infty} dx' e^{ik_n(x-x')} \\
&\times e^{-i[E(k_n)t/\hbar - \frac{k_n}{m}\int_0^t f(t')dt']} < x'|Ve^{-\epsilon|t|}|\varphi_n(K,k_n) >
\end{aligned} \tag{16}
$$

where $E(k_n) = \frac{\hbar^2 k_n^2}{2m} = E(K) + n\hbar w$. Making use of Eqs. (3) and (15), we have for $x \to +\infty$

$$
\begin{aligned}
< x|\phi(t) > \;\; \to \;\; & \sum_n \Big(-\frac{mi\sqrt{w}}{\hbar^2 k_n}\Big) e^{ik_n x} e^{-i[E(k_n)t/\hbar - \frac{k_n}{m}\int_0^t f(t')dt']} \\
& \times \int_t^{t+2\pi/w} dt' < \chi_{k_n}(t')|Ve^{-\epsilon|t'|}|\psi_K(t') >
\end{aligned}
$$

Substituting Eq. (6) into above equation and making use of Eqs. (5) and (9), finally we obtain

$$
\begin{aligned}
&< x|\phi(t) > \\
&\to \sum_n \Big(\frac{\alpha}{k_n}\Big) < x|\chi_{k_n}(t) > \int_t^{t+2\pi/w} dt'[< \chi_{k_n}(t')|Ve^{-\epsilon|t'|}|\chi_K(t') > \\
&+ e^{i\theta_{k_n}(t')} b_{k_n}(t')]
\end{aligned} \tag{17}
$$

where $\alpha = -\sqrt{2\pi w}\, im/\hbar^2$. Therefore Eq. (6) gives

$$
< x|\psi_K(t) > \to \sum_n T_n < x|\chi_{k_n}(t) >, \quad x \to +\infty
$$

where

$$
\begin{aligned}
T_n(t) \;\; = \;\; & \delta_{n0} + \frac{\alpha}{k_n}\int_t^{t+2\pi/w} dt'[< \chi_{k_n}(t')|Ve^{-\epsilon|t'|}|\chi_K(t') > \\
& + e^{i\theta_{k_n}(t')} b_{k_n}(t')]
\end{aligned} \tag{18}
$$

is the transmission coefficient for the electron to absorb n photons. Following the similar procedures, the reflection coefficient for absorbing n photons, $R_n(t)$, can also be obtained by letting $x \to -\infty$ in Eq. (16).

$$
\begin{aligned}
< x|\phi(t) > \;\; \to \;\; & \sum_n R_n(t) < x|\chi_{-k_n}(t) > \\
R_n(t) \;\; = \;\; & \frac{\alpha}{k_n}\int_t^{t+2\pi/w} dt'[< \chi_{-k_n}(t')|Ve^{-\epsilon|t'|}|\chi_K(t') > \\
& + e^{i\theta_{-k_n}(t')} b_{-k_n}(t')]
\end{aligned} \tag{19}
$$

Therefore, the exact results of the transmission and reflection coefficients are derived as given in Eqs. (18) and (19). Our work is the first detailed derivation of these coefficients for the electron scattering by atoms in the intense radiation field. Comparing these exact results with the heuristic guessed expressions given in the previous work (Eq. (12) of Ref. 10), it is found that some errors are made in the previous expressions and have been corrected in our results.

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REFERENCES

[1] J. I. Gersten and M. H Mittleman, Phys. Rev. A13, 123 (1976).
[2] F. W. Byson Jr. and C. J. Joachain, J. Phys. B17, L295 (1984).
[3] M. Gavrila, A. Maquet, and V. Veniard, Phys. Rev. A42, 236 (1990).
[4] N. M. Kroll and K. M. Watson, Phys. Rev. A8, 804 (1973).
[5] R. Shakeshaft, Phys. Rev. A28, 667 (1983).
[6] L. Rosenberg, Phys. Rev. **A26**, 132 (1982).
[7] R. Bhatt, B. Piraux, and K. Kurnett, Phys. Rev. A37, 98 (1988).
[8] M. Gavrila and J. I. Kaminski, Phys. Rev. Lett. 52, 613 (1984).
[9] R. Shakeshaft and M. Dorr, Z. Phys. D8, 255 (1988).
[10] R. Shakeshaft and C. S. Han, Phys. Rev. A38, 2163 (1988).
[11] C. S. Han, J. Phys. B23, L495 (1990).
[12] C. S. Han, Chin J. Phys. 28, 131 (1990).
[13] D. M. Volkov, Z. Phys. 94, 250 (1935).