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Dressing Effect on the Electron-Atom Scattering in Intense Laser Field

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The dressing effect on the electron-atom scattering in an intense laser field is investigated by solving the time-dependent Schrödinger equation in momentum space. It is found that the result of previous works in the literature is only a limiting case of ours. Our method gives a general formulation for the discussion of the dressing effect during the scattering process.

I. INTRODUCTION

Charged particle-atom scattering in the presence of radiation field is a fundamental process in many physical systems such as plasma heating by electro-magnetic radiation, gas breakdown, etc. During recent years, the availability of increasingly more powerful lasers in a wide range of frequencies has stimulated considerable interest in the study of multiphoton phenomena in such a process. Following the pioneer work of Kroll and Watson [1], most of the theoretical investigations on the laser-assisted electron-atom scattering are based on the perturbative method [2-4]. In the recent papers [5,6], we have formulated a non-perturbative method to study this problem by solving the Schrodinger equation in momentum space and it is found that the dynamics of the multiphoton process during the scattering can be well understood in this method. However, in these previous works we have simplified the formulation by assuming that the atomic target is structureless so that it can be considered to be unaffected by the laser field. It is obvious that this simplification is not valid when the intensity of the field is strong, or when the laser frequency is nearly resonant with that of an atomic transition. This "dressing" of atomic target state by the laser field for the elastic scattering have been recently studied by several authors [7-9] and some important modifications on the differential cross section are found. The extension of the above calculations to the case of inelastic collision was further studied by Francken et al [10]. The electron-atom scattering in a resonant laser field has also been investigated by

Unnikrishnan [11]. However, in all these calculations the perturbative method is used to study the problem and the atom-field interaction is treated by the first-order perturbation theory. In this paper we shall extend our non-perturbative method [6] to take into account the atomic degree of freedom and the dressing effect on the scattering process will be discussed.

II. FORMULATION

In our problem, the unbound electron, denoted as e(l), moving in the radiation field of a vector potential $\vec{A}(t)$ is incident on the atom with a bound electron, e(2). The Hamiltonian for the system can be written as

$$H = H_e + H_T + V \tag{1}$$

where H_e is for the unbound electron in the laser field,

$$H_e = \frac{p_1^2}{2m} + V_{ef} \tag{2}$$

with the electron-field interaction V_{ef} given as

$$V_{ef} = -\frac{e}{mc}\vec{A}(t) \cdot \vec{p}_1 + \frac{e^2 A^2}{2mc^2}. \tag{3}$$

H_T is the Hamiltonian for the atomic target in the laser field,

$$H_T = \frac{p_2^2}{2m} - \frac{Ze^2}{r_2} + V_{Tf} \tag{4}$$

with the target-field interaction V_{Tf} given as

$$V_{Tf} = -\frac{e}{mc}\vec{A}(t) \cdot \vec{p}_2 + \frac{e^2 A^2}{2mc^2}. \tag{5}$$

V is the unbound electron-atom interaction given by

$$V = -\frac{Ze^2}{r_1} + \frac{e^2}{|\vec{r}_1 - \vec{r}_2|} \tag{6}$$

As treated in Ref. 6, the $A^2(t)$ terms in Eqs. (3) and (5) can be removed by a trivial contact transformation. We first consider the solution for the Hamiltonian H_e, which is just a free electron scattering in the radiation field. The solution to the time-dependent Schrödinger equation was originally derived by Volkov [12] and can be written in the following form

$$H_e|\chi_{\vec{k}}(\vec{r}_1,t)>= i\hbar\frac{\partial}{\partial t}|\chi_{\vec{k}}(\vec{r}_1,t)>, \tag{7}$$

$$|\chi_k(\vec{r}_1,t)>=\exp\{-i[E_i t\hbar + \theta_{\vec{k}}(t)]\}|\vec{k}> \tag{8}$$

where E_i is the initial energy, $|k>$ is the eigenvector of $\vec{p}$ with the momentum eigenvalue $\hbar\vec{k}$ normalized as

$$\varphi_{\vec{k}}(\vec{r}_1) =< \vec{r}_1|\vec{k}>= (2\pi)^{-3/2}e^{i\vec{k}\cdot\vec{r}_1} \tag{9}$$

and the real phase $\theta_{\vec{k}}(t)$ is given by

$$\theta_{\vec{k}}(t) = \frac{1}{\hbar}\int_0^t dt'\left[\frac{\hbar^2k^2}{2m} - E_i - \frac{e}{mc}\vec{p}\cdot\vec{A}(t')\right] \tag{10}$$

Assuming a monochromatic linearly polarized laser field in dipole approximation and working in the Coulomb gauge, ie,

$$\begin{aligned}\vec{\mathcal{E}}(t) &= \vec{\mathcal{E}}_o \text{ sinwt}\\ A(t) &= \vec{a}\cos wt\end{aligned} \tag{11}$$

with $\vec{a} = -\frac{C\vec{\mathcal{E}}_o}{w}$, we have

$$\chi_{\vec{k}}(\vec{r}_1,t)= (2\pi)^{-3/2}\exp\{-i(\vec{k}\cdot\vec{r}_1 - \vec{k}\cdot\vec{\alpha}_o \text{ sinwt} - E_k t/\hbar)\} \tag{12}$$

where $\vec{\alpha}_o = \frac{e\vec{\mathcal{E}}_o}{mw^2}$ and $E_k = \frac{\hbar^2k^2}{2m}$.

For H_T, which describes the “dressed state” of the atomic target in the presence of the laser field,

$$H_T\Phi(\vec{r}_2,t)= i\hbar\frac{\partial}{\partial t}\Phi(\vec{r}_2,t) \tag{13}$$

This problem has been recently studied by Bhattacharya [13] using the parabolic coordinate representation. If the field with the electric field strength is much less than the atomic unit of field, namely, $\varepsilon_o << 5 \times 10^9 V/cm$, the dressed wave function for the hydrogen atom can also be obtained by using first-order perturbation theory [14] as

$$\begin{aligned}\Phi_m(\vec{r}_2,t) \;=\; & \exp(-iw_m t)\exp(-i\vec{a}\cdot\vec{r}_2)\Bigg[\psi_m(\vec{r}_2)\\ & - \sin wt\sum_{m_1}\frac{w_{m_1m}M_{m_1m}}{\hbar(w^2_{m_1m} - w^2)}\psi_{m_1}(\vec{r}_2)\\ & - i\cos wt\sum_{m_1}\frac{wM_{m_1m}}{\hbar(w^2_{m_1m} - w^2)}\psi_{m_1}(\vec{r}_2)\Bigg],\end{aligned} \tag{14}$$

where $\psi_m(\vec{r}_2)$ is the unperturbed atomic wave function with energy $E_m = \hbar w_m$ and $w_{mm'} = w_m - w_{m'}$. $M_{mm'}$ is the dipole matrix element given as

$$M_{mm'} = \vec{\varepsilon}_o \cdot < \psi_m | e\vec{r}_2 | \psi_{m'} > . \tag{15}$$

We shall use the wave function of the "unperturbed" system $H_o = H_e + H_T$ as our basis. Initially, the incident electron with incoming momentum $\vec{k}_i$ is in the state $\chi_{\vec{k}_i}(\vec{r}_1, t)$ and the atomic target in the state $\Phi_o(\vec{r}_2, t)$. The solution for the total Hamiltonian H in Eq. (1) can be written as

$$|\Psi(t)> = |\chi_{\vec{k}_i}(\vec{r}_1, t)\Phi_o(\vec{r}_2, t)> + |\phi(t)> . \tag{16}$$

Expand $|\phi(t)>$ in terms of the basis states,

$$|\phi(t)> = \sum_m \int d\vec{k} a_m(\vec{k}, t)\chi_{\vec{k}}(\vec{r}_1, t)\Phi_m(\vec{r}_2, t) \tag{17}$$

with the boundary condition $|\Psi(t)> \rightarrow |\chi_{\vec{k}_i}(\vec{r}_1, t)\Phi_o(\vec{r}_2, t)>$ as $t \rightarrow -\infty$, ie, $|\phi(-\infty)> \rightarrow 0$. Substituting Eqs. (16) and (17) into the Schrödinger equation $H|\Psi> = i\hbar\frac{\partial}{\partial t}|\Psi>$, an inhomogeneous integro-differential equation can be obtained for the coefficient $a_m(\vec{k}, t)$.

$$\begin{aligned} i\hbar\frac{d}{dt}a_m(\vec{k}, t) = \; & e^{i\theta_{\vec{k}}(t)}\Big[e^{-i\theta_{\vec{k}_i}(t)} < \vec{k}, \Phi_m(\vec{r}_2, t)|V|\vec{k}_i, \Phi_o(\vec{r}_2, t) > \\ & + b_m(\vec{k}, t)\Big] \end{aligned} \tag{18}$$

with

$$b_m(\vec{k}, t) = \sum_{m'} \int d\vec{k}' a_{m'}(\vec{k}', t) e^{-i\theta_{\vec{k}'}(t)} < \vec{k}, \Phi_m(\vec{r}_2, t)|V|\vec{k}', \Phi_{m'}(\vec{r}_2, t) > . \tag{19}$$

Here $a_m(\vec{k}, t)$ represents the scattering amplitude for the electron scattered with momentum $\hbar\vec{k}$ and the atomic target in the dressed state Φ_m. It can be seen that the dressing effect is clearly manifest in Eq. (18), in which the atomic target is excited from the initial state Φ_o to the final state Φ_m during the scattering of electron in the laser field. Moreover, the function $b_m(\vec{k}, t)$ on the right hand side of Eq. (18) gives the various contributions due to the intermediate atomic states during the scattering process as given in the summation terms in Eq. (19). We discuss several interesting points as follows.

It has been shown that [5,15], because of the phase factor $\exp[i\theta_{\vec{k}}(t)]$ on the right hand side of Eq. (18), the function $a_m(\vec{k}, t)$ varies rapidly with both k and t. On the other hand, $b_{\vec{k}}(t)$ varies relatively slowly with $\vec{k}$ and t. Thus, we can first ignore the effect of the slowly varying function $b_{\vec{k}}(t)$ and Eq. (18) becomes

$$i\hbar\frac{d}{dt}a_m(\vec{k},t) = e^{i\theta_{\vec{k}}(t)}e^{-i\theta_{\vec{k}_i}(t)} < \vec{k}, \Phi_m(\vec{r}_2,t)|V|\vec{k}_i, \Phi_o(\vec{r}_2,t) >$$

Using Eq. (8), the above equation gives

$$i\hbar\frac{d}{dt}a_m(\vec{k},t) = < \chi_{\vec{k}}(\vec{r}_1,t)\Phi_m(\vec{r}_2,t)|V|\chi_{\vec{k}_i}(\vec{r}_1,t)\Phi_o(\vec{r}_2,t) > \tag{20}$$

This result reduces to that of the previous work by Francken and Joachain (Eq. (7) of Ref. 8). Therefore, their result is just the limiting case of ours by putting $b_m(\vec{k},t)= 0$. This is, of course, an oversimplification. The correct treatment must also take into account the effect of $b_m(\vec{k},t)$, which we discuss in the following.

Eq. (18) can be rewritten as

$$\begin{aligned} i\hbar\frac{d}{dt}a_m(\vec{k},t) = \; & e^{i[\theta_{\vec{k}}(t)-\theta_{\vec{k}_i}(t)]} < \vec{k}, \Phi_m|V|\vec{k}_i, \Phi_o > \\ & + \sum_{m'} \int d\vec{k}' a_{m'}(\vec{k}',t) e^{i[\theta_{\vec{k}}(t)-\theta_{\vec{k}_i}(t)]} e^{-i[\theta_{\vec{k}'}(t)-\theta_{\vec{k}_i}(t)]} \\ & < \vec{k}, \Phi_m|V|\vec{k}', \Phi_{m'} > \end{aligned} \tag{21}$$

For a linearly polarized light given as in Eq. (1 1), we have

$$\begin{aligned} \theta_{\vec{k}}(t) = \; & \frac{1}{\hbar}\int_0^t dt'\left[\frac{\hbar^2k^2}{2m} - E_i - \frac{e\hbar}{mc}\vec{k}\cdot\vec{a}\cos wt'\right] \\ = \; & \frac{1}{\hbar}(E_k - E_i)t - \frac{e\hbar}{mcw}(\vec{k}\cdot\vec{a})\sin wt \end{aligned} \tag{22}$$

and

$$e^{i[\theta_{\vec{k}}(t)-\theta_{\vec{k}_i}(t)]} = e^{i(E_k-E_i)t/\hbar}e^{-i\xi_k \sin wt} \tag{23}$$

with

$$\xi_k = (\vec{k}-\vec{k}_i)\cdot\vec{\alpha}_o \tag{24}$$

Making use of the Fourier-Bessel expansion

$$e^{-i\xi_k \sin wt} = \sum_{n=-\infty}^{\infty} J_n(\xi_k)e^{-inwt} \tag{25}$$

and substituting Eqs. (23) into Eq. (21), we obtain

$$
\begin{aligned}
i\hbar\frac{d}{dt}a_m(\vec{k},t) = \; & e^{i(E_k-E_{ki})t/\hbar}\sum_n J_n(\xi_k)e^{-inwt} \\
& \times\int \varphi^*_{\vec{k}}(\vec{r}_1)\Phi^*_m(\vec{r}_2,t)V\varphi_{\vec{k}_i}(\vec{r}_1)\Phi_o(\vec{r}_2,t)d\vec{r}_1 d\vec{r}_2 \\
& +\sum_{m'}\int d\vec{k}' a_{m'}(\vec{k},t)e^{i(E_k-E_{k'})t/\hbar}\sum_n J_n(\xi_k)e^{-inwt}\sum_\ell J_\ell(\xi_{k'})e^{i\ell wt} \\
& \times\int \varphi^*_{\vec{k}}(\vec{r}_1)\Phi^*_m(\vec{r}_2,t)V\varphi_{\vec{k}'}(\vec{r}_1)\Phi_{m'}(\vec{r}_2,t)d\vec{r}_1 d\vec{r}_2
\end{aligned}
$$

Substituting Eq. (14) into the above equation, finally we obtain the scattering amplitude of the electron,

$$
\begin{aligned}
a_m(\vec{k},t) = \; & -\frac{i}{\hbar}\int_{-\infty}^{t} dt'\Bigg\{\sum_n e^{i(E_k-E_{k_i}+\hbar w_{mo}-n\hbar w)t'/\hbar}A_n(k_i0;km) \\
& +\sum_{m'}\sum_n\sum_\ell\int d\vec{k}' J_n(\xi_k)e^{i[E_k-E_{k'}+\hbar w_{mm'}-(n-\ell)\hbar w]t'/\hbar} \\
& \times a_{m'}(\vec{k}',t')A_\ell(k'm';km)\Bigg\}
\end{aligned}
\tag{26}
$$

where

$$
\begin{aligned}
A_\ell(k'm';km) = \; & J_\ell(\xi_{k'}) < \vec{k},\psi_m(\vec{r}_2)|V|\vec{k}',\psi_{m'}(\vec{r}_2) > \\
& +iJ'_\ell(\xi_{k'})\Bigg[\sum_{m_1}\frac{w_{m_1m'}M_{m_1m'}}{\hbar(w^2_{m_1m'}-w^2)} < \vec{k},\psi_m(\vec{r}_2)|V|\vec{k}',\psi_{m_1}(\vec{r}_2) > \\
& +\sum_{m_1}\frac{w_{m_1m}M_{m_1m}}{\hbar(w^2_{m_1m}-w^2)} < \vec{k},\psi_m(\vec{r}_2)|V|\vec{k}',\psi_{m'}(\vec{r}_2) > \Bigg] \\
& -\frac{i\ell}{\xi_{k'}}J_\ell(\xi_{k'})\Bigg[\sum_{m_1}\frac{wM_{m_1m'}}{\hbar(w^2_{m_1m'}-w^2)} < \vec{k},\psi_m(\vec{r}_2)|V|\vec{k}',\psi_{m_1}(\vec{r}_2) > \\
& +\sum_{m_1}\frac{wM_{m_1m}}{\hbar(w^2_{m_1m}-w^2)} < \vec{k},\psi_m(\vec{r}_2)|V|\vec{k}',\psi_{m'}(\vec{r}_2) > \Bigg]
\end{aligned}
\tag{27}
$$

If we first neglect the second term on the right-hand side of Eq. (26) and after carrying out the time integration, Eq. (26) reduces to the form

$$
a_m(\vec{k},\infty) = i\pi\sum_n \delta(E_k - E_{k_i} + \hbar w_{mo} - n\hbar w)A_n(k_i0;km) \tag{28}
$$

The δ function in Eq. (28) expresses that the incoming electron with momentum $\hbar\vec{k}_i$ is undergoing the scattering process with n photons absorbed and $\hbar w_{mo}$ the atomic transition energy from the initial state Φ_o to the final state Φ_m so that the outgoing electron with momentum $\hbar\vec{k}$ would have energy

$$E_k = E_{k_i} - \hbar w_{mo} + n\hbar w$$

This expression is just the conservation of energy for the process. The main contribution of the dressing effect of the atomic target comes from the second term on the right-hand side of Eq. (26). First we note that the quantity $A_\ell(k'm'; km)$ gives the amplitude for the process in which the atomic target transits from the states $m' - m$ while ℓ photons are exchanged. The summations in each set of parentheses in Eq. (27) give the contributions from various intermediate states of the target. It is also clear to see from Eq. (26) that $a_{m'}(\vec{k}', t')$ represents the scattering amplitude at time t' with electron momentum $\hbar\vec{k}'$ after having absorbed ℓ photons and the atomic target having made the transition from the initial state Φ_o to the intermediate state m' so that the electron energy is given by $E_{k'} = E_{k_i} - \hbar w_{m'o} + \ell\hbar w$. Then it reabsorbs $(n - \ell)$ photons and the atomic target transits from the state m' to the final state m. The net result is that at time t, the electron has absorbed n photons and the atom has made transition from the initial state Φ_o to the final state Φ_m and the scattering amplitude is represented by $a_m(\vec{k}, t)$. In this way, the "dressing" of the atomic target is naturally involved in the scattering process through the amplitude $A_\ell(k'm'; km)$. We can also see that our method is a general extension of the previous works and the dressing effect on electron-atom scattering in an intense field is clearly described in our formulation.

It is worth to note that the effect of the function $b_m(\vec{k}, t)$ has also been investigated recently for the case of the above-threshold ionization (ATI) [16]. It was found that the dramatic suppression of the low-energy peaks and the "peak switching" effect in AT1 multiphoton process can be well explained when the function $b_m(\vec{k}, t)$ is included. For the case of scattering process studied here, the function $b_m(\vec{k}, t)$ again plays an important role with the dressing effect clearly manifested through the scattering amplitude $A_\ell(k'm'; km)$ as in Eq. (26). Since our method is analytic and non-perturbative in nature, we believe that the inclusion of the function $b_m(\vec{k}, t)$ will give significant modifications on the differential cross sections obtained by the previous works. Our method also provides a suitable way to compare those approximations [1,8] commonly used in the literature and the regions of validity of these approximations can also be investigated. Furthermore, the scattering process for the non-resonance case has been extensively studied so far, but the resonance scattering processes are less understood and only studied either by using the expansion in the power series of w [17] or using the two-level model of the rotating-wave approximation [11,18]. Recently, it is also found in experiment [19] that some substructures are shown in the curve of ionization signal peaks. It is believed that these substructures come from the resonance effect due to the atom. Our method should be an appropriate approach to study these resonance phenomena. Detailed analysis is in progress and will be reported later.

III. CONCLUSION

We have studied the dressing effect on the scattering of electron by atoms in the presence of intense laser field by using a non-perturbative method to solve the time-dependent Schrödinger equation in momentum space. It is found that the result of previous works in the literature is only a limiting case of ours by neglecting the slowly varying function $b_m(\vec{k},t)$. As the function $b_m(\vec{k},t)$ is included, our method gives a general formulation for the discussion of the dressing effect on the electron-atom scattering in the intense field and the dressing effect on the scattering process is clearly manifested in this formulation.

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