CSMA/CD protocol for time-constrained communication on bus networks

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Indexing terms: Bus networks, CSMA/CD, Time-constrained communications

Abstract: A multiple access protocol for transmitting time-constrained packets on bus networks is presented. Based on carrier sense multiple access/collision detection (CSMA/CD) protocol, a new protocol is developed and its performance evaluated by computer simulation. An approximate analysis model for this protocol is also presented. Numerical results indicate that the new protocol achieves a better performance, and the approximate analysis model is also effective.

1 Introduction

In multiaccess/broadcast systems, each station is connected to a common communication medium (such as a bus) through an interface to listen to all transmissions and copy packets that are addressed to it. Since no more than one transmission can be carried on the bus at a time, the stations have to share the bus by means of a multiple access protocol. Over two decades, a wide variety of protocols for multiple access, packet-switched communication have been presented [1–5]. One of the most notable protocols is the carrier sense multiple access with collision detection (CSMA/CD) [1, 6, 7]. CSMA/CD protocol is adopted in Ethernet, which is the most widespread used local area network in the world.

Note that CSMA/CD is not suitable for transmitting time-constrained messages because it cannot bound the transmission delay of message. However, in many multiaccess systems, there often exist time-constrained messages, such as real-time voice, short interactive data, acknowledgement, network real-time control packets and so on. These time-constrained messages must be received by the destination station before their deadlines or they are lost. Such a communication system is usually called a real-time communication system or a time-constrained communication system [8–11]. For many time-constrained communication applications, a critical performance measure is the percentage of messages successfully transmitted within the time constraint. To maximise this percentage (or minimise the rate of messages lost), a special protocol to manage effectively the transmission of time-constrained messages is desired.

Zhao and Ramamritham [10] have developed a virtual time CSMA protocol for real-time communication. Recently, Zhao, Stankovic and Ramamritham present another time-constrained communication protocol, called window protocol [11]. However, those protocols are complicated and of higher overhead in the transmission. In this paper, we present a modified CSMA/CD protocol, named the dynamic p_i-persistent CSMA/CD protocol, for real-time communication systems. The proposed protocol is simple, has low overhead, and is suitable for transmitting time-constrained messages.

Paper 9413I (E7), first received 19th December 1991 and in final revised form 9th November 1992

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2 Dynamic p_i-persistent CSMA/CD

2.1 System model

We make the following assumptions in constructing the system model of bus network with time-constrained communication

(*a*) The time-axis is slotted. The slot duration τ is equal to maximum propagation delay between any two stations and is assumed to be the unit of time. All stations are synchronised and forced to start transmission only at the beginning of the slot.

(*b*) Packets, assumed to be of fixed length, require a transmission time of T slots. In this system, all packets are time-constrained packets. The deadline d_v is the time by which packet v must be received by its destination. The laxity $l_v(t)$ of packet v at time t is the maximum amount of time that the transmission of packet v can be delayed at time t. Therefore

$$l_v(t) = d_v - T - t$$

Every time-constrained packet is generated with initial laxity value L.

(*c*) There are M identical stations in the system, and each of them has a single buffer. Once a station generates a packet for transmission, the packet is retained in buffer until it is transmitted successfully, or until its laxity has become zero. Let each empty station, which does not have a buffered packet, have an arrival with probability g in any slot where $0 < g < 1$.

2.2 Protocol description

Before stating dynamic p_i-persistent CSMA/CD protocol for time-constrained communication, consider the uniprocessor scheduling problem. Two problems, time-constrained communication on bus networks and uniprocessor scheduling, are quite similar. The uniprocessor scheduling algorithms are used for allocation

This work was supported by the National Science Council of the Republic of China under grant NSC 80-0408-E009-29.

of the serially-used processor to a set of processes, and the protocols of time-constrained communication deal with allocation of a common bus to the ready packets [11]. When all the task or process characteristics are known *a priori*, minimum-deadline-first and minimum-laxity-first scheduling policies are optimal. Owing to this fact, the dynamic p_i-persistent CSMA/CD protocol is designed such that the packets with lower laxity can get higher probability to transmit.

The dynamic p_i-persistent CSMA/CD protocol works similar to the p-persistent CSMA/CD protocol. In dynamic p_i-persistent CSMA/CD protocol, the transmission probability of a ready packet depends on two parameters: the laxity of the ready packet, and a time window X of the system. The time window is used to reduce the number of collisions in a fixed time when the system becomes heavily loaded. So that the packets with lower laxity can get higher transmission probability, the transmission probability p_i, where i is the laxity value of the packet, is determined as

$$p_i = \begin{cases} \max\left\{p, \left(\frac{1}{i+1}\right)^c\right\} & \text{if } i \leqslant X \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

where the protocol parameters c and p $(0 < p < 1)$ can be identified according to the system load and message laxity. Thus, the most of packets with lower laxity can be transmitted before the packets with higher laxity and then the rate of packets lost can be reduced.

The operations of dynamic p_i-persistent CSMA/CD protocol are summarised as follows:

(i) When the system is started, every station senses the channel and sets $X = \delta$ where δ is a power of 2, say 2^k.

(ii) For each time slot, if the channel is idle, every station sets its X value as follows:

(*a*) if the previous slot is collision, then set $X = \max\{1, (X/2)\}$

(*b*) otherwise, set $X = \min\{\delta, 2X\}$

(iii) If a station has a packet ready to transmit, it checks the channel status.

(*a*) If the channel is idle, the station checks its packet's laxity value i. If $i < 0$, the station aborts the ready packet because the packet cannot be transmitted successfully before its deadline. Otherwise, determine p_i according to eqn. 1. Then the station transmits the packet with probability p_i, and with probability $1 - p_i$, it defers the packet until next slot and repeat step 3.

(*b*) If the channel is busy, the station waits until the channel is idle, i.e. at the end of the current transmission, and apply step 2.

(iv) If a collision is detected during transmission the station that is transmitting the packet immediately ceases its transmission and waits for the channel to return to idle. Then go to step 2.

Note that X is chosen as a power of two in consideration of implementation. Although the time window X is locally maintained in each station, the values of X are all equal under normal operation. This is because they are derived from the globally available channel status. When the collision occurs, each station realises that two or more stations attempt to transmit the packets so that the time window should be reduced to resolve the collision. From eqn. 1, only the stations in which their laxity values are not greater than the reduced time window can get a nonzero transmission probability. By this way, the collision can be resolved fast. In addition, p_i is set to be $\max\{p, (1/i+1)^c\}$ when $i \leqslant X$. The constant p is active whenever laxity value i is large. This can avoid that p_i is too small due to the large laxity value.

3 Approximate performance analysis model

In this Section, we analyse the throughput and rate of packets lost of dynamic p_i-persistent CSMA/CD. The channel can be divided into idle and busy periods. An idle period, denoted by I, is defined as the time in which the channel is idle, no packets are waiting for transmission, and $X = \delta$. When any packet arrives in I period, the next slot is said to be a beginning of the busy period. The busy period, denoted by B, ends if no packets are waiting for transmission and $X = \delta$. Furthermore, let U be the time spent for successful transmission in a busy period B.

From the above definitions, the system state alternates between idle periods I and busy periods B. It is clear that the channel throughput S can expressed as

$$S = \frac{\bar{U}}{\bar{B} + \bar{I}} \tag{2}$$

where $\bar{Y}$ denotes the expectation of a random variable Y, e.g. $\bar{U} = E[U]$.

Recall that there are M identical stations in this system. Each empty station has an arrival with probability g in any slot, where $0 < g < 1$. Thus, the duration of an idle period I is geometrically distributed with

$$\text{Prob}\{I = k \text{ slots}\} = (1-g)^{M(k-1)}[1-(1-g)^M] \qquad k = 1, 2, \ldots$$

and

$$\bar{I} = \frac{1}{1-(1-g)^M} \tag{3}$$

For simplicity of analysis, assume that the station with a ready packet checks the channel status before it starts to count down the laxity value. If the channel is checked in an idle state, the station begins to count down laxity value i. Otherwise the station holds laxity value $i = L$ until the channel state becomes idle. With this assumption, all ready packets have laxity L when the system leaves from idle state. Thus, the system's state-space can be reduced. The influence of this assumption is very small when the system's packet arrival rate is low and the packet's laxity value is high.

Let $\pi_m(x)$ be the probability that we have m arrivals among M stations in x slots, given that $m \geqslant 1$ [12]. That is

$$\pi_m(x) = \frac{1}{1-(1-g)^{xM}}\binom{M}{m}[1-(1-g)^x]^m(1-g)^{x(M-m)} \qquad m = 1, 2, \ldots, M$$

In any time slot, define the system state as $(n_L, n_{L-1}, \ldots, n_0, X)$ where n_i, $n_i \geqslant 0$, represents the number of ready packets with laxity value i and X represents system's time window. Let N be the total number of ready packets, i.e. $N = \sum_{i=0}^{L} n_i$. For notational simplicity, let $\boldsymbol{n} = (n_L, n_{L-1}, \ldots, n_0)$ and $(\boldsymbol{n}, X) = (n_L, n_{L-1}, \ldots, n_0, X)$.

The channel in the idle period means the system in state $(\mathbf{0}, \delta)$ where δ is the initial value of X. We assume that the system starts from state $(\mathbf{0}, \delta)$. If there is any packet ready, then the system transits to state $(s, 0, 0, \ldots, 0, \delta)$, where s is the number of the ready packets at this slot. Otherwise, the system remains at state $(\mathbf{0}, \delta)$. That is, if there is any packet ready, the system transits from I

period to B period. The mean time, in which system starts from $(s, 0, 0, \ldots, 0, \delta)$ and finally returns to state $(\mathbf{0}, \delta)$, is denoted as $B(s, 0, 0, \ldots, 0, \delta)$. Let $P_{(s, 0, 0, \ldots, 0, \delta)}$ be the probability that the system transits from state $(\mathbf{0}, \delta)$ to state $(s, 0, 0, \ldots, 0, \delta)$. Then

$$P_{(s, 0, 0, \ldots, 0, \delta)} = \pi_s(1)$$

and the expected value of B period can be written as

$$\bar{B} = \sum_{s=1}^{M} P_{(s, 0, 0, \ldots, 0, \delta)} B(s, 0, 0, \ldots, 0, \delta) \tag{4}$$

Similarly, let $U(s, 0, 0, \ldots, 0, \delta)$ be the mean utilised time in which the system starts from state $(s, 0, 0, \ldots, 0, \delta)$ and returns to $(\mathbf{0}, \delta)$. The mean value of U can be computed by

$$\bar{U} = \sum_{s=1}^{M} P_{(s, 0, 0, \ldots, 0, \delta)} U(s, 0, 0, \ldots, 0, \delta) \tag{5}$$

The evaluations of $B(s, 0, 0, \ldots, 0, \delta)$ and $U(s, 0, 0, \ldots, 0, \delta)$ are summarised in the Appendix (Section 7.1). From eqns. 3, 4 and 5, the throughput S can be obtained.

Let random variable D be the number of packets lost in a busy period B. Then, it is clear that the ratio of packets lost ρ can be expressed as

$$\rho = \frac{\bar{D}}{\bar{D} + \dfrac{\bar{U}}{T}}$$

where $\bar{D}$ and $\bar{U}$ are the expectation values of D and U, respectively, and T is the packet length. Let $D(\mathbf{n}, X)$ be the mean number of loss in which the system starts from state $(\mathbf{n}, X)$ and returns to state $(\mathbf{0}, \delta)$. Similarly to eqn. 4, the $\bar{D}$ can be written as

$$\bar{D} = \sum_{s=1}^{M} P_{(s, 0, 0, \ldots, 0, \delta)} D(s, 0, 0, \ldots, 0, \delta) \tag{6}$$

The evaluations of $D(s, 0, \ldots, 0, \delta)$, $s = 1, \ldots, M$, are given in Section 7.2. From eqns. 5 and 6, the ratio of packets lost ρ can be found.

4 Performance evaluation

4.1 Simulation results

To compare dynamic p_i-persistent CSMA/CD protocol with the window protocol, the simulation model is parameterised by the distributions of packet arrivals, packet lengths, and packet laxities. The packets arrive as a Poisson process with parameter λ. Packet lengths are exponential distributed with mean $\bar{T} = 100$, or mean $\bar{T} = 10$. Packet laxities are uniformly distributed in the interval $[0, 2\theta]$, where θ is the average laxity. At the end of each simulation, the ratio of packets lost ρ_l will be collected. The definition of ρ_l is

$$\rho_l = \frac{N_l}{N_l + N_t}$$

where N_l is the total number of packets lost, and N_t is the total number of packets successfully transmitted.

The following values of parameters are assumed in the simulations.

(*a*) Two cases of normalised end-to-end delay (α) are considered, i.e. $\alpha = \bar{T}^{-1} = 0.01$ and $\alpha = \bar{T}^{-1} = 0.1$.

(*b*) The system load (SL) is defined as $SL = \lambda \cdot \bar{T}$ and SL changes from 0.1 to 0.5, to 1.0 and to 2.0.

(*c*) The mean of packet laxity varies from 1 to 10^4.

(*d*) The δ, initial value of window X, is set to 2^{10}, and the initial window size of window protocol is 10^4. The choice of initial value of window size in the window protocol is of almost no influence to the performance of the protocol [11].

(*e*) p_i is assumed to be

$$p_i = \begin{cases} \max\left\{0.05, \left(\dfrac{1}{i+1}\right)^{0.5}\right\} & \text{if } i \leqslant X \\ 0 & \text{otherwise} \end{cases}$$

where i is the laxity of packet.

The numerical results are shown in Figs. 1 and 2. Those Figures plot the percentage of packet loss against the

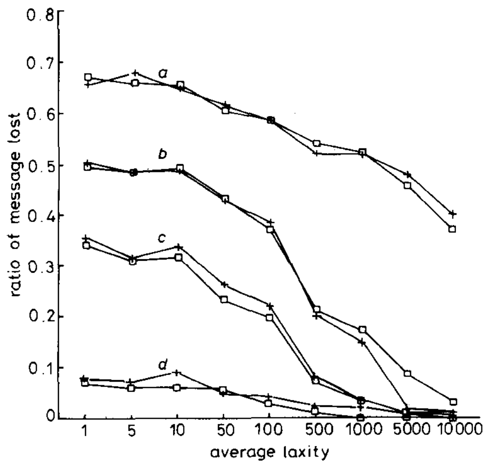

Fig. 1 *Rate of message lost against average laxity of dynamic p_i-persistent CSMA/CD protocol and the window protocol ($\alpha = 0.01$)*
□ dynamic p_i
+ window protocol
a $SL = 2.0$
b $SL = 1.0$
c $SL = 0.5$
d $SL = 0.1$

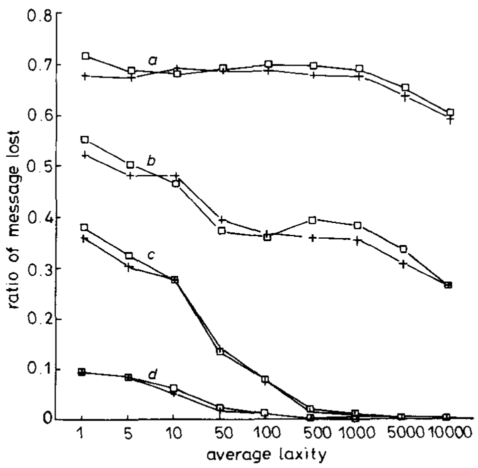

Fig. 2 *Rate of message lost against average laxity of dynamic p_i-persistent CSMA/CD protocol and the window protocol ($\alpha = 0.1$)*
□ dynamic p_i
+ window protocol
a $SL = 2.0$
b $SL = 1.0$
c $SL = 0.5$
d $SL = 0.1$

mean of packet laxity in a logarithmic scale. In Reference 11, the window protocol has been shown to perform better than the virtual time CSMA/CD protocol which in turn has been shown to work well and be better than a general CSMA/CD protocol [10]. From Figs. 1 and 2, we know that the performance of dynamic p_i-persistent CSMA/CD protocol is as good as the performance of window protocol. Thus, the dynamic p_i-persistent CSMA/CD protocol also achieves a better performance.

4.2 Comparisons between simulation and analytical model

The simulation model and analytical model are parameterised by a finite number of stations, fixed packet transmission time and fixed initial laxity value. The station number $M = 10$, packet transmission time $T = 3$, and initial value $L = 5$ are chosen. The packet arrival probability g ranges from 0.001 to 0.02. The corresponding system load varies from 0.03 to 0.6. The numerical results of simulation and analytical model are summarised in Table 1, which shows the system throughput and the rate of packets lost ρ for simulation and analytical results. From Table 1, notice that the approximate analysis model is valid when the system's packet arrival rate is low.

Table 1: Throughput and rate of packets lost for simulation and analytical results

Arrival prob. g†	Throughput		Rate of packets lost	
	simulation	analysis	simulation	analysis
0.001	0.030293	0.029358	0.013699	0.014428
0.002	0.058019	0.057374	0.028141	0.028809
0.003	0.085438	0.083963	0.044936	0.043145
0.004	0.109258	0.109061	0.060856	0.057433
0.005	0.133377	0.132630	0.071041	0.071668
0.006	0.156297	0.154461	0.090115	0.085483
0.007	0.177596	0.175173	0.104387	0.099947
0.008	0.198520	0.194208	0.119721	0.113967
0.009	0.219116	0.211824	0.130062	0.127893
0.010	0.237526	0.228091	0.141713	0.141711
0.011	0.255175	0.243087	0.159145	0.155408
0.012	0.271885	0.256891	0.175882	0.168973
0.013	0.286974	0.269587	0.191924	0.182394
0.014	0.299694	0.281253	0.210277	0.195659
0.015	0.315414	0.291968	0.222682	0.208760
0.016	0.329153	0.301805	0.236465	0.221686
0.017	0.340913	0.310832	0.254624	0.234429
0.018	0.354279	0.319113	0.262244	0.246983
0.019	0.363353	0.326709	0.280931	0.251341
0.020	0.374633	0.333674	0.292465	0.271497

† The corresponding system load is from 0.03 to 0.6

5 Conclusions

We have proposed the dynamic p_i-persistent CSMA/CD protocol for time-constrained communication. The protocol differs from traditional CSMA/CD in the sense that it uses the packet's laxity to determine the transmission probability p_i. This transmission probability is used to implement the minimum-laxity-first transmission policy. Comparing with the window protocol [11], the dynamic p_i-persistent CSMA/CD protocol has almost the same performance as that of the window protocol. However, the dynamic p_i-persistent CSMA/CD protocol is simpler than the window protocol. This is because the minimum-laxity-first transmission policy is implemented by computing the transmission probability p_i in our protocol instead of using the complicate window approach. Because of its simpler characteristic, we are able to obtain a mathematical model to approximate the performance of dynamic p_i-persistent CSMA/CD protocol. This approximate analysis model has been verified by simulation. The simulation results agree with the analytical results when the packet arrival rate is low and initial laxity value L is greater than the packet transmission time T.

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7 Appendix

7.1 Evaluation of $B(s, 0, 0, \ldots, 0, \delta)$ and $U(s, 0, 0, \ldots, 0, \delta)$

Let $P^I_{(\boldsymbol{n}, X)}$ be the probability given that the system is in state $(\boldsymbol{n}, X)$, and the next slot is an idle slot. Then

$$P^I_{(\boldsymbol{n}, X)} = \begin{cases} 0 & \text{if } n_0 > 0 \\ \prod_{j=1}^{\min(X, L)} (1 - p_j)^{n_j} & \text{otherwise} \end{cases}$$

Let $P^{T_i}_{(\boldsymbol{n}, X)}$ be the probability given that the system is in state $(\boldsymbol{n}, X)$ and a packet with laxity value i will be transmitted successfully in this time. $P^{T_i}_{(\boldsymbol{n}, X)}$ can be determined by

$$P^{T_i}_{(\boldsymbol{n}, X)} = \begin{cases} 0 & \text{if } n_0 > 1 \text{ or } i > X \\ 0 & n_0 = 1 \text{ and } i \neq 0 \\ \prod_{j=1}^{\min(X, L)} (1 - p_j)^{n_j} & n_0 = 1 \text{ and } i = 0 \\ n_i p_i (1 - p_i)^{n_i - 1} \prod_{j=1, j \neq i}^{\min(X, L)} (1 - p_j)^{n_j} & \text{otherwise} \end{cases}$$

Let $P^c_{(\boldsymbol{n}, X)}$ be the probability given that the system is in state $(\boldsymbol{n}, X)$ and a collision occurs at this time:

$$P^c_{(\boldsymbol{n}, X)} = \begin{cases} 0 & \text{if } N \leqslant 1 \\ 1 - P^I_{(\boldsymbol{n}, X)} - \sum_{i=0}^{\min(X, L)} P^{T_i}_{(\boldsymbol{n}, X)} & \text{otherwise} \end{cases}$$

Let $\pi_m^n(x)$ be the probability that there are m arrivals among n stations in x slots. Then

$$\pi_m^n(x) = \binom{n}{m}[1 - (1 - g)^x]^m (1 - g)^{x(n-m)}$$

$$m = 1, 2, \ldots, n \quad n = 1, 2, \ldots, M$$

Note that the packet transmission time is T slots. So the length of a successful transmission period $T + 1$ slots where the extra slot is needed for propagation delay. In case of a collision, the length of a collision period is $\gamma + 1$ slots, where γ is the duration between collision occurs and all stations stop transmission. Because one propagation delay is needed before interference signal reaches all stations, set $\gamma = 1$.

Let $P^y_{(\boldsymbol{n}, X), (\boldsymbol{n}_m, X')}$ represent the transition probability that the system transits from $(\boldsymbol{n}, X)$ to $(\boldsymbol{n}_m, X')$ owing to the channel occurring state y where y can represent the symbols I, T_i, c, and the system having m new arrivals. $P^I_{(\boldsymbol{n}, X), (\boldsymbol{n}_m, X')}$, $P^{T_i}_{(\boldsymbol{n}, X), (\boldsymbol{n}_m, X')}$ and $P^c_{(\boldsymbol{n}, X), (\boldsymbol{n}_m, X')}$, are given as follows:

$$P^I_{(\boldsymbol{n}, X), (\boldsymbol{n}_m, X')} = P^I_{(\boldsymbol{n}, X)} \pi_m^{M-N}(1)$$

where $\boldsymbol{n} = (n_L, n_{L-1}, \ldots, n_0)$, $\boldsymbol{n}_m = (m, n_L, n_{L-1}, \ldots, n_1)$ and $X' = \min\{\delta, 2X\}$.

$$P^{T_i}_{(\boldsymbol{n}, X), (\boldsymbol{n}_m, X')} = P^{T_i}_{(\boldsymbol{n}, X)} \pi_m^{M-N}(T + 1)$$

where $\boldsymbol{n} = (n_L, n_{L-1}, \ldots, n_0)$, $\boldsymbol{n}_m = (m, 0, \ldots, 0, n_L, n_{L-1}, \ldots, n_{T+1})$ and $X' = \min\{\delta, 2X\}$.

$$P^c_{(\boldsymbol{n}, X), (\boldsymbol{n}_m, X')} = P^c_{(\boldsymbol{n}, X)} \pi_m^{M-N}(2)$$

where $\boldsymbol{n} = (n_L, n_{L-1}, \ldots, n_0)$, $\boldsymbol{n}_m = (m, 0, n_L, n_{L-1}, \ldots, n_2)$ and $X' = \max\{1, (X/2)\}$.

Now the values of $B(n, 0, \ldots, 0, \delta)$ and $U(n, 0, \ldots, 0, \delta)$ can be computed.

$$\begin{aligned}
B(n, 0, \ldots, 0, \delta) &= \sum_{m=0}^{M-n} P^I_{(n, 0, 0, \ldots, 0, \delta), (m, n, 0, \ldots, 0, \delta)} \cdot [1 + B(m, n, 0, \ldots, 0, \delta)] \\
&+ \sum_{m=0}^{M-n} P^{T_L}_{(n, 0, 0, \ldots, 0, \delta), (m, 0, \ldots, 0, n-1, 0, \ldots, 0, \delta)} \cdot [(T + 1) + B(m, 0, \ldots, 0, n - 1, 0, \ldots, 0, \delta)] \\
&+ \sum_{m=0}^{M-n} P^c_{(n, 0, 0, \ldots, 0, \delta), [m, 0, n, 0, \ldots, 0, (\delta/2)]} \cdot \{2 + B[m, 0, n, 0, \ldots, 0, (\delta/2)]\} \qquad (7) \\
&= P^I_{(n, 0, \ldots, 0, \delta)} \cdot 1 + \sum_{m=0}^{M-n} P^I_{(n, 0, 0, \ldots, 0, \delta), (m, n, 0, \ldots, 0, \delta)} \cdot B(m, n, 0, \ldots, 0, \delta) \\
&+ P^{T_L}_{(n, 0, 0, \ldots, 0, \delta)} \cdot (T + 1) \\
&+ \sum_{m=0}^{M-n} P^{T_L}_{(n, 0, 0, \ldots, 0, \delta), (m, 0, \ldots, 0, n-1, 0, \ldots, 0, \delta)} \cdot B(m, 0, \ldots, 0, n - 1, 0, \ldots, 0, \delta) \\
&+ P^c_{(n, 0, 0, \ldots, 0, \delta)} \cdot 2 \\
&+ \sum_{m=0}^{M-n} P^c_{(n, 0, 0, \ldots, 0, \delta), [m, 0, n, 0, \ldots, 0, (\delta/2)]} \cdot B[m, 0, n, 0, \ldots, 0, (\delta/2)] \qquad (8)
\end{aligned}$$

To understand why eqn. 7 is correct consider, for instance

$$\sum_{m=0}^{M-n} P^{T_L}_{(n, 0, 0, \ldots, 0, \delta), (m, 0, \ldots, 0, n-1, 0, \ldots, 0, \delta)} \cdot [(T + 1) + B(m, 0, \ldots, 0, n - 1, 0, \ldots, 0, \delta)]$$

and reason as follows. If the system starts from $(n, 0, 0, \ldots, 0, \delta)$ and transmits a packet successfully, it spends $T + 1$ slots and then enters into state $(m, 0, \ldots, 0, n - 1, 0, \ldots, 0, \delta)$. Once the system enters state $(m, 0, \ldots, 0, n - 1, 0, \ldots, 0, \delta)$, its expected additional time until it returns to state $(\mathbf{0}, \delta)$ is $B(m, 0, \ldots, 0, n - 1, 0, \ldots, 0, \delta)$. The argument behind the other terms in eqn. 7 is similar. In a similar way to eqn. 8, the expression of $U(n, 0, \ldots, 0, \delta)$ can be obtained as follows

$$\begin{aligned}
U(n, 0, \ldots, 0, \delta) &= \sum_{m=0}^{M-n} P^I_{(n, 0, 0, \ldots, 0, \delta), (m, n, 0, \ldots, 0, \delta)} \cdot U(m, n, 0, \ldots, 0, \delta) \\
&+ P^{T_L}_{(n, 0, 0, \ldots, 0, \delta)} \cdot T \\
&+ \sum_{m=0}^{M-n} P^{T_L}_{(n, 0, 0, \ldots, 0, \delta), (m, 0, \ldots, 0, n-1, 0, \ldots, 0, \delta)} \cdot U(m, 0, \ldots, 0, n - 1, 0, \ldots, 0, \delta) \\
&+ \sum_{m=0}^{M-n} P^c_{(n, 0, 0, \ldots, 0, \delta), [m, 0, n, 0, \ldots, 0, (\delta/2)]} \cdot U[m, 0, n, 0, \ldots, 0, (\delta/2)] \qquad (9)
\end{aligned}$$

The values of $B(\boldsymbol{n}, X)$s in eqn. 8 can be recursively determined by the following equations.

$$B(0, 0, \ldots, 0, \delta) = 0 \qquad (10)$$

$$\begin{aligned}
B(\boldsymbol{n}, X) &= P^I_{(\boldsymbol{n}, X)} \cdot 1 + \sum_{m=0}^{M-N} P^I_{(\boldsymbol{n}, X), (\boldsymbol{n}_m, X')} \cdot B(\boldsymbol{n}_m, X') \\
&+ \sum_{i=0}^{\min(X, L)} \left[P^{T_i}_{(\boldsymbol{n}, X)} \cdot (T + 1) + \sum_{s=0}^{M-n} P^{T_i}_{(\boldsymbol{n}, X), (\boldsymbol{n}_s, X')} \cdot B(\boldsymbol{n}_s, X') \right] \\
&+ P^c_{(\boldsymbol{n}, X)} \cdot 2 + \sum_{t=0}^{M-N} P^c_{(\boldsymbol{n}, X), (\boldsymbol{n}_t, Y')} \cdot B(\boldsymbol{n}_t, Y') \qquad (11)
\end{aligned}$$

where $X' = \min\{\delta, 2X\}$ and $Y' = \max\{1, (X/2)\}$. Similarly, the values of $U(\boldsymbol{n}, X)$ in eqn. 9 can be determined recursively as follows

$$U(0, 0, \ldots, 0, \delta) = 0 \qquad (12)$$

$$\begin{aligned}
U(\boldsymbol{n}, X) &= \sum_{m=0}^{M-N} P^I_{(\boldsymbol{n}, X), (\boldsymbol{n}_m, X')} \cdot U(\boldsymbol{n}_m, X') \\
&+ \sum_{i=0}^{\min(X, L)} \left[P^{T_i}_{(\boldsymbol{n}, X)} \cdot T + \sum_{s=0}^{M-n} P^{T_i}_{(\boldsymbol{n}, X), (\boldsymbol{n}_s, X')} \cdot U(\boldsymbol{n}_s, X') \right] \\
&+ \sum_{t=0}^{M-N} P^c_{(\boldsymbol{n}, X), (\boldsymbol{n}_t, Y')} \cdot U(\boldsymbol{n}_t, Y') \qquad (13)
\end{aligned}$$

where $X' = \min\{\delta, 2X\}$ and $Y' = \max\{1, (X/2)\}$.

7.2 Evaluation of $D(n, 0, \ldots, 0, \delta)$

$$
\begin{aligned}
&D(n, 0, \ldots, 0, \delta) \\
&= \sum_{m=0}^{M-n} P^{I}_{(n, 0, 0, \ldots, 0, \delta), (m, n, 0, \ldots, 0, \delta)} \cdot D(m, n, 0, \ldots, 0, \delta) \\
&\quad + \sum_{m=0}^{M-n} P^{T_L}_{(n, 0, 0, \ldots, 0, \delta), (m, 0, \ldots, 0, n-1, 0, \ldots, 0, \delta)} \\
&\qquad \cdot D(m, 0, \ldots, 0, n-1, 0, \ldots, 0, \delta) \\
&\quad + \sum_{m=0}^{M-n} P^{c}_{(n, 0, 0, \ldots, 0, \delta), [m, 0, n, 0, \ldots, 0, (\delta/2)]} \\
&\qquad \cdot D[m, 0, n, 0, \ldots, 0, (\delta/2)]
\end{aligned} \tag{14}
$$

The values of $D(0, 0, \ldots, 0, \delta)$ and $D(\boldsymbol{n}, X)$ in eqn. 14 can be determined recursively as follows, in a similar way to eqns. 10 and 11.

$$
D(0, 0, \ldots, 0, \delta) = 0 \tag{15}
$$

$$
\begin{aligned}
D(\boldsymbol{n}, X) &= P^{I}_{(\boldsymbol{n}, X)} \cdot n_0 + \sum_{m=0}^{M-N} P^{I}_{(\boldsymbol{n}, X), (\boldsymbol{n}_m, X')} \cdot D(\boldsymbol{n}_m, X') \\
&\quad + \sum_{i=0}^{L} \Bigg[P^{T_i}_{(\boldsymbol{n}, X)} \cdot \sum_{k=0}^{T} n_k \\
&\qquad + \sum_{s=0}^{M-n} P^{T_i}_{(\boldsymbol{n}, X), (\boldsymbol{n}_s, X')} \cdot D(\boldsymbol{n}_s, X') \Bigg] \\
&\quad - \sum_{i=0}^{T} P^{T_i}_{(\boldsymbol{n}, X)} + P^{c}_{(\boldsymbol{n}, X)} \cdot (n_1 + n_0) \\
&\quad + \sum_{t=0}^{M-N} P^{c}_{(\boldsymbol{n}, X), (\boldsymbol{n}_t, Y')} D(\boldsymbol{n}_t, Y')
\end{aligned} \tag{16}
$$

where $X' = \min\{\delta, 2X\}$ and $Y' = \max\{1, (X/2)\}$.