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Material characterization of laminated composite plates via static testing

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Abstract

A minimization method for material characterization of laminated composite plates using static test results is presented. Mechanical responses such as strains and displacements are measured from the static tests of the laminated composite plates. The finite element method is used to analyse the deformation of the laminated composite plates. An error function is established to measure the differences between the experimental and theoretical mechanical responses of the laminated composite plates. The identification of the material elastic constants of the laminated composite plates is formulated as a constrained minimization problem in which the elastic constants are determined by making the error function a global minimum. A number of examples are given to illustrate the feasibility and applications of the proposed method. © 2000 Elsevier Science Ltd. All rights reserved.

Keywords: Composite laminate; Material characterization; Identification; Minimization method; Finite element method

1. Introduction

Recently composite laminates have been used extensively in the mechanical and aerospace industries, especially for the fabrication of high performance structures. To ensure high reliability of the structures, the actual behaviors of the laminated composite parts in service must be accurately predicted and carefully monitored. The attaining of the actual behavioral predictions of the structures depends on the correctness of the elastic constants of the structures. As well-known, there are many methods for manufacturing laminated composite components [1,2] and different manufacturing or curing processes may yield different mechanical properties of the components. Furthermore, the material properties determined from standard specimens tested in laboratory may deviate significantly from those of actual laminated composite components manufactured in factory. On the other hand, laminated composite structures subject to dynamic loads or used in severe environments may experience progressive stiffness reduction or material degradation which will finally lead to the failure of the structures. It has been pointed out that accurate determination of current stiffness or material properties of a laminated composite structure can help prevent sudden failure of the structure [3]. Therefore, the determination of realistic material or mechanical properties of laminated composite components has become an important topic of research. In the past two decades, a number of non-destructive evaluation techniques have been proposed for the determination of material properties of or damages in laminated composite parts [4–7]. Nevertheless, these techniques have their own limitations or specific difficulties when in use. On the other hand, a number of researchers have presented methods to identify or improve the analytical system matrices of a structure using vibration test data [8–12]. For instance, Berman and Nagy [8] developed a method which used measured normal modes and natural frequencies to improve an analytical mass and stiffness matrix model of a structure. Their method could find minimum changes in the analytical model to make it exactly agree with the set of measured modes and frequencies. Kam and his associates [9–12] developed methods to identify the element bending stiffnesses of beam structures using measured natural frequencies and mode shapes or displacements alone.

In this paper, a non-destructive evaluation method is presented for the determination of material elastic constants of laminated composite plates. The method is

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based on the minimization of the sum of the differences between the predicted and measured mechanical responses of a composite plate. A global minimization technique together with an appropriate bounding method for determining the starting points and the search direction are used to solve the minimization problem from which the material constants can be identified. Static tests of several laminated composite plates have been performed and the results used to verify the accuracy and illustrate the applications of the method.

2. Deformation analysis of laminated composite plate

Consider a rectangular plate of area $a \times b$ and constant thickness h subject to transverse load $p(x,y)$ as shown in Fig. 1. The plate is composed of a finite number of layer groups in which each layer group contains several orthotropic layers of same fiber angle and uniform thickness. The x and y coordinates of the plate are taken in the mid-plane of the plate. The displacement field is assumed to be of the form:

$$\begin{aligned} u_1(x,y,z) &= u_0(x,y) + z \cdot \psi_x(x,y), \\ u_2(x,y,z) &= v_0(x,y) + z \cdot \psi_y(x,y), \\ u_3(x,y,z) &= w(x,y), \end{aligned} \quad (1)$$

where u_1, u_2, u_3 are displacements in the x, y, z directions, respectively, and u_0, v_0, w the associated mid-plane displacements; ψ_x and ψ_y are shear rotations.

The constitutive equations of a shear deformable laminated composite plate can be written as

$$\begin{bmatrix} N_1 \\ N_2 \\ Q_y \\ Q_x \\ N_6 \\ M_1 \\ M_2 \\ M_6 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & 0 & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & 0 & 0 & A_{26} & B_{12} & B_{22} & B_{26} \\ 0 & 0 & A_{44} & A_{45} & 0 & 0 & 0 & 0 \\ 0 & 0 & A_{45} & A_{55} & 0 & 0 & 0 & 0 \\ A_{16} & A_{26} & 0 & 0 & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & 0 & 0 & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & 0 & 0 & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & 0 & 0 & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \cdot \begin{bmatrix} u_{0,x} \\ v_{0,y} \\ w_{,y} + \psi_y \\ w_{,x} + \psi_x \\ u_{0,y} + v_{0,x} \\ \psi_{x,x} \\ \psi_{y,y} \\ \psi_{x,y} + \psi_{y,x} \end{bmatrix}, \quad (2)$$

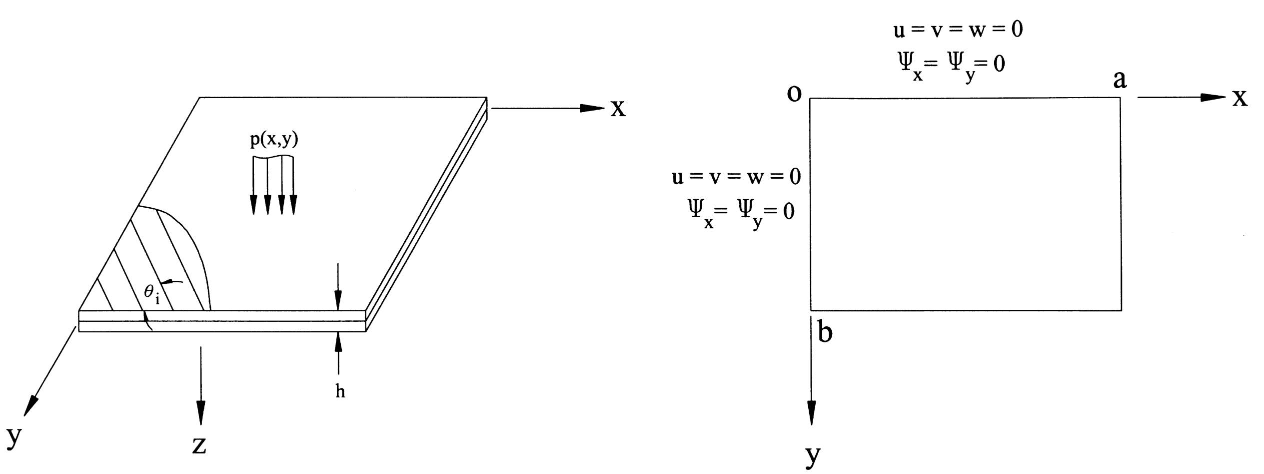
where $N_1, N_2, \ldots, M_6$ are stress resultants; A_{ij}, B_{ij} and D_{ij} are material components; the comma before a subscript denotes the partial derivative with respect to the subscript. The material components are given by

$$(A_{ij}, B_{ij}, D_{ij}) = \int_{-h/2}^{h/2} Q_{ij}^{(m)} (1, z, z^2)\, dz \quad (i,j = 1,2,6) \quad (3a)$$

and

$$A_{ij} = k_\alpha \cdot k_\gamma \cdot \overline{A}_{ij}, \quad \overline{A}_{ij} = \int_{-h/2}^{h/2} Q_{ij}^{(m)}\, dz \quad (i,j = 4,5;\ \alpha = 6 - i;\ \gamma = 6 - j). \quad (3b)$$

The stiffness coefficients $Q_{ij}^{(m)}$ depend on the material properties and orientation of the mth layer group. The parameters k_i are shear correction factors which are



(a) Loading condition (b) Boundary condition

Fig. 1. Laminated composite plate.

determined using the expressions given by Whitney [13]. In the plate analysis, the finite element developed by Kam and Chang [14] is adopted to evaluate the deformation of the plate. The element contains five degrees-of-freedom (three displacements and two slopes, i.e., shear rotations) per node. In the evaluation of the element stiffness matrix, a quadratic element of a serendipity family and the reduced integration are used. The element has been used in the static analysis of both thin and moderately thick laminated composite plates and very good results for strains and displacements can be obtained. The accuracy of the finite element in predicting strains was also investigated via an experimental approach [15]. A square $[0_2°/90_2°/0_2°/90_2°]_s$ graphite/epoxy laminate with clamped edges was loaded at the center and strains at the center of the bottom surface of the laminate were measured in the experiment. The strain data were compared with those predicted by the finite element method. It was found that the use of a 3 × 3 mesh over a quarter plate could yield very good results. In the following identification of material constants, strains and deflections at some particular points on a plate are defined as deformational parameters.

3. Determination of material constants

The problem of material constants identification is formulated as a minimization problem. In mathematical form it is stated as

$$\begin{aligned} &\text{Minimize} \quad e(\underline{x}) = (\underline{D^*})^t(\underline{D^*}) \\ &\text{subject to} \quad x_i^{\mathrm{L}} \leqslant x_i \leqslant x_i^{\mathrm{U}}, \quad i = 1, \ldots, 5, \end{aligned} \tag{4}$$

where $\underline{x} = [E_1, E_2, \nu_{12}, G_{12}, G_{23}]$ the material constants; $\underline{D^*}$ is an N ×1 vector containing the differences between the measured and predicted values of the deformational parameters; $e(\underline{x})$ is an error function measuring the sum of differences between the predicted and measured data; $x_i^{\mathrm{L}}, x_i^{\mathrm{U}}$ are the lower and upper bounds of the material constants. It is noted that the lower and upper bounds of the material constants are chosen in such a way that the lower bound of E_1 is larger than the upper bounds of E_2 and G_{12}. The elements in $\underline{D^*}$ are expressed as

$$D_i^* = \frac{D_{\mathrm{p}i} - D_{\mathrm{m}i}}{D_{\mathrm{m}i}}, \quad i = 1, \ldots, N, \tag{5}$$

where $D_{\mathrm{p}i}$, $D_{\mathrm{m}i}$ are predicted and measured values of deformation parameters, respectively. The above problem of Eq. (4) is then converted into an unconstrained minimization problem by creating the following general augmented Lagrangian:

$$\underline{\Psi}\left(\underline{\tilde{x}}, \underline{\mu}, \underline{\eta}, r_p\right) = e(\underline{x}) + \sum_{j=1}^{5} [\mu_j z_j + r_p z_j^2 + \eta_j \phi_j + r_p \phi_j^2] \tag{6}$$

with

$$\begin{aligned} &z_j = \max\left[g_j(\tilde{x}_j), \frac{-\mu_j}{2r_p}\right], \\ &g_j(\tilde{x}_j) = \tilde{x}_j - \tilde{x}_j^{\mathrm{U}} \leqslant 0, \\ &\phi_j = \max\left[H_j(\tilde{x}_j), \frac{-\eta_j}{2r_p}\right], \\ &H_j(\tilde{x}_j) = \tilde{x}_j^{\mathrm{L}} - \tilde{x}_j \leqslant 0, \quad j = 1, \ldots, 5, \end{aligned} \tag{7}$$

where μ_j, η_j, r_p are multipliers; $\max[^*,^*]$ takes on the maximum value of the numbers in the bracket. The modified design variables $\underline{\tilde{x}}$ are defined as

$$\underline{\tilde{x}} = \left[\frac{E_1}{\alpha_1}, \frac{E_2}{\alpha_2}, \nu_{12}, \frac{G_{12}}{\alpha_3}, \frac{G_{23}}{\alpha_4}\right], \tag{8}$$

where α_i are the normalization factors. The update formulas for the multipliers μ_j, η_j and r_p are

$$\begin{aligned} &\mu_j^{n+1} = \mu_j^n + 2r_p^n z_j^n, \\ &\eta_j^{n+1} = \eta_j^n + 2r_p^n \phi_j^n, \quad j = 1, \ldots, 5, \end{aligned}$$

$$r_p^{n+1} = \begin{cases} \gamma_0 r_p^n & \text{if } r_p^{n+1} < r_p^{\max}, \\ r_p^{\max} & \text{if } r_p^{n+1} \geqslant r_p^{\max}, \end{cases} \tag{9}$$

where the superscript n denotes iteration number; γ_0 is a constant; $r_p^{\max}$ is the maximum value of r_p. The parameters $\mu_j^0, \eta_j^0, r_p^0, \gamma_0$ and $r_p^{\max}$ are chosen as

$$\begin{aligned} &\mu_j^0 = 1.0, \quad \eta_j^0 = 1.0, \quad j = 1, \ldots, 5, \\ &r_p^0 = 0.4, \quad \gamma_0 = 1.25, \quad r_p^{\max} = 100. \end{aligned} \tag{10}$$

The constrained minimization problem of Eq. (6) has thus become the solution of the following unconstrained problem:

$$\text{Minimize} \quad \underline{\Psi}(\underline{\tilde{x}}, \underline{\mu}, \underline{\eta}, r_p). \tag{11}$$

The solution of the above unconstrained optimization problem is obtained by using the previously proposed unconstrained multi-start global optimization algorithm [16,17]. In the adopted optimization algorithm, the objective function to be minimized is treated as the potential energy of a traveling particle and the search trajectories for locating the global minimum are derived from the equation of motion of the particle in a conservative force field. The design variables, i.e., material constants, that make the potential energy of the particle, i.e., objective function, the global minimum constitute the solution of the problem. In the minimization process, a series of starting points are selected at random from the region of interest and the lowest local minimum along the search trajectory initiated from each starting point is determined. A Bayesian argument is then used to establish the probability of the current overall minimum value of the objective function being the global minimum, given the number of starts and the number of times this value has been achieved. The multi-start procedure is terminated once a target probability, typically 0.99, has been exceeded.

4. Experimental investigation

The composite materials under consideration are T300/2500 graphite/epoxy produced by Torayca, Japan. The properties of the graphite/epoxy material are first determined experimentally in accordance with the relevant ASTM specifications [18]. Each material constant was determined from tests using five specimens. The mean values and coefficients of variation of the experimentally determined material constants are given as follows:

$$E_1 = 124.68 \text{ GPa } (2.75\%), \quad E_2 = 9.6 \text{ GPa } (3.24\%),$$
$$G_{12} = 8.64 \text{ GPa } (2.8\%), \quad G_{23} = 2.32 \text{ GPa } (6.82\%),$$
$$\nu_{12} = 0.33 \ (5.1\%). \tag{12}$$

For experimental investigation, a number of square symmetric laminates, namely, $[45_2°/0_3°/-45_2°/0_3°/45_2°]_s$ and $[0_2°/90_2°/0_2°/90_2°]_s$ of dimensions 14 cm × 14 cm were manufactured and subjected to static tests in accordance with the test procedure described in Ref. [19]. The lamina thickness for the laminated plates is 0.125 mm. A schematic description of the experimental setup is shown in Fig. 2 in which the laminate is clamped at all edges and the actual dimensions of the laminate are 10 cm × 10 cm. The laminated plates were subjected to two types of loadings, namely, a center point load or a uniformly distributed load. A number of displacement transducers (LVDT) and strain gauges were placed beneath the bottom surface of the laminate for measuring the deformational parameters of the laminate. The load–displacement and load–strain curves of the laminates were constructed using the data measured from the displacement transducer (LVDT) and strain gauges, respectively. The points on the bottom surface of the laminated plate at which the displacements and strains were measured is shown in Fig. 3.

5. Results and discussions

The aforementioned non-destructive evaluation method will be applied to the material characterization of the laminated composite plates which have been tested. The upper and lower bounds of the material constants are chosen based on experience.

$$40 \leqslant E_1 \leqslant 400 \text{ GPa}, \quad 0 \leqslant E_2 \leqslant 40 \text{ GPa},$$
$$0 \leqslant G_{12} \leqslant 40 \text{ GPa}, \quad 0 \leqslant G_{23} \leqslant 40 \text{ GPa}, \tag{13}$$
$$0 \leqslant \nu_{12} \leqslant 0.5.$$

The modified design variables are obtained via the use of the following normalization factors:

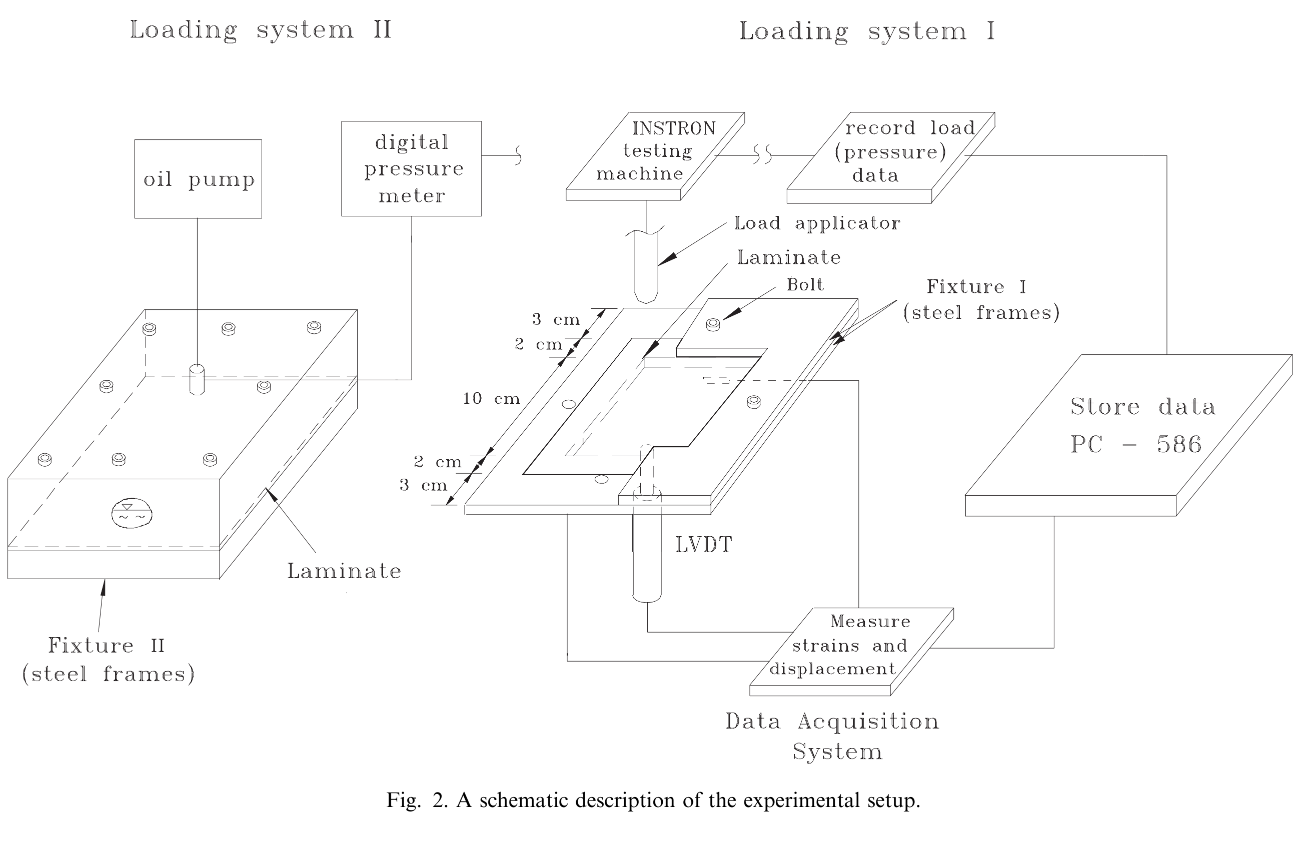


Fig. 2. A schematic description of the experimental setup.

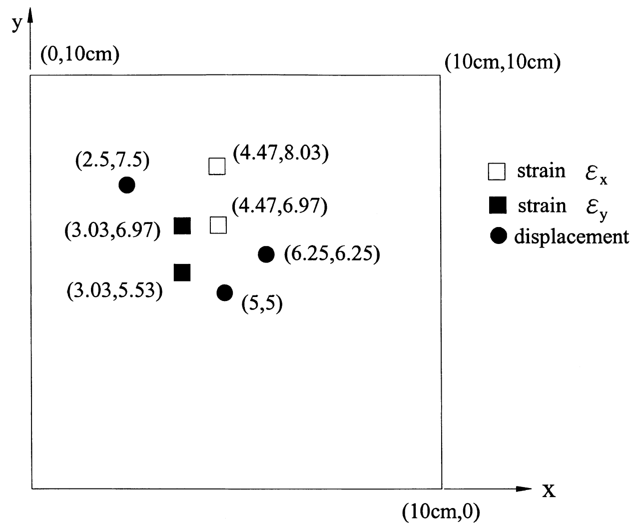


Fig. 3. Locations on laminated plates for the measurements of displacements and strains.

$$\alpha_1 = 1000, \qquad \alpha_i = 100 \quad (i = 2, 3, 4). \tag{14}$$

Three deformational parameters are used in the present identification method to determine the material constants of the laminated composite plates. About eight starting points have been randomly selected to obtain the global minimum in the identification process. The predicted material constants as well as the percentage differences between the theoretically predicted and experimentally determined material constants for different cases are listed in Tables 1 and 2. It is noted that the present method can produce results of acceptable accuracy for the cases under consideration. In particular, for the laminated plates subjected to a point load, excellent results can be obtained for the predictions of the material constants and in general their errors are equal to or less then 9%. As for the $[0_2°/90_2°/0_2°/90_2°]_s$ plates subjected to uniform loads, the error for the prediction of G_{23} can be as large as 26.7%. The cause of the relatively large errors in the prediction of G_{23} is due to the small contribution of transverse shear deformation to the magnitudes of the deformational parameters when the plate is under uniform load. Due to the facts that the value of Poisson's ratio is small and it has small effects on the deformation of the laminated composite plates, the error in the prediction of ν_{12} tend to be large when compared with those of E_1, E_2 and G_{12}. After all it is obvious that point load is more appropriate than uniform load for use in the identification of the five material constants. It is also noted that if the present normalization and

Table 1
Material constant identification of laminated composite plates subjected to a uniform load

Plate lay-up	Measured deformational parameters	Identified material constants
$[45_2°/0_3°/-45_2°/0_3°/45_2°]_s$	d (5 cm, 5 cm)[a] = 0.12 mm	E_1 = 125.1 GPa (0.34%)[b], E_2 = 9.6 GPa (0%)
	ε_x (4.47 cm, 6.97 cm) = 0.112×10^{-3}	G_{12} = 8.6 GPa (0.46%), G_{23} = 2.5 GPa (7.8%)
	ε_y (3.03 cm, 5.53 cm) = 0.14×10^{-3}	ν_{12} = 0.294 (10.9%)
$[0_2°/90_2°/0_2°/90_2°]_s$	d (5 cm, 5 cm) = 0.348 mm	E_1 = 123.67 GPa (0.8%), E_2 = 9.74 GPa (1.46%)
	ε_x (4.47 cm, 6.97 cm) = 0.368×10^{-3}	G_{12} = 8.65 GPa (0.12%), G_{23} = 2.79 GPa (20.2%)
	ε_y(3.03 cm, 5.53 cm) = 0.33×10^{-3}	ν_{12} = 0.36 (9%)
	d(2.5 cm, 7.5 cm) = 0.125 mm	E_1 = 124.7 GPa (0.016%), E_2 = 9.57 GPa (0.33%)
	d(5 cm, 5 cm) = 0.348 mm	G_{12} = 7.87 GPa (8.9%), G_{23} = 2.94 GPa (26.7%)
	d(6.25 cm, 6.25 cm) = 0.275 mm	ν_{12} = 0.39 (18.2%)

[a] The values in the parentheses denote coordinates.
[b] The values in the parentheses denote percentage difference between predicted and measured data.

Table 2
Material constant identification of laminated composite plates subjected to a point load

Plate lay-up	Measured deformational parameters	Identified material constants
$[45_2°/0_3°/-45_2°/0_3°/45_2°]_s$	d (5 cm, 5 cm)[a] = 0.595 mm	E_1 = 124.3 GPa (0.3%)[b], E_2 = 9.8 GPa (2.1%)
	ε_x (4.47 cm, 8.03 cm) = 3.5×10^{-4}	G_{12} = 8.7 GPa (1.1%), G_{23} = 2.13 GPa (8.2%)
	ε_y (3.03 cm, 6.97 cm) = 1.03×10^{-4}	ν_{12} = 0.3 (9%)
$[0_2°/90_2°/0_2°/90_2°]_s$	d (5 cm, 5 cm) = 1.63 mm	E_1 = 124.2 GPa (0.41%), E_2 = 9.8 GPa (2.1%)
	ε_x (4.47 cm, 6.97 cm) = 1.506×10^{-3}	G_{12} = 8.2 GPa (4.68%), G_{23} = 2.3 GPa (0.9%)
	ε_y (3.03 cm, 5.53 cm) = 1.89×10^{-3}	ν_{12} = 0.325 (1.5%)
	d (2.5 cm, 7.5 cm) = 0.352 mm	E_1 = 125.1 GPa (0.34%), E_2 = 9.82 GPa (2.3%)
	d (5 cm, 5 cm) = 1.63 mm	G_{12} = 7.89 GPa (8.7%), G_{23} = 2.13 GPa (8.2%)
	d (6.25 cm 6.25 cm) = 1.01 mm	ν_{12} = 0.31 (6%)

[a] The values in the parentheses denote coordinates.
[b] The values in the parentheses denote percentage difference between predicted and measured data.

bounding techniques were not adopted in the material constants identification process, erroneous results would be obtained or there would be difficulties in making the solution converge.

6. Conclusion

A method for non-destructive evaluation of material constants of laminated composite structures was presented. Five material constants were identified via the minimization of the error function which was used to measure the differences between the theoretical and experimental deformations. A global minimization algorithm together with an appropriate normalization technique were used to determine the global minimum. A number of laminated composite plates subjected to different loading conditions were tested and three measured deformational parameters were used in the present method for material constants identification. The study showed that the present method could produce reasonably good results. It was also demonstrated that loading condition could affect the accuracies of the identified values of the material constants and the use of point load was better than the use of uniform load. The present method is general and can be extended to the material constants identification of other types of structures.

Acknowledgements

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