Observations of multipass transverse modes in an axially pumped solid-state laser with different fractionally degenerate resonator configurations

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We experimentally show, for the first time to our knowledge, that a diode-pumped Nd:YVO_4 laser can operate with multipass transverse (MPT) modes that self-reproduce after several round trips in a plano–concave cavity that has fractionally degenerate resonator configurations when the pump beam waist is sufficiently smaller than that of the fundamental cavity mode. The MPT mode is found to exhibit multiple beam waists located at different positions and to experience a lower pumping threshold than the single-pass transverse mode. With off-axis pumping, the N-pass transverse mode forms a symmetric pattern for even N and an asymmetric pattern for odd N. This result can be explained as being due to the introduction of MPT modes but not to superposition of the standard cavity modes. © 2001 Optical Society of America

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1. INTRODUCTION

Lasers with various geometrically stable resonators can usually support definite transverse modes, such as Hermite–Gaussian and Laguerre–Gaussian modes, which reproduce themselves after a round trip and hence are also referred to as single-pass transverse (SPT) modes. However, the round-trip ray matrix T of a confocal resonator is easily determined to be $T = -I$, and $T^2 = I$ is the identity matrix. Thus any starting field distribution that has inverse symmetry with respect to the optic axis of the resonator will be exactly reproduced after a round trip and any arbitrary ray will return to its initial position and direction after two round trips. This resonator is referred to as a self-imaging, 1/2-degenerate system[1] and is capable of supporting a more-or-less arbitrary transverse beam pattern even if it significantly deviates from the standard Gaussian profile. This property has been applied for optically synthesizing various laser waveforms[2–5] and for optimizing the extraction efficiency of solid-state lasers.[6,7]

By ray analysis of the resonators, however, a paraxial resonance equation[8] yields the mirror separations of a two-mirror cavity in which any arbitrary rays repeat themselves after an integer number (say N) of return transits. It has been argued that a set of paraxial closed ray paths that is complete in N round trips might also be regarded as a mode of the resonator. We describe in this paper how an axially pumped solid-state laser that consists of a two-mirror cavity with fractionally (K/N) degenerate resonator configurations in which the paraxial resonance equation is satisfied may select an unusual transverse mode rather than the standard cavity mode observed at a slightly different mirror separation. By investigating the effects of off-axis pump on the laser with these degenerate resonator configurations, we found that a symmetric pattern forms for even N and an asymmetric pattern forms for odd N. These results may be accounted for simply by the introduction of multipass transverse (MPT) modes[9] that self-reproduce after several round trips in terms of the ray matrix analysis but not by the superposition of standard cavity modes. To demonstrate the MPT modes in a straightforward way, we observed the output laser beam after it propagated through a lens by translating a beam profiler along the optical axis. We found as usual a single beam waist for the SPT mode but multiple waists for the MPT modes. Inasmuch as the MPT mode is capable of adjusting itself for better overlap with the pump beam, a lower pump threshold can be attained when the waist size of the pump beam is sufficiently smaller than that of the fundamental cavity mode. We demonstrated this theoretically, based on spatially dependent rate equation analysis, and experimentally with a diode-pumped Nd:YVO_4 laser that has a plano–concave cavity.

2. FRACTIONALLY DEGENERATE RESONATOR CONFIGURATIONS

Consider a resonator formed by two mirrors with radii of curvature R_1 and R_2 separated by some lossless optical elements with the transfer matrix

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix},$$

and let the reference plane be the optical ray just leaving mirror 1. The round-trip ray matrix T is

$$T = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 2aG_2 - 1 & 2bG_2 \\ 2(2aG_1G_2 - G_1 - a^2G_2)/b & 4G_1G_2 - 2aG_2 - 1 \end{bmatrix}, \quad (1)$$

with G parameters $G_1 = a - b/R_1$ and $G_2 = d - b/R_2$. The ray matrix after N round trips is obtained from Sylvester's theorem[8]:

$$T^N = \begin{bmatrix} A & B \\ C & D \end{bmatrix}^N = \frac{1}{\sin\theta}\begin{bmatrix} A\sin N\theta - \sin(N-1)\theta & B\sin\theta \\ C\sin N\theta & D\sin N\theta - \sin(N-1)\theta \end{bmatrix}, \quad (2)$$

with $\cos\theta = (A + D)/2 = 2G_1G_2 - 1$. If the condition $\theta = 2\pi K/N$ is satisfied, where N and K are integers with $0 \leqslant K \leqslant N/2$ to prevent duplicate values for $\cos\theta$, then we obtain $T^N = I$. A resonator with G_1G_2 parameters satisfying this condition will be referred to as the cavity with the fractionally (K/N) degenerate resonator configuration. MPT modes that self-reproduce after N round trips may therefore exist in addition to the SPT modes in this resonator. When N is an even number, the MPT mode forms an inverse symmetry beam pattern with respect to the optic axis in the resonator, as asserted by the relation $T^{N/2} = -I$, which follows from Eq. (2). In a plano–concave cavity with $R_1 = \infty$, the K/N-degenerate resonator configurations have cavity lengths L given by $L = (R_2/2)[1 - \cos(2\pi K/N)]$, which was referred to in Ref. 8 as a paraxial resonance equation for the case of a plano–concave cavity. In addition, the resonant frequency of the TEM$_{qmn}$ mode can be given by $\nu_{qmn} = (c/2L)[q + (1 + m + n)(1/\pi)\cos^{-1}(G_1G_2)^{1/2}] = q(c/2L) + (1 + m + n)(\theta/2\pi)(c/2L) = q(\Delta\nu_l) + (n + m + 1)(\Delta\nu_t)$, where m and n are transverse mode indices, q is the axial mode index, $\Delta\nu_l$ is the axial mode spacing, and $\Delta\nu_t$ is the transverse mode spacing. It is interesting to note that, for the fractionally degenerate resonator configurations, $\Delta\nu_l/\Delta\nu_t = 2\pi/\theta = N/K$, this is also referred to the transverse mode degeneracy at which the cavity mode comprises N sets of degenerate transverse modes.

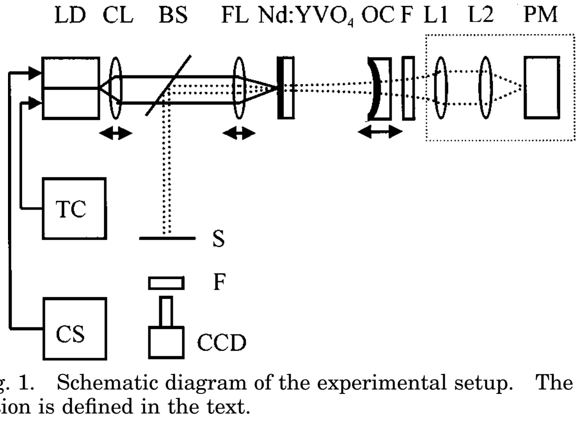

Fig. 1. Schematic diagram of the experimental setup. The notation is defined in the text.

3. EXPERIMENTAL SETUP

A schematic of the experimental setup is shown in Fig. 1, where a diode-pumped Nd:YVO$_4$ laser with a plano–concave cavity is used. A 1-mm-thick Nd:YVO$_4$ laser crystal that was coated at its surface facing the pumping beam for less than 5% reflection at 808 nm and greater than 99.8% reflection at 1064 nm was used as a flat end mirror. The second surface of the crystal was antireflection coated at 1064 nm. The output coupler (OC) is a concave mirror with radius of curvature $R = 80$ mm and 90% reflection at 1064 nm. It was mounted upon a translatable stage to facilitate continuous adjustment of the cavity length and consequently determination of the resonator configuration. The pump beam from a commercially available laser diode (LD) with a microlens to reduce perpendicular divergence was collimated by an objective lens (CL) with a numerical aperture of 0.47 and focused onto the Nd:YVO$_4$ crystal by another objective lens (FL) with a focal length of 12 mm. This arrangement yielded a pump beam waist located close to the flat end mirror with a diameter of ~14 μm in the vertical direction and ~25 μm in the horizontal direction. Throughout the experiments the waist size of the pump beam was much less than that of the fundamental cavity beam. We kept the laser diode operating at a constant current and temperature by a using a current source (CS) and a temperature controller (TC) to ensure stable pumping wavelength and power. Because the wavelength of the laser diode is 806.62 nm at an operating power of 1 W, higher output power will lead to a shift of pumping wavelength toward the absorption peak of the Nd:YVO$_4$ crystal, and more pump power will be absorbed by the Nd:YVO$_4$ crystal. Approximately 74–89% of the pump beam is absorbed by the Nd:YVO$_4$ crystal when the pump power increases from 10 to 800 mW. We used a pair of lenses (L1 and L2) to image the laser beam waist to an optical powermeter (PM) to measure the total output power.

4. TRANSVERSE MODES IN DIFFERENT RESONATOR CONFIGURATIONS

To observe the transverse modes of the laser we magnified and imaged the waist patterns with an objective lens (FL; Fig. 1), a dichromatic beam splitter (BS) inserted between the focusing and collimating lenses, and a 1064-nm filter (F), onto a screen (S), then recorded them with a CCD camera. In general, a fundamental cavity mode

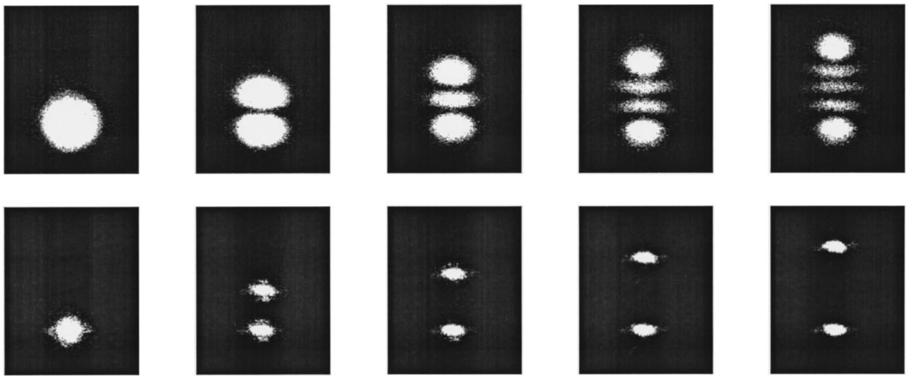

Fig. 2. Sequence of transverse mode patterns for a Nd:YVO$_4$ laser observed at the semiconfocal cavity (bottom row) and away from it (upper row) with on-axis pumping and the offset between the optic axis and the pump beam increasing from left to right.

with a Gaussian profile could be observed when an on-axis pump beam was applied to the laser crystal. Nevertheless, we found that when the cavity length was adjusted near 40 mm, corresponding to a semiconfocal cavity or a 1/4-degenerate cavity configuration, the waist pattern suddenly shrank several times to a small elliptical spot similar to the pattern of the pump beam observed on the Nd:YVO$_4$ crystal. To study the effect of off-axis pumping on the cavity mode, we transversely moved the concave mirror and hence the optic axis of the resonator with respect to the pump beam such that the higher-order spatial modes could be excited without breaking the resonator eigenmodes. We found that when the cavity length was adjusted away from the 1/4-degenerate resonator configuration the higher-order Hermite–Gaussian modes would oscillate in succession, as shown in the upper row of Fig. 2. We can see that from left to right the mode pattern successively changes from TEM$_{00}$ with on-axis pumping, to TEM$_{01}$, TEM$_{02}$, and then TEM$_{04}$ as the offset between the optic axis and the pump beam increases. In contrast, the effect of off-axis pumping on the semiconfocal cavity, shown in the lower row of Fig. 2, is to cause an inverse image of the shrinking transverse mode spot to be formed at the other side of the optic axis. The first figure at the bottom left shows that the mode pattern has a spot size smaller than the TEM$_{00}$ pattern for on-axis pumping. With increasing offset, the mode pattern consists of two shrinking spots, whose separation increases rather than changes to higher-order transverse modes. In addition, by inserting a knife-edge into the resonator to block the cavity modes slightly and introduce excess loss, we found that the higher-order Hermite–Gaussian mode changed into a lower-order mode but the semiconfocal resonator mode was elongated along the direction in which the knife-edge moved. We can conveniently explain these results by introducing MPT modes that are capable of supporting arbitrary beam patterns, even those that deviate significantly from standard Gaussian profiles, or by the superposition of various sets of degenerate cavity eigenmodes.

To discover whether any one of the fractionally degenerate resonator configurations other than a semiconfocal cavity would also support MPT modes, we investigated the waist pattern as a function of the cavity length. We found that the phenomenon of a shrinking laser beam waist[10] did indeed occur at those cavity lengths specified by the K/N-degenerate resonator configurations with low values of N. The transverse mode with fractionally degenerate resonator configurations under off-axis pumping is again quite different in behavior from a standard cavity mode. Figure 3 shows a sequence of waist patterns observed for the concave mirror moving perpendicularly to the optical axis and found at the $1/N$-degenerate resonator configurations with some low values of N ($N = 3, \ldots, 6$). For $1/N$-degenerate resonator configurations with even N, the pattern looks like that which occurs at the semiconfocal cavity. In the case of $1/N$-degenerate resonator configurations with odd N, however, the inverse image of the original shrinking spot as observed in the semiconfocal cavity is replaced by a divergent pattern with concentric rings. These rings may result from the diffraction of the laser beam in the cavity off the gain volume that has a smaller diameter than the cavity mode. Patterns that show no rotation symmetries and occur in fractionally degenerate resonator configurations with odd N can no longer be constructed from any superposition of the standard cavity modes that match the curvature of end mirror and have a certain rotational symmetry with respect to the optic axis of the resonator. We again can easily account for these results by introducing the MPT modes and taking into account the fact that a negative identity matrix does not exist in $1/N$-degenerate cavities with odd N.

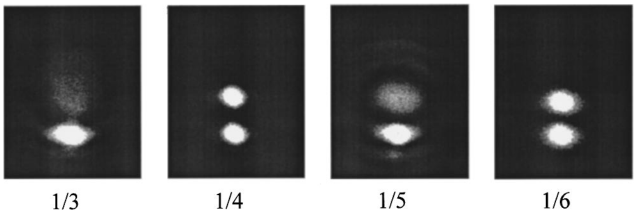

Fig. 3. Sequence of transverse mode patterns for an off-axis pumped Nd:YVO$_4$ laser observed at $1/N$-degenerate resonator configurations with some low values of N ($N = 3, 4, 5, 6$ from left to right).

5. OBSERVATIONS OF MULTIPASS TRANSVERSE MODES

To demonstrate the MPT modes in a straightforward way, we used an experimental arrangement such as that shown in Fig. 4. We placed a transform lens with a focal length of 52 mm at a distance $d_0 = 105$ mm behind the output mirror. We employed a beam profiler at a distance d_i from the lens to observe the transformed beam distribution. We observed as usual a single beam waist for the SPT modes away from the fractionally degenerate resonator configurations. In exceptional cases we found multiple beam waists when the cavity length was adjusted near fractionally degenerate resonator configurations. In general, we found N beam waists for $1/N$-degenerate resonator configurations with odd N and $N/2$ waists with even N. Figure 5(a) illustrates how the MPT mode propagates within a cavity for a 1/3-degenerate resonator configuration, corresponding to a cavity length $L = 60$ mm and $G_1G_2 = 1/4$. An equivalent lens guide cavity is shown in Fig. 5(b). In Figs. 5(c), 5(d), and 5(e) we depict the forward propagation of the MPT mode within a cavity and the transformation of its output through the transform lens for the first, second, and third passes, respectively. The beam waist for the first pass located at the crystal face (the flat mirror) was imaged at a distance d_1 from the lens [see Fig. 5(c)]; that of the second pass at the curved mirror was transformed to d_2, as shown in Fig. 5(d). Figure 5(e) shows that the beam waist of the third pass at the curved mirror directed toward and then reflected by the flat mirror (dotted portion at the left in this figure) was transformed to d_3. Figure 6 shows the experimental observation of beam spots at distances of d_3 [Figs. 6(a) and 6(d)], d_1 [Figs. 6(b) and 6(e)], and d_2 [Figs. 6(c) and 6(f)] behind the lens when the cavity length is adjusted as 61 mm for the SPT mode and 60 mm for the MPT mode, respectively, with $N = 3$. A single beam waist at $d_2 = 103$ mm for a cavity length of 61 mm (the SPT mode) and three beam waists, at $d_1 = 76$ mm, $d_2 = 103$ mm, and $d_3 = 68$ mm for a cavity length of 60 mm (the MPT mode with $N = 3$) can be found. The locations of the beam waists observed in this experiment are consistent with the positions expected according to the resonator calculations as well as the lens formula.

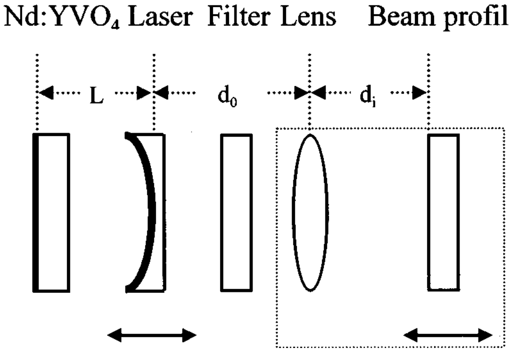

Fig. 4. Experimental arrangement used for measuring the transformation of the laser output through a lens.

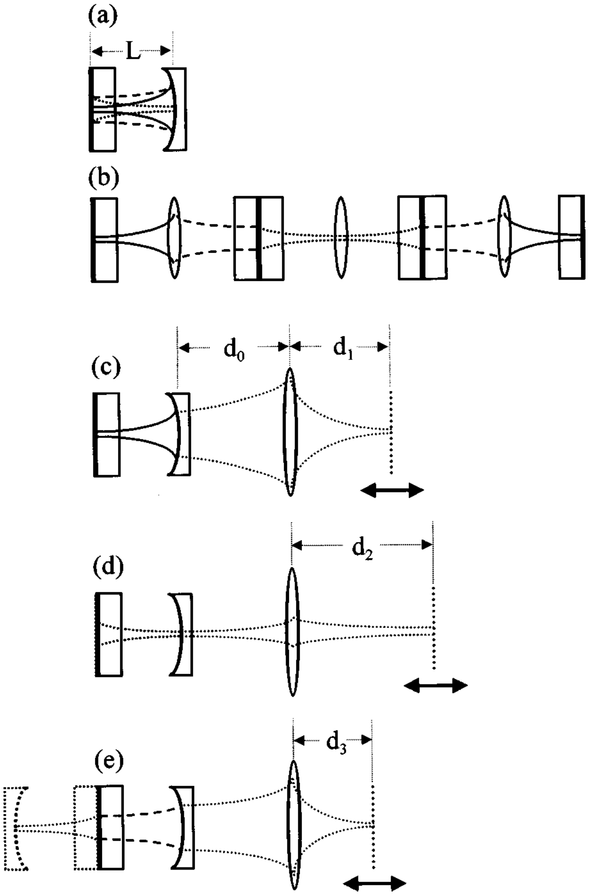

Fig. 5. Schematic illustrating the MPT mode for a cavity with the 1/3-degenerate resonator configuration. Beam propagation (a) within a plano–concave cavity and (b) along an equivalent lens-guide cavity. Forward propagation of the MPT mode within a cavity and transformation of its output through the transform lens for (c) the first pass, (d) the second pass, and (e) the third pass.

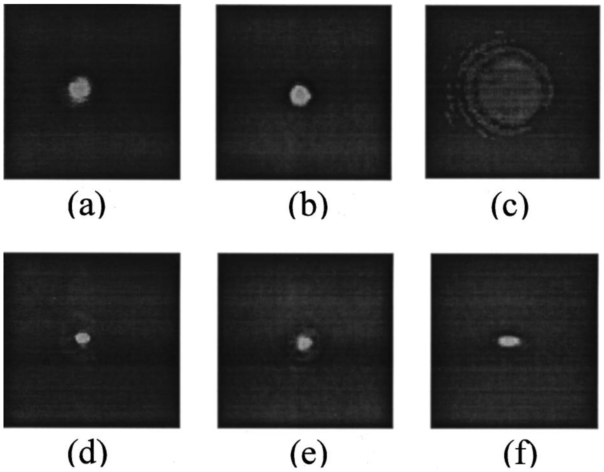

Fig. 6. Transformed beam spots observed at distances of (a) d_3, (b) d_1, and (c) d_2 after the lens for the SPT mode and (d) d_3, (e) d_1, and (f) d_2 for the MPT ($N = 3$) mode. Because the original beam spot of (c) is too dark to be seen, we nonlinearly adjusted its gamma value to make it visible.

6. THRESHOLD ESTIMATION FOR MULTIPASS TRANSVERSE MODES

The following questions may be raised now: Why does the laser have to choose the MPT modes instead of the standard cavity modes? How would the MPT modes have lower lasing thresholds than the standard cavity modes that are available even at fractionally degenerate resonator configurations? By use of a rate-equation analysis in which the spatial variations of both the pump beam and the cavity field were taken into account, the threshold pump power P_{th} of an axially pumped laser was derived[11] to be $P_{\mathrm{th}} = A_e \gamma I_{\mathrm{sat}} / \eta_p J_1$, where A_e is the effective area of the mode, γ is the total logarithmic loss per pass, I_{sat} is the saturation intensity, η_p is the pumping efficiency, and $J_1 = \int_a \epsilon g \mathrm{d}V$ takes into account the spatial overlap of pump and mode distribution.[11] Assuming a Gaussian distribution for both the pump beam and the fundamental cavity mode with effective spot sizes of w_p and w inside the laser crystal, we can calculate J_1 as $J_1 = 2A_e / \pi(w^2 + w_p^2)$. For a semiconfocal cavity, corresponding to 1/4-degenerate resonator configuration, its ray matrices after one, two, and four round trips are

$$T = \begin{bmatrix} 0 & R/2 \\ -2/R & 0 \end{bmatrix},$$

$T^2 = -I$, and $T^4 = I$, respectively. This resonator is also referred to as a Fourier-transform system,[1] in which beam patterns in the consecutive round trips form Fourier-transform pairs. The spot sizes of the MPT mode at the laser crystal in the semiconfocal cavity can accordingly be expressed as $w_1 = aw$ (at beam waist) and $w_2 = w/a$ for the alternate round trips, where a is a proportional constant of less than 1. The threshold pump power P_{th} of the MPT mode is therefore given by $P_{\mathrm{th}}(a) = \pi \gamma I_{\mathrm{sat}}[w_p^4 + (a^2 + 1/a^2)w^2 w_p^2 + w^4]/\eta_p[2w_p^2 + (a^2 + 1/a^2)w^2]$, which reduces in the case of a SPT mode to $a = 1$. The difference in the threshold pump powers for the MPT mode and for the SPT mode is $P_{\mathrm{th}}(a) - P_{\mathrm{th}}(1) = \pi \gamma I_{\mathrm{sat}}[(a^2 + 1/a^2 - 2)w^2(w_p^2 - w^2)]/2\eta_p[2w_p^2 + (a^2 + 1/a^2)w^2]$. We can see that if the waist size of pump beam is less than that of fundamental cavity mode the MPT modes will then have a lower laser threshold than the SPT mode. The dependence of output power on the pump power for equivalent cavity lengths both to and 1 mm longer than the 1/4-degenerate resonator configuration was measured and is shown in Fig. 7. As was theoretically expected, the laser with a semiconfocal cavity has a lower (approximately a factor of 5) pump power at threshold than away from it. It is worth mentioning that if the MPT mode were not taken into account, the difference in spot size and hence in laser threshold between the laser with semiconfocal and 1-mm-longer cavities would be negligible. Similarly, we can estimate the threshold pump power of the MPT modes for the laser with other fractionally degenerate resonator configurations. We obtain conclusions similar to those for the semiconfocal cavity, except that we have to reduce pump spot size w_p further to a smaller value, for instance, $(1/\sqrt{3})w$ for $N = 3$ (or 6) and even smaller for $N = 5$ (or 10), with the factor of the wave-front match neglected. By measuring the output power as a function of the $G_1 G_2$ parameters under on-axis pump power slightly above threshold of the MPT mode of the semiconfocal cavity, we observed an increase in the output power when the cavity length approached fractionally degenerate resonator configurations. Figure 8 shows that some narrow power peaks occur at the fractionally degenerate resonator configurations that correspond to $K/N = 1/3$, 3/10, 1/4, 1/5, and 1/6 from left to right. When the pump power increases further, more discrete power peaks were found at the fractionally degenerate resonator configurations involving higher values of N. The lower pump threshold that occurs at the cavity length corresponding to fractionally degenerate resonator configurations is in good agreement with the theoretical expectation when we take into account the property of the MPT modes that permits the selection of a laser beam distribution that better fits the pump profile.

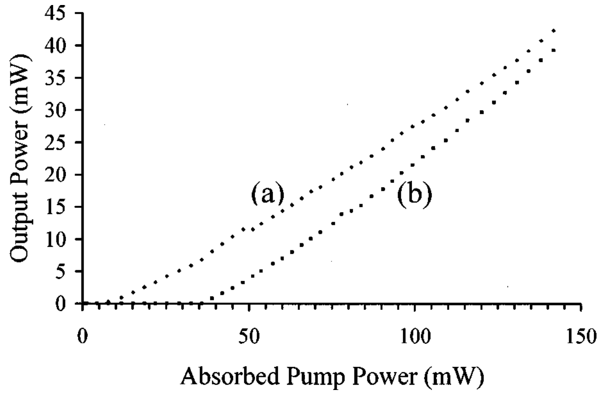

Fig. 7. Output power as a function of absorbed pump power measured for cavity lengths (a) equivalent to a 1/4-degenerate resonator configuration and (b) 1 mm longer than it.

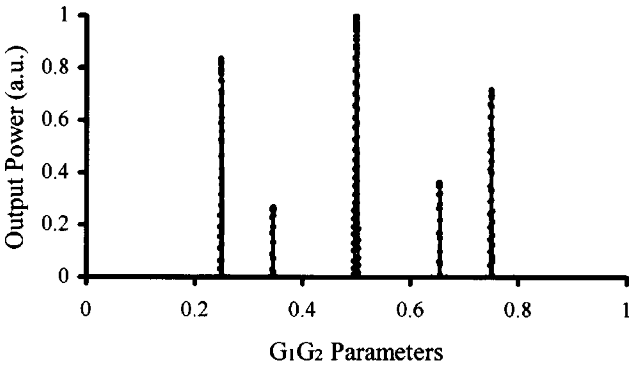

Fig. 8. Dependence of the output power on the resonator $G_1 G_2$ parameters with pump power slightly above the threshold of the MPT mode in a semiconfocal cavity. Some narrow power peaks occur at the cavity with $G_1 G_2$ parameters corresponding to discrete fractionally degenerate resonator configurations (K/N = 1/3, 3/10, 1/4, 1/5, and 1/6 from left to right).

7. CONCLUSIONS

We have shown that multipass transverse modes that self-reproduce after several round trips may exist in a simple two-mirror laser that has cavity lengths that correspond to fractionally degenerate resonator configurations. MPT modes with the ability to adjust themselves for a better fit with the pump beams could then be excited in a diode-pumped Nd:YVO$_4$ laser by virtue of a relatively low pump threshold if the waist size of the pump beam were much less than that of the fundamental cavity mode. With off-axis pumping, these MPT modes form symmetric mode patterns for even N and asymmetric ones for odd N, which can be explained in terms of ray matrix analysis but not by the superposition of standard cavity modes. Moreover, we used a lens to transform the output laser beam and found multiple beam waists for the MPT modes and a single waist for the SPT modes. As a result, we suggest that the factor of MPT modes should be taken into account in the design and application of axially pumped solid-state lasers, particularly for those that have fractionally degenerate resonator configurations characterized by low values of N.

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