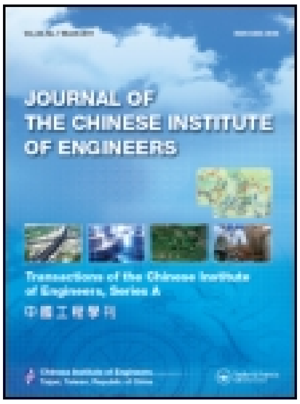

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Decoupling precompensation and optimal decoupling

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DECOUPLING PRECOMPENSATION AND OPTIMAL DECOUPLING

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Key Words: admissible decoupling precompensator, optimal decoupling, cost.

ABSTRACT

We study decoupling control under a unity-feedback configuration for linear multivariable plants. We parametrize the set of all admissible decoupling precompensators. With the parametrization, it is shown that decoupling controller design is equivalent to a set of SISO controller designs. The parametrization is also used to establish a necessary and sufficient condition for the existence of a stable decoupling controller. Optimal decoupling controller design is proposed and the cost of decoupling is discussed.

I. INTRODUCTION

Decoupling controller design has been studied by many authors (Desoer and Gundes, 1986, Lin and Hsieh, 1991, and Vardulakis, 1987). The proposed approaches are either based on a coprime factorization of the plant or on the interpolation condition at the poles and zeros of the plant. A conceptually decoupling controller can also be designed by first diagonalizing the plant by means of a precompensator and design controller for the diagonalized plant. Of course for the approach to work, care must be taken to avoid unstable pole-zero cancellation, otherwise the diagonalized plant would not be stabilizable. In this paper, we consider the latter approach. We study the first description of admissible decoupling precompensators, those which maintain stabilizability. For 2-input 2-output systems this has been given in Linnemann and Maier (1993) and for the general square plant case in Wang (1992). Compared with that given in Wang (1992), our description is simple and easier to compute. The simplicity of our description is because we check the stability of the appropriate closed-loop transfer matrix for internal stability of the feedback system, as is now usually done, instead of deriving conditions based on avoidance of unstable pole-zero cancellations in the MIMO sense (Anderson and Gevers, 1981). The simple description allows us to give very simple necessary and sufficient conditions for the existence of stable decoupling controllers. It is also used in the discussion of optimal decoupling and the cost of decoupling. It is shown that if the design objective is to minimize a weighted mixed sensitivity, then the design of an optimal decoupling controller reduces to a set of optimal SISO mixed sensitivity designs.

It has been noted in Desoer and Gundes (1986) and Morari and Zafiriou (1989) that decoupling may (and usually does) increase the multiplicities of nonminimum phase zeros of the loop transfer matrix, thus it further limits the achievable sensitivity and induces a cost of decoupling. The decoupling requirement usually also increases the multiplicities of unstable poles of the loop transfer matrix (Linnemann and Maier, 1993) and thus further limits the achievable robustness with respect to multiplicative

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uncertainty. In this paper the cost of decoupling is defined as the difference between the achievable optimal performance indices with and without decoupling constraint. It is argued that for stable and minimum phase plants the cost is zero; for stable nonminimum phase or minimum phase unstable plants the cost is moderate; and for unstable and nonminimum phase plants the cost is usually high.

The paper is organized as follows. The unity-feedback system under consideration together with some preliminary results is given section II. Section III describes the set of admissible decoupling precompensators and gives necessary and sufficient conditions for the existence of stable decoupling controllers. Section IV discusses optimal decoupling and the cost of decoupling. Section V is a brief conclusion.

II. PRELIMINARIES

Consider the unity-feedback system $S(P,C)$ shown in Fig. 1, where $P\in \mathbb{R}_{po}(s)^{n\times n}$ is the plant, $C\in \mathbb{R}_{p}(s)^{n\times n}$ is the controller, (u_1,u_2) is the input and (y_1,y_2) is the output. We assume that P is nonsingular so that the inverse $P^{-1}\in \mathbb{R}(s)^{n\times n}$ exists. Let $u:=[u_1^T\ u_2^T]^T$ and $y:=[y_1^T\ y_2^T]^T$. The closed-loop transfer matrix $H_{yu}\in \mathbb{R}_p(s)^{2n\times 2n}$ and is given by

$$H_{yu}=\begin{bmatrix} H_{y_1u_1} & H_{y_1u_2} \\ H_{y_2u_1} & H_{y_2u_2}\end{bmatrix}=\begin{bmatrix} C(I+PC)^{-1} & -CP(I+CP)^{-1} \\ PC(I+PC)^{-1} & P(I+CP)^{-1}\end{bmatrix}. \tag{1}$$

We say that the system $S(P,C)$ is (internally) stable and C is a stabilizing controller for P if H_{yu} is stable, i.e., $\mathcal{P}[H_{yu}]\subset\mathbb{C}_-$; the system is decoupled and C is a decoupling controller for P if C stabilizes P and the I/O map[1] $H_{y_2u_1}$ is nonsingular and diagonal. We assume throughout that a decoupling controller exists for P. A necessary and sufficient condition is given in Lin (1998).

Lemma 1. (Lin, 1998) For the system $S(P,C)$ with H_{yu} given in (1), if $H_{y_1u_1}$ and $H_{y_2u_2}$ are stable then $\mathcal{P}_+[H_{y_2u_1}]\subset(\mathcal{P}_+[P]\cap\mathcal{Z}_+[P])$ and $\mathcal{P}_+[H_{y_1u_2}]\subset(\mathcal{P}_+[P]\cap\mathcal{Z}_+[P])$. ■

Proposition 1. Consider the feedback system $S(P,C)$ shown in Figure 1. Suppose the plant P is *diagonal* and that C is a stabilizing controller achieving the I/O map $H=[H_{ij}]$, that is, $PC(I+PC)^{-1}=H$. Under these conditions, there is a diagonal stabilizing controller C_d such that $PC_d(I+PC_d)^{-1}=\text{diag}[H_{ii}]$. ■

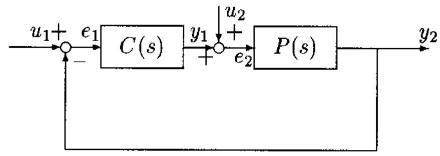

Fig. 1 Unity-feedback system $S(P,C)$

Comment. Thus if a diagonal plant can not be stabilized by a diagonal controller, it can not be stabilized by any (linear time-invariant) controller.

Proof. Since $H=PC(I+PC)^{-1}$, in term of H, the closed-loop transfer matrix in (1) becomes

$$H_{yu}=\begin{bmatrix} P^{-1}H & -P^{-1}HP \\ H & (1-H)P\end{bmatrix} \tag{2}$$

and is stable. Let $H_d=\text{diag}[H_{ii}]$. Since P is diagonal, it follows that $P^{-1}H_d$, $P^{-1}H_dP$ and $(I-H_d)P$ are all stable. Hence with the diagonal controller $C_d=P^{-1}H_d(I-H_d)^{-1}$, the feedback system is stable and the I/O map is $H_d=\text{diag}[H_{ii}]$. ■

Proposition 2. (Dickman and Sivan, 1985) Let $A\in\mathbb{C}^{n\times n}$ and $A_D=\text{diag}[a_{ii}]$, where a_{ii} is the ith diagonal element of A. Then $\overline{\sigma}(A)\geq\overline{\sigma}(A_D)$. ■

For later use, we write

$$P=\left[\frac{Z_{ij}}{P_{ij-}P_{ij+}}\right] \tag{3}$$

where Z_{ij}, P_{ij-}, $P_{ij+}\in\mathbb{R}[s]$ are mutually coprime, P_{ij+} is monic, $\mathcal{Z}[P_{ij+}]\subset\mathbb{C}_+$ and $\mathcal{Z}[P_{ij-}]\subset\mathbb{C}_-$; and write

$$P^{-1}=\left[\frac{N_{ij}}{D_{ij-}D_{ij+}}\right] \tag{4}$$

where N_{ij}, D_{ij-}, $D_{ij+}\in\mathbb{R}[s]$ are mutually coprime, D_{ij+} is monic, $\mathcal{Z}[D_{ij+}]\subset\mathbb{C}_+$ and $\mathcal{Z}[D_{ij-}]\subset\mathbb{C}_-$.

Let

$$P_{i+}=\text{ the monic least common multiple of } \{P_{ij+}\}_{j=1}^{n} \tag{5}$$

and

$$D_{i+}=\text{ the monic least common multiple of } \{D_{ij+}\}_{i=1}^{n} \tag{6}$$

and γ_j be the relative degree of the jth column of P^{-1}. Since $P\in\mathbb{R}_{po}(s)^{n\times n}$, $\gamma_j>0$. Existence of a decoupling

[1]For convenience, we call the transfer matrix $H_{y_2u_1}$ the I/O map of the feedback system.

controller implies that, for i=1, ..., n, the polynomials D_{i+} and P_{i+} are coprime (Lin, 1998). Let H= diag$[\frac{D_{i+}\beta_i}{\alpha_i(s)}]$, where β_i, $\alpha_i \in \mathbb{R}[s]$, α_i is Hurwitz, and

$$P_{i+}|(\alpha_i - D_{i+}\beta_i), \quad i=1, ..., n \tag{7}$$

and

$$\deg(\alpha_j) - \deg(\beta_j) \geq \gamma_j + \deg(D_{j+}), \ j=1, ..., n \tag{8}$$

Then H is a decoupled I/O map of the system $S(P,C)$ (Lin, 1998).

III. ADMISSIBLE DECOUPLING PRECOMPENSATORS

As we mentioned, the design of a decoupling controller can be decomposed into two steps: find a decoupling precompensator that open-loop decouples the plant and then design a SISO feedback controller for each of the decoupled channels. The resulting decoupling controller combines the precompensator and the design (SISO) controller. Design of SISO controllers is relatively simple; however the cascade connection of the plant and a decoupling precompensator may not be internally stabilizable due to unstable pole-zero cancellations (Anderson and Gevers, 1981). A proper decoupling precompensator is said to be *admissible* if its cascade connection with the plant maintains stabilizability. Clearly the existence of an admissible decoupling precompensator is equivalent to the existence of a decoupling controller. Since if the diagonal plant is stabilizable, then it can be stabilized by a diagonal controller.

In this section, we construct an admissible decoupling precompensator and give a simple characterization of all admissible decoupling precompensators.

Let the strictly proper P be given and let P_{i+} and D_{i+} be as defined in (5) and (6) respectively. Consider the unity-feedback system $S(P,C)$ shown in Fig. 1 with the controller

$$C = P^{-1}\text{diag}[\frac{D_{i+}\beta_i}{\alpha_i - D_{i+}\beta_i}] \tag{9}$$

where the polynomials α_i and β_i satisfy conditions (7) and (8) and α_i is Hurwitz. Thus C is a decoupling controller for P which achieves the decoupled I/O map H=diag$[\frac{D_{i+}\beta_i}{\alpha_i}]$. Let

$$F = P^{-1}\text{diag}[\frac{D_{i+}}{P_{i+}(s+1)^{\mu_i}}] \tag{10}$$

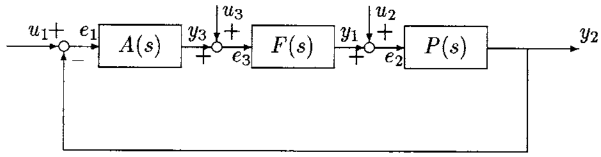

Fig. 2 The feedback system $S(P,F,A)$

and

$$A = F^{-1}C = \text{diag}[\frac{P_{i+}(s+1)^{\mu_i}\beta_i}{\alpha_i - D_{i+}\beta_i}] = \text{diag}[\frac{(s+1)^{\mu_i}\beta_i}{f_i}] \tag{11}$$

where γ_i>0 is the relative degree of the ith column of P^{-1}, $\mu_i=\gamma_i-\deg(P_{i+})+\deg(D_{i+})$ and $f_i=\frac{\alpha_i - D_{i+}\beta_i}{P_{i+}}$ is, by (7), a polynomial. It follows that F is a proper rational matrix and each column of F has at least one element that is not strictly proper. It also follows from (8) that A is a proper rational matrix.

Now consider the feedback system $S(P,F,A)$ shown in Fig. 2. Since PF is diagonal and C=FA, we have decomposed the decoupling controller C into a cascade connection of an open-loop decoupling precompensator F and a diagonal controller A. Note that the I/O map $H_{y_2u_1}$ of the system $S(P,F,A)$ is diag $[\frac{D_{i+}\beta_i}{\alpha_i}]$, the same as the I/O map of the system $S(P,C)$.

We show that the system $S(P,F,A)$ is also (internally) stable and thus F is an admissible decoupling precompensator for P (since the cascade connection of F and P is stabilizable). The closed-loop transfer matrix from $[u_1^T\, u_2^T\, u_3^T]^T$ to $[y_1^T\, y_2^T\, y_3^T]^T$ is given by[2]

$$H_{yu} = \begin{bmatrix} C(I+PC)^{-1} & -CP(I+CP)^{-1} & F(I+APF)^{-1} \\ PC(I+PC)^{-1} & P(I+CP)^{-1} & PF(I+APF)^{-1} \\ A(I+PC)^{-1} & -AP(I+CP)^{-1} & -APF(I+APF)^{-1} \end{bmatrix} \tag{12}$$

where we have used C=FA. The system $S(P,F,A)$ is stable if and only if the transfer matrix H_{yu} is stable (Callier and Desoer, 1982). Since $S(P,C)$ is stable, $H_{y_iu_j}$, i, j=1, 2 are stable, we thus only have to check the stability of the remaining 5 block entries. By computation,

$$H_{y_3u_1} = \text{diag}[\frac{\beta_i(s+1)^{\mu_i}P_{i+}}{\alpha_i}],$$

[2]With a slight abuse of notation we have used H_{yu} for the closed-loop transfer matrices of $S(P,C)$ and $S(P,F,A)$

$$H_{y_3u_2} = -\operatorname{diag}[\frac{\beta_i(s+1)^{\mu_i}P_{i+}}{\alpha_i}]P \,,$$

$$H_{y_2u_3} = \operatorname{diag}[\frac{D_{i+}f_i}{\alpha_i(s+1)^{\mu_i}}] \,,$$

$$H_{y_1u_3} = P^{-1}\operatorname{diag}[\frac{D_{i+}f_i}{\alpha_i(s+1)^{\mu_i}}] \,.$$

The transfer matrices $H_{y_3u_1}$ and $H_{y_2u_3}$ are stable since the polynomials α_i are Hurwitz by choice; $H_{y_3u_2}$ is stable since every unstable pole in the ith row of P is cancelled by zeros of P_{i+}; $H_{y_1u_3}$ is stable since every unstable pole in the ith column of P^{-1} is cancelled by zeros of D_{i+}; and finally $H_{y_3u_3}$=-$H_{y_2u_1}$ is also stable. We thus have the following result.

Theorem 1. The transfer matrix F defined in (10) is an admissible decoupling precompensator for P. ■

Since PF=diag$[\frac{D_{i+}}{P_{i+}(s+1)^{\mu_i}}]$, it is clear that if R is a diagonal matrix with diagonal entries $r_i \in \mathbb{R}_p(s)$ such that, for each i, r_i and $\frac{D_{i+}}{P_{i+}}$ have no pole-zero cancellations in $\mathbb{C}_+$, then

$$G=FR \tag{13}$$

is also an admissible decoupling precompensator for P. In fact every admissible decoupling precompensator for P is described by (13) for some diagonal proper R satisfying the $\mathbb{C}_+$-coprimeness condition above. Let us see why.

If R is not proper, then the transfer matrix G, defined in (13), is not proper since every column of F has at least one entry which is not strictly proper. We show below that if $\mathbb{C}_+$ pole-zero cancellations between r_i and D_{i+}/P_{i+} occur in forming G, then the cas-cade connection of G and P can not be stabilized by any diagonal controller and in view of Proposition 1 by *any* controller.

Let A:=diag$[n_i/d_i]$ be a proper diagonal controller where n_i, $d_i \in \mathbb{R}[s]$ are coprime and R:=diag$[\frac{s_i}{t_i}]$ be a proper diagonal rational matrix where s_i, $t_i \in \mathbb{R}[s]$ are coprime. Suppose that for some k, $1 \le k \le n$, there is a $\mathbb{C}_+$-cancellation between D_{k+} and t_k. Hence for some $\lambda \in \mathbb{C}_+$, $D_{k+}(\lambda)=t_k(\lambda)=0$. We show that either $H_{y_1u_3}$ or $H_{y_1u_1}$ of the system $S(P,G,A)$[3] has a $\mathbb{C}_+$-pole at λ in the kth column. Since

$$G = FR = P^{-1}\operatorname{diag}[\frac{D_{i+}s_i}{P_{i+}(s+1)^{\mu_i}t_i}] \quad \text{and } A = \operatorname{diag}[\frac{n_i}{d_i}] \,,$$

by computations,

$$H_{y_1u_3} = G(I+APG)^{-1}$$
$$= P^{-1}\operatorname{diag}[\frac{D_{i+}s_i d_i}{d_iP_{i+}(s+1)^{\mu_i}t_i + D_{i+}s_in_i}]$$
$$= P^{-1}\operatorname{diag}[D_{i+}]\operatorname{diag}[\frac{s_id_i}{d_iP_{i+}(s+1)^{\mu_i}t_i + D_{i+}s_in_i}] \tag{14}$$

Now $[d_kP_{k+}(s+1)^{\mu_k}t_k+D_{k+}s_kn_k](\lambda)=0$ since $t_k(\lambda)=D_{k+}(\lambda)=0$. By the definition of D_{i+}, we know that at least one entry in the kth column of $\{P^{-1}\text{diag}[D_{i+}]\}(\lambda)$ is nonzero. It follows from (14) that $H_{y_1u_3}$ has a pole at λ in the kth column unless $d_k(\lambda)=0$, since s_k and t_k are coprime. But if $d_k(\lambda)=0$, then $n_k(\lambda)\neq 0$ and $H_{y_1u_1}$ has a pole at λ in the kth column since

$$H_{y_1u_1} = P^{-1}\operatorname{diag}[D_{i+}]\operatorname{diag}[\frac{s_in_i}{d_iP_{i+}(s+1)^{\mu_i}t_i + D_{i+}s_in_i}]$$

Therefore the system $S(P,G,A)$ is not stable. Similar computations show that if there is a cancellation between P_{k+} and s_k at $\lambda \in \mathbb{C}_+$, then either $H_{y_2u_2}$ or $H_{y_3u_2}$ has a pole at λ in the kth row. We have thus established the following result.

Theorem 2. Assume that a decoupling controller for the plant P exists. Let F be as defined in (10). Under these conditions, $G \in \mathbb{R}_p(s)^{n\times n}$ is an admissible decoupling precompensator for P if and only if G=Fdiag$[\frac{s_i}{t_i}]$, for some s_i, $t_i \in \mathbb{R}[s]$, $1 \le i \le n$, such that
(i) s_i and P_{i+} have no common zero in $\mathbb{C}_+$;
(ii) t_i and D_{i+} have no common zero in $\mathbb{C}_+$; and
(iii) deg(s_i)≤deg(t_i) ■

Comments:

(a) Theorem 2 completely characterizes the set of all admissible decoupling precompensators by the class of proper diagonal rational matrices satisfying the $\mathbb{C}_+$-coprimeness conditions (i) and (ii).
(b) It is important to note that some decoupling precompensators may introduce additional $\mathbb{C}_+$-poles and $\mathbb{C}_+$-zeros into the system and thus further limit the achievable performance of the feedback system. Thus, in design, the decoupling precompensator should contain only $\mathbb{C}_+$-poles and $\mathbb{C}_+$-zeros that are absolutely necessary for decoupling while maintaining stabilizability.
(c) From (10) and that F^{-1}=diag$[\frac{P_{i+}(s+1)^{\mu_i}}{D_{i+}}]P$, every $\mathbb{C}_+$-pole of F is a $\mathbb{C}_+$-pole of P and every $\mathbb{C}_+$-zero of F is a $\mathbb{C}_+$-zero of P, it thus follows that the

[3]This is the feedback system shown in Fig. 2 with the precompensator F replaced by G and the diagonal controller A as defined.

$\mathbb{C}_+$-poles and $\mathbb{C}_+$-zeros of F are necessary. Thus the decoupling precompensator F may increase the multiplicities of the $\mathbb{C}_+$-poles and $\mathbb{C}_+$-zeros of the plant, and by Theorem 2 this possible increase in multiplicities is shared by all admissible decoupling precompensators. This increase in multiplicities of $\mathbb{C}_+$-zeros further limits the achievable sensitivity and is regarded as the cost of decoupling (Desoer and Gundes, 1986). The effect of these 'pinned' zeros is also discussed in Morari and Zafiriou (1989). We note that the increase in multiplicities of $\mathbb{C}_+$-poles of the plant may reduce the achievable robustness with respect to multiplicative uncertainty.

The description in Theorem 2 also allows us to give very simple necessary and sufficient conditions for the existence of stable decoupling controllers.

Theorem 3. Assume that a decoupling controller for P exists. There exists a stable decoupling controller for P if and only if

(a) The open-loop decoupling precompensator F defined in (10) is stable; and

(b) For $1 \le i \le n$, the rational functions $\frac{D_{i+}}{P_{i+}}$ satisfy the parity interlacing property (Vidyasagar, 1985).

Proof. (*Sufficiency*) Since $\frac{D_{i+}}{P_{i+}}$ satisfies the parity interlacing property, the SISO plant represented by $\frac{D_{i+}}{P_{i+}(s+1)^{\mu_i}}$ can be stabilized by a stable controller $A_i(s)$ under the unity-feedback configuration. Thus with $A(s)=\mathrm{diag}[A_i(s)]$ the system $S(P,F,A)$ is stable. Since F is stable by assumption, it follows that $C:=FA$ is a stable decoupling controller for P.

(*Necessity*) We note first that P can be decoupled by a stable controller if and only if there is a stable decoupling precompensator G so that the cascade connection of G and P is stabilized under unity-feedback configuration by the controller $A(s)=I$, the identity matrix. If F is not stable, say, F has a pole at $\lambda \in \mathbb{C}_+$ in the kth column, then necessarily $P_{k+}(\lambda)=0$ since $F=P^{-1}\mathrm{diag}[D_{i+}]\mathrm{diag}[\frac{1}{P_{i+}(s+1)^{\mu_i}}]$ and $P^{-1}\mathrm{diag}[D_{i+}]$ is analytic in $\mathbb{C}_+$. By Theorem 2 if G is an admissible decoupling precompensator then G must also have a pole at λ in the kth column. Thus there is no stable admissible decoupling precompensator for P. Hence P can not be decoupled by a stable controller.

Suppose that, for some k, $1 \le k \le n$, $\frac{D_{k+}}{P_{k+}}$ does not satisfy the parity interlacing property and the system $S(P,F,A)$ is stable, where $A(s)=\mathrm{diag}[A_i(s)]$, then $A_k^{-1}(s)$ is not stable, that is, for some $\gamma \in \mathbb{C}_+$, $A_k^{-1}(\gamma)=0$. Now consider $A^{-1}F^{-1}=\mathrm{diag}[\frac{A_i^{-1}}{D_{i+}}]\mathrm{diag}[P_{i+}(s+1)^{\mu_i}]P$. By the definition of P_{i+}, $\mathrm{diag}[P_{i+}(s+1)^{\mu_i}]P$ is analytic in $\mathbb{C}_+$; by Theorem 2, closed-loop stability implies that $D_{k+}(\gamma)\neq 0$. Thus the kth column of $(FA)^{-1}(\gamma)$ is zero, and hence the decoupling controller FA has a pole at γ and it follows that every decoupling controller is unstable.

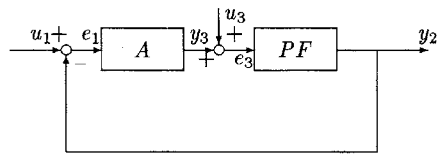

Fig. 3 Unity-feedback system $S(PF,A)$

The following Theorem shows that with decoupling precompensator F, decoupling controller design reduces to a set of SISO designs.

Theorem 4. Assume that a decoupling controller exists for P. Consider the unity-feedback system $S(PF,\overline{A})$ shown in Fig. 3, where F is defined in (10), $PF=\mathrm{diag}[\frac{D_{i+}}{P_{i+}(s+1)^{\mu_i}}]$ and $\overline{A}=\mathrm{diag}[\frac{n_i}{d_i}]$ is a diagonal controller. If $S(PF,\overline{A})$ is stable then the system $S(P,F,\overline{A})$ is stable.

Proof. Consider the feedback system $S(P,F,\overline{A})$ shown in Fig. 2 by replacing A with $\overline{A}$, where $\overline{A}=\mathrm{diag}[\frac{n_i}{d_i}]$. Stability of $S(PF,\overline{A})$ implies that the diagonal transfer matrices $H_{y_j u_i}$, i=1, 3, j=2, 3 are stable. Simple computations show that the transfer matrices $H_{y_2u_2}$, $H_{y_3u_2}$, $H_{y_1u_1}$, $H_{y_1u_3}$ are stable. Since $H_{y_2u_2}$ and $H_{y_1u_1}$ (the diagonal entries) are stable, it follows from Lemma 1 that the $\mathbb{C}_+$-poles of $H_{y_1u_2}$ form a subset of $(\mathcal{P}_+[P]\cap\mathcal{Z}_+[P])$. Since P satisfies the necessary and sufficient conditions for existence of a decoupling controller (Lin, 1998), $H_{y_1u_2}$ is analytic on $(\mathcal{P}_+[P]\cap\mathcal{Z}_+[P])$ and thus is stable. ■

Comment. Given a diagonal controller A defined in (11), we see that

$$(s+1)^{\mu_i}\beta_i D_{i+}+f_i P_{i+}(s+1)^{\mu_i}=\alpha_i(s+1)^{\mu_i} \quad (15)$$

Since $\alpha_i(s+1)^{\mu_i}$ is Hurwitz and $\deg(\alpha_i(s+1)^{\mu_i}) \ge \deg(f_i\, P_{i+}(s+1)^{\mu_i})$, A defined in (11) is a diagonal controller for PF that satisfies a Bezout identity. Thus, we can find all diagonal controllers for PF by parameterizations, and the set of all I/O maps of $S(PF,A)$ are the same as the set of all decoupled I/O maps of $S(P,C)$. Consequently, the decoupling controller design is reduced to a set of SISO designs.

IV. OPTIMAL DECOUPLING AND THE COST OF DECOUPLING

Based on the discussions following Theorem

4, decoupling controller design for P essentially reduces to design of SISO controllers for the plant $\frac{D_{i+}}{P_{i+}(s+1)^{\mu_i}}$. Thus if the criterion for optimality is defined based on the achievable diagonal sensitivity and/or diagonal I/O map (i.e. the complementary sensitivity), then optimal decoupling design reduces to a set of optimal SISO designs. To give an example, let us consider the H_∞ mixed sensitivity design problem.

Let W_1 and W_2 be stable, proper, minimum phase rational functions[4]. Consider the feedback system $S(P,C)$. The optimal design problem is to find a decoupling controller C which minimizes

$$\left\| \begin{matrix} W_1 H_{e_1 u_1} \\ W_2 H_{y_2 u_1} \end{matrix} \right\|_\infty$$

The problem is equivalent to finding stabilizing controllers $A_i(s)$ for the plant $G_i := \frac{D_{i+}}{P_{i+}(s+1)^{\mu_i}}$ so that

$$\max_i \sup_\omega \sqrt{\left| W_1(1+G_iA_i)^{-1}(j\omega)\right|^2 + \left| W_2 G_i A_i (1+G_iA_i)^{-1}(j\omega)\right|^2}$$

is minimized. Thus the optimal decoupling design problem is solved by solving n SISO optimal mixed sensitivity problems.

To make the discussions on the cost of decoupling quantitative and precise we pose the design problem as one of achieving optimal weighted sensitivity. The cost of decoupling is defined as the difference between the achievable optimal weighted sensitivity with and without the decoupling constraint. To be more precise, consider the system $S(P,C)$ shown in Fig. 1. Let $W(s)$ be a stable proper minimum-phase rational function. Let

$$J_0 := \inf\{\|W(I+PC)^{-1}\|_\infty | C \in \mathcal{C}\} \text{ and}$$

$$J_D := \inf\{\|W(I+PC)^{-1}\|_\infty | C \in \mathcal{C}_D]$$

where $\mathcal{C}$ is the set of all stabilizing controllers for P and $\mathcal{C}_D$ is the set of all decoupling controllers for P. The cost of decoupling is defined as $J := J_D - J_0$. Note that $J \geq 0$. Clearly,

$$J_D := \inf\{\|W(I+PFA)^{-1}\|_\infty | A = \text{diag}[A_i]$$

is a stabilizing controller for the plant PF}

And in view of Propositions 1 and 2,

$$J_D := \inf\{\|W(I+PFA)^{-1}\|_\infty | K$$

is a stabilizing controller for the plant PF}

Thus the cost of decoupling J is in fact the difference between the achievable optimal weighted sensitivity with the plant P and that with the precompensated (diagonal) plant PF.

Theorem 5. If decoupling precompensator F defined in (10) is stable, minimum phase and biproper, then the cost of decoupling is zero. ■

Proof. Since F and F^{-1} are proper, F and F^{-1} are both stable. Let C be any stabilizing controller for P so that $(I+PC)^{-1} = [h_{ij}]$. It can be checked that the feedback system $S(P, F, F^{-1}C)$ is internally stable and has the same I/O map and sensitivity map as the the system $S(P,C)$ has. Thus the controller $F^{-1}C$ is a stabilizing controller for the diagonal plant PF. By Proposition 1 there is a diagonal stabilizing controller for the plant PF achieving the diagonal sensitivity map diag$[h_{ii}]$. It follows from Proposition 2 that for every stabilizing controller C there corresponds a decoupling controller which yields no greater sensitivity. Thus the cost of decoupling as defined must be zero.

As noted in Section IV, the precompensator F introduce neither new $\mathbb{C}_+$-poles nor new $\mathbb{C}_+$-zeros into the cascade connection, but may increase the multiplicities of the $\mathbb{C}_+$-poles and $\mathbb{C}_+$-zeros of P. The precompensation also changes the geometry of $\mathbb{C}_+$-poles and $\mathbb{C}_+$-zeros, which is the main reason for the large cost of decoupling, especially when the poles and zeros are close in the complex plane. To give a justification of the claim, consider the plant[5] P which has a simple $\mathbb{C}_+$-pole p_0 and $\mathbb{C}_+$-zero z_0. Write

$$P(s) = \frac{R}{s-p_0} + U(s) \text{ and } P(s)^{-1} = \frac{S}{s-z_0} + V(s)$$

[4]We choose scalar weighting functions for simplicity, the conclusion holds if diagonal weighting matrices are used.

[5]The feedback systems under consideration are assumed to be stable throughout this section.

where $U(s)$ and $V(s)$ are analytic at $\mathbb{C}_+$. Let

$$\mathcal{N}_{p_0}=\text{range}(R) \text{ and } \mathcal{N}_{z_0}=\text{null space}(P(z_0)^T)$$

be respectively the *right null space* associated with pole p_0 and the *left null space* associated with zero z_0 (Boyd and Desoer, 1985). We have the following

Fact 1. $\mathcal{N}_{z_0}=\text{range}(S^T)$. ■

Proof. We have for all s, $[S+(s-z_0)V(s)]P(s)=(s-z_0)I$, in particular, $SP(z_0)=0$ and thus $P(z_0)^TS^T=0$. Therefore range$(S^T)\subset\mathcal{N}_{z_0}$. Suppose range$(S^T)\neq\mathcal{N}_{z_0}$, then there is a nonzero $v\in\mathcal{N}_{z_0}$ such that $Sv=0$. But

$$\| v \|^2 = v^TP(s)P(s)^{-1}v = v^TP(s)[\frac{S}{(s-z_0)} + V(s)]v$$

$$= v^TP(s)V(s)v = v^TP(z_0)V(z_0)v = 0$$

This contradicts v is nonzero. Thus range$(S^T)=\mathcal{N}_{z_0}$.

A lower bound of the optimal weighted sensitivity is given by the MIMO Zames and Francis Inequality (Boyd and Desoer, 1985)

$$\left\| H_{e_1u_1}W \right\|_\infty \geq \max\{\left| W(z_0) \right|, \cos\angle(\mathcal{N}_{p_0}, \mathcal{N}_{z_0})\left| \frac{z_0+\overline{p}_0}{z_0-p_0} \right| \left| W(z_0) \right|\} \tag{16}$$

where $\cos\angle(\mathcal{N}_{p_0}, \mathcal{N}_{z_0}):=\max\{|u^Tv| \,|\, u\in\mathcal{N}_{z_0}, v\in\mathcal{N}_{p_0}, \|u\|_2=\|v\|_2=1\}$ is the cosine of the angle between the spaces $\mathcal{N}_{z_0}$ and $\mathcal{N}_{p_0}$.

Now consider the precompensated plant

$$PF = \text{diag}[\frac{D_{i+}}{P_{i+}(s+1)^{\mu_i}}]. \tag{17}$$

Let R_i be the ith row of R and S_i be the ith column of S. Clearly,

$$D_{i+} = \begin{cases} 1 & \text{if } S_i = 0 \\ s-z_0 & \text{if } S_i \neq 0 \end{cases} \quad P_{i+} = \begin{cases} 1 & \text{if } R_i = 0 \\ s-p_0 & \text{if } R_i \neq 0 \end{cases} \tag{18}$$

Let $\mathcal{N}'_{p_0}$ and $\mathcal{N}'_{z_0}$ be respectively the right null space associated with p_0 and the left null space associated with z_0 for PF. Let $I_{p_0}=\{i|1\leq i\leq n, R_i\neq 0\}$, and $I_{z_0}=\{i|1\leq i\leq n, S_i\neq 0\}$.

It follows from Fact 1 that

$$\mathcal{N}'_{p_0}=\{\sum_{i\in I_{p_0}} \alpha_i e_i \,\Big|\, \alpha_i\in\mathbb{C}\} \text{ and } \mathcal{N}'_{z_0}=\{\sum_{i\in I_{z_0}} \alpha_i e_i \,\Big|\, \alpha_i\in\mathbb{C}\}$$

where e_i is the ith element of the standard basis for $\mathbb{C}^n$. Clearly,

$$\cos\angle(\mathcal{N}'_{z_0},\mathcal{N}'_{p_0}) = \begin{cases} 0 & \text{if } I_{p_0}\cap I_{z_0}=\varnothing \\ 1 & \text{if } I_{p_0}\cap I_{z_0}\neq\varnothing \end{cases}$$

It is also clear that if $I_{p_0}\cap I_{z_0}=\varnothing$, then $\cos\angle(\mathcal{N}'_{z_0},\mathcal{N}'_{p_0})=0$. In other words, if $\cos\angle(\mathcal{N}_{z_0},\mathcal{N}_{p_0})\neq 0$, then $\cos\angle(\mathcal{N}'_{z_0},\mathcal{N}'_{p_0})=1$. Note also that $\dim(\mathcal{N}'_{z_0})\geq\dim(\mathcal{N}_{z_0})$ and $\dim(\mathcal{N}'_{p_0})\geq\dim(\mathcal{N}_{p_0})$, the possible increase in multiplicities of poles and zeros. A lower bound of the optimal weighted decoupling sensitivity is given by

$$\left\| H_{e_1u_1}W \right\|_\infty \geq \max\{\left| W(z_0) \right|, \cos\angle(\mathcal{N}'_{p_0}, \mathcal{N}'_{z_0})\left| \frac{z_0+\overline{p}_0}{z_0-p_0} \right| \left| W(z_0) \right|\} \tag{20}$$

where $H_{e_1u_1}$ is diagonal. The lower bound of the optimal weighted decoupling sensitivity is larger than the lower bound of the optimal weighted sensitivity.

If the pole and zero are close, that is, $|p_0-z_0|$ is small and the subspaces $\mathcal{N}_{p_0}$ and $\mathcal{N}_{z_0}$ are nearly parallel, that is, $\cos\angle(\mathcal{N}_{p_0},\mathcal{N}_{z_0})$ is almost 1, then the cost of decoupling could be very large, since the lower bound of the weighted sensitivity is greatly increased. On the other hand, if the plant is stable then the cost of decoupling is expected to be moderate since the lower bound on the achievable weighted sensitivity, estimated by (16), does not increase as a consequence of decoupling.

Consider the plant P which has two simple $\mathbb{C}_+$-poles p_1 and p_2, and one simple $\mathbb{C}_+$-zero z_0. Write

$$P(s) = \frac{R^1}{s-p_1} + \frac{R^2}{s-p_2} + U(s) \quad \text{and}$$

$$P(s)^{-1} = \frac{S}{s-z_0} + V(s) \tag{21}$$

where $U(s)$ and $V(s)$ are analytic at $\mathbb{C}_+$. Let

$$\mathcal{N}_{p_j}=\text{range}(R^j) \text{ and } \mathcal{N}_{z_0}=\text{null space}(P(z_0)^T)$$

be the right null space associated with pole p_j, j=1, 2, and the left null space associated with zero z_0, respectively. $\mathcal{N}_{z_0}=\text{range}(S^T)$ by Fact 1.

A generalized lower bound of the optimal weighted sensitivity by the plant P and its inverse in (21) is given by the MIMO Zames and Francis Inequality (Boyd and Desoer, 1985)

$$\left\| H_{e_1u_1}W \right\|_\infty \geq \max_j\{\left| W(z_0) \right|, \cos\angle(\mathcal{N}_{p_j}, \mathcal{N}_{z_0})\left| \frac{z_0+\overline{p}_j}{z_0+p_j} \right| \left| W(z_0) \right|,$$

$$\cos\angle(\mathcal{N}_{p_1}\cap\mathcal{N}_{p_2}, \mathcal{N}_{z_0})\prod_j \left| \frac{z_0+\overline{p}_j}{z_0-p_j} \right| \left| W(z_0) \right|\} \tag{22}$$

where $\cos\angle(\mathcal{N}_{p_1}\cap\mathcal{N}_{p_2},\ \mathcal{N}_{z_0})$:=max$\{|u^Tv| |v\in \mathcal{N}_{z_0},\ u\in \mathcal{N}_{p_1}\cap\mathcal{N}_{p_2},\ \|u\|_2=\|v\|_2=1\}$.

Consider the precompensated plant given in (17). Let R_i^j be the ith row of R^j, j=1, 2 and S_i be the ith column of S. D_{i+} is defined as the same as in (18). And P_{i+} satisfies

$$R_i^j \quad \begin{cases} P_{i+}(p_j)\neq 0 & if\ R_i^j=0 \\ P_{i+}=(s-p_j)\tilde{P}_{i+} & if\ R_i^j\neq 0 \end{cases},$$

where $\tilde{P}_{i+}\in IR[s]$ and $\tilde{P}_{i+}(p_0)\neq 0$

Let $\mathcal{N}'_{p_j}$ and $\mathcal{N}'_{z_0}$ be respectively the right null space associated with p_j, j=1, 2, and the left null space associated with z_0 for PF. Let $I_{p_j}=\{i|1\le i\le n,\ R_i^j\neq 0\}$. Let

$$\mathcal{N}'_{p_j}=\left\{\sum_{i\in I_{p_j}}\alpha_i e_i \,\middle|\, \alpha_i\in \mathbb{C}\right\}$$

Clearly,

$$\cos\angle(\mathcal{N}'_{z_0},\ \mathcal{N}'_{p_j})=\begin{cases}0 & if\ I_{p_j}\cap I_{z_0}=\varnothing \\ 1 & if\ I_{p_j}\cap I_{z_0}\neq\varnothing\end{cases}$$

where $\mathcal{N}'_{z_0}$ is the same as defined in (19). It is also clear that if $I_{p_j}\cap I_{z_0}=\varnothing$, then $\cos\angle(\mathcal{N}_{z_0},\mathcal{N}_{p_j})=0$. In other words, if $\cos\angle(\mathcal{N}'_{z_0},\mathcal{N}'_{p_j})\neq 0$, then $\cos\angle(\mathcal{N}'_{z_0},\mathcal{N}'_{p_j})=1$. Note also that $\dim(\mathcal{N}'_{z_0})\ge\dim(\mathcal{N}_{z_0})$ and $\dim(\mathcal{N}'_{p_j})\ge\dim(\mathcal{N}_{p_j})$, the possible increase in multiplicities of poles and zeros. A generalized lower bound of the optimal weighted decoupling sensitivity is given by

$$\left\|H_{e_1u_1}W\right\|_\infty$$

$$\ge \max_j\{\left|W(z_0)\right|,\ \cos\angle(\mathcal{N}'_{p_j},\ \mathcal{N}'_{z_0})\left|\frac{z_0+\overline{p}_j}{z_0-p_j}\right|\left|W(z_0)\right|,$$

$$\cos\angle(\mathcal{N}'_{p_1}\cap\mathcal{N}'_{p_2},\ \mathcal{N}_{z_0})\prod_j\left|\frac{z_0+\overline{p}_j}{z_0-p_j}\right|\left|W(z_0)\right|\} \qquad (23)$$

where $H_{e_1u_1}$ is diagonal.

In summary, if the design objective is to minimize a weighted sensitivity, then for stable minimum phase plants that have a unimodular decoupling precompensator, the cost of decoupling is zero; for stable non-minimum phase plants the cost is expected to be moderate; for unstable non-minimum phase plants the cost of decoupling is, in general, high and could be very high if there are poles and zeros close in the complex plane.

The same comments apply if the design objective is to minimize a weighted complementary sensitivity for robustness, except that we expect the cost for minimum phase unstable plants to be moderate instead. Finally since Foo and Postlethwaite (1984) showed

$$\left\|\begin{matrix}W_1H_{y_2u_1}\\ W_2H_{e_1u_1}\end{matrix}\right\|_\infty \ge \frac{1}{2}\left(\left\|W_1H_{y_2u_1}\right\|_\infty+\left\|W_2H_{e_1u_1}\right\|_\infty\right)$$

the comments apply to the case of minimizing mixed sensitivity as well.

Example 1. Consider the plant (Foo and Postlethwaite, 1984)

$$P(s)=\begin{bmatrix}\dfrac{-\alpha(s-5)}{s-4} & \dfrac{s-5}{s+1}\\ \dfrac{1}{s-4} & \dfrac{1}{s+1}\end{bmatrix}$$

By computation,

$$P(s)^{-1}=\begin{bmatrix}\dfrac{-(s-4)}{(\alpha+1)(s-5)} & \dfrac{-\alpha(s-4)}{(\alpha+1)}\\ \dfrac{-(s-1)}{(\alpha+1)(s-5)} & \dfrac{-(s+1)}{(\alpha+1)}\end{bmatrix}$$

Note that $P(s)$ is nonsingular for all $\alpha>0$. Let the scalar weighting function $W(s)=(1+0.1s)/(1+s)$. The subspace $\mathcal{N}_{p_0}$, associated with the pole at p_0=4, is one dimensional and spanned by $[\alpha\ 1]^T$, the subspace $\mathcal{N}_{z_0}$, associated with the zero at z_0=5, is one dimensional and spanned by $[1\ 0]^T$ and

$$\cos\angle(\mathcal{N}_{z_0},\ \mathcal{N}_{p_0})=\frac{|\alpha|}{\sqrt{1+\alpha^2}}$$

We also have $P_{1+}=s-4$, $P_{2+}=s-4$, $D_{1+}=s-5$ and $D_{2+}=1$. Thus $\dim(\mathcal{N}'_{z_0})=\dim(\mathcal{N}_{z_0})=1$ and $2=\dim(\mathcal{N}'_{p_0})>\dim(\mathcal{N}_{p_0})=1$. Without decoupling,

$$\left\|WH_{e_1u_1}\right\|_\infty\ge\max\{0.25,\ 2.25\frac{|\alpha|}{\sqrt{1+\alpha^2}}\}$$

and with decoupling, $\|WH_{e_1u_1}\|_\infty\ge 2.25$. Thus the cost of decoupling increases as α decreases. Decoupling has the effect of aligning the null space associated with unstable poles and zeros.

Example 2. Consider the plant

$$P(s)=\begin{bmatrix}\dfrac{1}{s+1} & 0 & \dfrac{1}{s+3}\\ \dfrac{s-5}{s+1} & \dfrac{s-5}{s-4} & 0\\ 0 & \dfrac{1}{s-4} & \dfrac{1}{s+3}\end{bmatrix}$$

By computation,

$$P(s)=\begin{bmatrix} \frac{1}{s+1} & 0 & \frac{1}{s+3} \\ \frac{s-5}{s+1} & \frac{s-5}{s-4} & 0 \\ 0 & \frac{1}{s-4} & \frac{1}{s+3} \end{bmatrix}$$

Let the scalar weighting function $W(s)=(s+200)/100(s+2)$. The subspace $\mathcal{N}_{p_0}$, associated with the pole at $p_0=4$, is spanned by $[0\ \text{-}1\ 1]^T$. The subspace $\mathcal{N}_{z_0}$, associated with the zero at $z_0=5$, is spanned by $[0\ 1\ 0]^T$. $W(z_0)=0.2929$. And $\cos\angle(\mathcal{N}_{z_0}, \mathcal{N}_{p_0})=\frac{1}{\sqrt{2}}$. We have $P_{1+}=1$, $P_{2+}=s-4$, $P_{3+}=s-4$, $D_{1+}=1$, $D_{2+}=s-5$ and $D_{3+}=1$. $1=\dim(\mathcal{N}'_{z_0})=\dim(\mathcal{N}_{z_0})=1$ and $2=\dim(\mathcal{N}'_{p_0})>\dim(\mathcal{N}_{p_0})=1$. Without decoupling,

$$\| WH_{e_1u_1}\|_\infty \geq \cos\angle(\mathcal{N}_{p_0}, \mathcal{N}_{z_0})\left|\frac{z_0+\overline{p}_0}{z_0-p_0}\right| |W(z_0)|$$

$$=\frac{9}{\sqrt{2}}|W(z_0)| = 1.8637$$

and with decoupling,

$$\| WH_{e_1u_1}\|_\infty \geq \left|\frac{5+4}{5-4}\right| |W(z_0)| = 9|W(z_0)| = 2.6357$$

In Matlab simulation, the optimal weighted sensitivity without decoupling is 1.8761; and the optimal weighted sensitivity with decoupling is 2.6358.

Example 3. Consider the plant

$$P(s)=\begin{bmatrix} \frac{s+3}{s-2} & -\frac{2(s+4)}{s-3} & 0 \\ \frac{s+3}{s-2} & \frac{3(s+4)}{s-3} & -\frac{4(s-4)}{s+5} \\ -\frac{s+3}{s-2} & \frac{s+4}{s-3} & \frac{s-4}{s+5} \end{bmatrix}$$

By computation,

$$P(s)^{-1}=\begin{bmatrix} \frac{7(s-2)}{s+3} & \frac{2(s-2)}{s+3} & \frac{8(s-2)}{s+3} \\ \frac{3(s-3)}{s+4} & \frac{s-3}{s+4} & \frac{4(s-3)}{s+4} \\ \frac{4(s+5)}{s-4} & \frac{s+5}{s-4} & \frac{5(s+5)}{s-4} \end{bmatrix}$$

Let the scalar weighting function $W(s)=(s+200)/100(s+2)$. The respective subspaces $\mathcal{N}_{p_j}$, $j=1, 2$, associated with the poles at $p_1=2$ and at $p_2=3$, are spanned by $[1\ 1\ \text{-}1]^T$ and $[\text{-}2\ 3\ 1]^T$. The subspace $\mathcal{N}_{z_0}$, associated with the zero at $z_0=4$, is spanned by $[4\ 1\ 5]^T$. The subspaces $\mathcal{N}_{p_j}$, $j=1, 2$, are orthogonal with the subspace $\mathcal{N}_{z_0}$, thus $\cos\angle(\mathcal{N}_{z_0},\mathcal{N}_{p_j})=0$, $j=1, 2$. We have $P_{1+}=(s-2)(s-3)$, $P_{2+}=(s-2)(s-3)$, $P_{3+}=(s-2)(s-3)$, $D_{1+}=s-4$, $D_{2+}=s-4$ and $D_{3+}=s-4$. $3=\dim(\mathcal{N}'_{z_0})>\dim(\mathcal{N}_{z_0})=1$ and $3=\dim(\mathcal{N}'_{p_j})>\dim(\mathcal{N}_{p_j})=1$, $j=1, 2$. Without decoupling,

$$\| WH_{e_1u_1}\|_\infty \geq |W(z_0)| = 0.34$$

and with decoupling,

$$\| WH_{e_1u_1}\|_\infty \geq \left|\frac{4+2}{4-2}\right|\left|\frac{4+3}{4-3}\right| |W(z_0)| = 21|W(z_0)| = 7.14$$

In Matlab simulation, the optimal weighted sensitivity without decoupling is 0.34; and the optimal weighted sensitivity with decoupling is 7.14.

V. CONCLUSIONS

In this paper, we give a simple description of admissible decoupling precompensators. The description provides an alternative to independently designing each decoupled I/O channel by designing an SISO controller for each ‘equivalent SISO plant’ and in particular to the design of optimal decoupling controllers. The discussion on the cost of decoupling provides useful information to determine whether decoupling is worthwhile. We note however that there are other considerations when deciding whether a decoupling controller should be used at all (Morari and Zafiriou, 1989).

NOMENCLATURE

$\mathbb{C}$	the field of complex numbers
$\mathbb{C}_-$	$\{s\in\mathbb{C}\mid Re(s)<0\}$; equiv. the open left half of the complex plane
$\mathbb{C}_+$	$\{s\in\mathbb{C}\mid Re(s)\geq 0\}$; equiv. the closed right half of the complex plane
$\mathbb{R}$	the field of real numbers
$\mathbb{R}[s]$	the ring of polynomials in s with real coefficients
$\mathbb{R}(s)$	the field of rational functions in s with real coefficients
$\mathbb{R}_p(s)$, $\mathbb{R}_{po}(s)$	the ring of proper, resp. strictly proper, rational functions in s with real coefficients
$\deg(f/g)$	$\deg(f)-\deg(g)$ for $f,g\in\mathbb{R}[s]$
$\deg(v(s))$	the largest relative degree of $v_i(s)$, $1\leq i\leq n$, if $v(s)=[v_1(s)\ \dots\ v_n(s)]^T\in\mathbb{R}(s)^n$
$\text{diag}[h_i]$	the $n\times n$ matrix with h_i as its ith

	diagonal element
$f\|g$	f divides g; equiv. $g=fh$ for some $h\in \mathbb{R}[s]$
$\mathcal{P}[H]$	the set of all poles of H in $\mathbb{C}$ for $H(s)\in \mathbb{R}(s)$
$\mathcal{P}_+[H]$	the set of all poles of H in $\mathbb{C}_+$
$\mathcal{Z}[H]$	the set of all zeros of H in $\mathbb{C}$
$\mathcal{Z}_+[H]$	the set of all zeros of H in $\mathbb{C}_+$
$\|\|H\|\|_\infty$	$\sup_{\omega\in IR}\overline{\sigma}(H(j\omega))$, H is a stable rational matrix

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解耦預先補償與最佳解耦

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摘　要

在線性多變數控制系統的單一回授系統的架構下，本文將探討解耦控制。我們建立所有可行解耦補償器的參數化解的集合，利用這個解耦補償器的參數化解，我們可以証明多變數解耦控制器的設計是等於對多個單一輸入單一輸出的控制器設計。這個參數化解也可用來建立穩定解耦控器存在的充分與必要條件。我們也提出最佳的解耦控制器設計並探討解耦控制所必須付出的代價。

關鍵詞：可行解耦補償器，最佳解耦，代價。