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DEA Malmquist productivity measure: Taiwanese semiconductor companies

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Abstract

In this research we employ data envelopment analysis (DEA) to measure the Malmquist productivity of semiconductor packaging and testing firms in Taiwan from 2000 to 2003. Malmquist productivity has three components: the measurement of technical change, the measurement of the frontier forward shift, and the measurement of the frontier backward shift of a company over two consecutive periods. This approach not only reveals patterns of productivity change and presents a new interpretation along with the managerial implication of each Malmquist component, but also identifies the strategy shifts of individual companies based upon isoquant changes. Therefore, one can judge with greater accuracy whether or not such strategy shifts are favorable and promising. We use slacks-based measurement (SBM) and Super-SBM models to obtain more accurate measurements. Comparison is made between the results from SBM/Super-SBM and CCR models.

Keywords: Data envelopment analysis; Malmquist productivity; Super-SBM

1. Introduction

DEA is a multiple input–output efficient technique that measures the relative efficiency of decision-making units (*DMUs*) using a linear programming based model. The technique is non-parametric because it requires no assumption about the weights of the underlying production function. DEA was originally proposed by Charnes et al. (1978) and this model is commonly referred to as a CCR model. The DEA frontier *DMUs* are those with maximum output levels for given input levels or with minimum input levels for given output levels. DEA provides efficiency scores for individual units as their technical efficiency measure, with a score of one assigned to the frontier (efficient) units.

Färe et al. (1992, 1994a) developed the DEA-based Malmquist productivity index by CCR model. The DEA-based Malmquist productivity is a combined index that can be extended to measure the productivity change of *DMUs* over time. It has been applied in many ways, as described in Färe et al. (1994b), Grifell-Tatjé and Lovell (1996), Fulginiti and Perrin (1997), Löthgren and Tambour (1999), Herrero and Pascoe (2004), Wei (2006) and others. The two components embedded in Malmquist productivity, measuring the changes in technology frontier and technical efficiency, are also further examined in this research. By the technology frontier shift (*FS*), the development or decline of all *DMUs* is able to measure. Technical efficiency

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change (*TEC*) is used to measure the change in technical efficiency. It is also a measure of how much closer to the frontier the company (*DMU*) is when crossing the two consecutive times. We define *TEC* and Malmquist productivity as R_3 and R_4, respectively, in Section 4.1 for the performance measurement.

Chen and Ali (2004) applied the DEA Malmquist productivity measure to the computer industries by the CCR model to assess the four distance functions of Malmquist productivity. Moreover, they discovered more information about the two components that obscure in the Malmquist productivity index. We define them as R_1 and R_2 in Section 3 for the performance measurement in this research and account for the attributes. Their approach not only reveals patterns of productivity change and presents a new interpretation along with the managerial implication of each component, but also identifies the strategy shifts of individual *DMU*s in a particular time period. They determined whether such strategy shifts were favorable and improving.

However, the ratio efficiency θ_0^* by the CCR model is not able to take account of slacks. For instance, the optimal solution $\theta_0^* = 1$ might be with positive slacks. In the DEA Malmquist productivity, the DMU_0 is regarded as efficient but actually, it should be regarded as inefficient. Therefore, it is important to observe both the ratio efficiency and the slacks. Some attempts have been made to unify θ_0^* and slacks into a scalar measure.

Charnes et al. (1985) developed the additive model of DEA, which deals directly with input excess and output shortfalls. But this model has no scalar measure (ratio efficiency) per se. Thus, although this model can discriminate between efficient and inefficient *DMU*s by the existence of slacks, it has no means of gauging the depth of inefficiency, similar to θ_0^* in the CCR model.

Tone (2001) developed a slacks-based measure (SBM) of efficiency in DEA, which takes account of scalar measure and slacks. Further, Tone (2002) developed a SBM of super efficiency (Super-SBM) in DEA for discriminating between efficient *DMU*s. Super efficiency measures the degree of superiority that efficient DMU_0 possesses against other *DMU*s.

To extend the investigation on influence from slacks to Malmquist productivity index, Chen (2003) proposed a non-radial Malmquist productivity index, which is able to eliminate possible inefficiency represented by the non-zero slacks to measure the productivity change of three Chinese major industries. Instead, we employ the SBM and Super-SBM models in this research. In addition to *TEC* (R_3) and Malmquist productivity (R_4) which existed in the traditional Malmquist productivity measurement, we also investigate the two components—R_1 and R_2 proposed by Chen and Ali (2004) to interpret a more detailed management implication. The next section reviews how the DEA-based Malmquist productivity index works. We also present the Malmquist productivity approach.

2. DEA Malmquist productivity index

Färe et al. (1992) construct the DEA-based Malmquist productivity index as the geometric mean of the two Malmquist productivity indices of Caves et al. (1982): one measures the change in efficiency and the other measures the change in the frontier technology. The frontier technology, determined by the efficient frontier, is estimated using DEA for a set of *DMU*s.

There are n *DMU*s under comparison for their performance. Let x_{ij} and y_{rj} denote the value of the ith input $(i = 1,\ldots,m)$ and the rth output $(r = 1,\ldots,s)$ of DMU_j $(j = 1,\ldots,n)$, respectively. The slack variables for the ith input and the rth output are, respectively, represented by s_i^- and s_r^+, which indicate the *input excess* and *output shortfall*, respectively. The variable λ_j denotes the weight of DMU_j while assessing the performance θ_0 of the object DMU_0.

Instead of a radial-based model, we now use the SBM model and explain the reason for the substitution. A notation with '*' in superscript indicates it is the optimal solution. We must first know two proved theorems: (I) The optimal SBM ρ_0^* is not greater than the optimal CCR θ_0^*, and (II) A *DMU* (x_{i0},y_{r0}) is CCR-efficient, if only if DMU_0 is SBM-efficient. Moreover, because the CCR score is a radical measure and takes no account of slacks, the particular DMU_0 may have an efficiency score $\theta_0^* = 1$ although it has a shortfall $s_r^{+*} \geqslant 0$, but an inefficiency score $\rho_0^* \leqslant 1$ for SBM measure when the factor is taken into account. In this case, we can reduce the misleading result with the SBM measure. On the other hand, the SBM score $\rho_0^* = 1$ guarantees the particular *DMU* has the more precise efficiency score. Tone (2004) discusses the differences between the slack-based and radial-based approaches in depth.

Let $D^a(x_0^b, y_0^b)$ denote the relative efficiency of a particular DMU_0 in period b against the performance of those DMUs in period a. There are four possible pairs (a,b) for analysis of the Malmquist productivities, (t,t), $(t+1,\ t)$, $(t,t+1)$ and $(t+1,t+1)$. Hence, there are four distances to be measured, $D^t(x_0^t, y_0^t)$, $D^{t+1}(x_0^t, y_0^t)$, $D^t(x_0^{t+1}, y_0^{t+1})$, and $D^{t+1}(x_0^{t+1}, y_0^{t+1})$, and they are denoted as the efficiency score ρ_{10}^*, ρ_{20}^*, ρ_{30}^* and ρ_{40}^*, respectively. Let x_{i0}^t and y_{r0}^t denote DMU_0's ith input and rth output, respectively, in time period t. Employing the SBM model introduced in Tone (2001), the following model (M1) is used to measure the relative efficiencies of DMU_0 for (a,b) equal to (t,t) or $(t+1,t+1)$.

$$\rho_{q0}^* = Min \quad D^a(x_0^b, y_0^b) = k - \frac{1}{m}\sum_{i=1}^{m}(S_i^-/x_{i0}^b),$$

$$q = 1 \text{ and } 4.$$

$$S.t. \quad k + \frac{1}{s}\sum_{r=1}^{s}(S_r^+/y_{r0}^b) = 1,$$

$$kx_{i0}^b = \sum_{j=1}^{n} x_{ij}^b k\lambda_j + S_i^-, \quad i = 1,2,\ldots,m, \qquad \text{(M1)}$$

$$ky_{r0}^b = \sum_{j=1}^{n} y_{rj}^b k\lambda_j - S_r^+, \quad r = 1,2,\ldots,s,$$

$$\lambda_j \geqslant 0,\ j = 1,2,\ldots,n;\ k \geqslant 0;\ S_i^- \geqslant 0,$$

$$i = 1,2,\ldots,m;\ S_r^+ \geqslant 0,\ r = 1,2,\ldots,s.$$

The optimal solutions λ_j^*, k^*, S_i^{-*}, S_r^{+*}, ρ_{q0}^* are obtained. Further, the excess and the shortfall can be obtained indirectly: $s_i^{-*} = S_i^{-*}/k^*$, $s_r^{+*} = S_r^{+*}/k^*$. For instance, ρ_{10}^* is the relative efficiency score. The values $\hat{x}_{i0}^b = x_{i0}^b - s_i^{-*}$, $i = 1–m$, and $\hat{y}_{r0}^b = y_{r0}^b + s_r^{+*}$, $r = 1–s$ are its projection points on the efficient frontier constructed by the DMUs performed in period a.

If $\rho_{q0}^* = 1$, we employ the Super-SBM model introduced in Tone (2002) to measure the distance of DMU_0 to the frontier that is constructed by the other DMUs. The following model (M2) is used to compute the distance π_{q0}^*. Its projection point on the frontier is obtained $(\overline{X}_0^b,\ \overline{Y}_0^b)$ where $\overline{X}_0^b = (\bar{x}_{i0}^b, i = 1-m)$ and $\overline{Y}_0^b = (\bar{y}_{r0}^b, r = 1-s)$. $\bar{x}_{i0}^b = \tilde{x}_{i0}^{b*}/\tau^*$, $\bar{y}_{r0}^b = \tilde{y}_{r0}^{b*}/\tau^*$.

$$\pi_{q0}^* = Min \quad \frac{1}{m}\sum_{i=1}^{m}\frac{\tilde{x}_{i0}^b}{x_{i0}^b}, \quad q = 1 \text{ and } 4.$$

$$S.t. \quad 1 = \frac{1}{s}\sum_{r=1}^{s}\frac{\tilde{y}_{r0}^b}{y_{r0}^b},$$

$$\tilde{x}_{i0}^b \geqslant \sum_{j=1,\neq 0}^{n} x_{ij}^a \Lambda_j^a, \quad i = 1,2,\ldots,m,$$

$$\tilde{y}_{r0}^b \leqslant \sum_{j=1,\neq 0}^{n} y_{rj}^a \Lambda_j^a, \quad r = 1,2,\ldots,s,$$

$$\tilde{x}_{i0}^b \geqslant \tau x_{i0}^b, \quad i = 1,2,\ldots,m,$$

$$0 \leqslant \tilde{y}_{r0}^b \leqslant \tau y_{r0}^b, \quad r = 1,2,\ldots,s,$$

$$\Lambda_j^a \geqslant 0, \quad j = 1,2,\ldots,n; \ \tau > 0. \qquad \text{(M2)}$$

The mixed period measures, $(a,b) = (t+1,\ t)$, which is defined as ρ_{20}^* for each DMU_0, is computed as the optimal value to the following SBM model (M3). In particular, the object DMU_0 is also included in the production possibility set. The model is also used for the second mixed period measures ρ_{30}^* where $(a,b) = (t,t+1)$.

$$\rho_{q0}^* = Min \quad D^a(x_0^b, y_0^b) = k - \frac{1}{m}\sum_{i=1}^{m}(S_i^-/x_{i0}^b),$$

$$q = 2 \text{ and } 3.$$

$$S.t. \quad k + \frac{1}{s}\sum_{r=1}^{s}(S_r^+/y_{r0}^b) = 1,$$

$$kx_{i0}^b = \sum_{j=1}^{n} x_{ij}^b k\lambda_j + x_{i0}^b k\lambda_{n+1} + S_i^-,$$

$$i = 1,2,\ldots,m,$$

$$ky_{r0}^b = \sum_{j=1}^{n} y_{rj}^b k\lambda_j + y_{r0}^b k\lambda_{n+1} - S_r^+,$$

$$r = 1,2,\ldots,s,$$

$$\lambda j \geqslant 0,\ j = 1,2,\ldots,(n+1);\ k \geqslant 0;$$

$$S_i^- \geqslant 0,\ i = 1,2,\ldots,m;$$

$$S_r^+ \geqslant 0,\ r = 1,2,\ldots,s. \qquad \text{(M3)}$$

If $\rho_{q0}^* = 1$, employ the following Super-SBM model (M4) to measure the super-efficiency score π_{q0}^*.

$$\pi_{q0}^* = Min \quad \frac{1}{m}\sum_{i=1}^{m}\frac{\tilde{x}_{i0}^b}{x_{i0}^b}, \quad q = 2 \text{ and } 3.$$

$$S.t. \quad 1 = \frac{1}{s}\sum_{r=1}^{s}\frac{\tilde{y}_{r0}^b}{y_{r0}^b},$$

$$\tilde{x}^b_{i0} \geqslant \sum_{j=1}^{n} x^a_{ij} \Lambda^a_j, \quad i = 1, 2, \ldots, m,$$
$$\tilde{y}^b_{r0} \leqslant \sum_{j=1}^{n} y^a_{rj} \Lambda^a_j, \quad r = 1, 2, \ldots, s,$$
$$\tilde{x}^b_{i0} \geqslant \tau x^b_{i0}, \quad i = 1, 2, \ldots, m,$$
$$0 \leqslant \tilde{y}^b_{r0} \leqslant \tau y^b_{r0}, \quad r = 1, 2, \ldots, s, \qquad \text{(M4)}$$

$\Lambda^a_j \geqslant 0, \quad j = 1, 2, \ldots, n; \quad \tau > 0.$

The efficiency score ρ^*_{q0} is replaced by the value π^*_{q0}.

Therefore ρ^*_{10}, ρ^*_{20}, ρ^*_{30}, and ρ^*_{40} fall into one of the three ranges: >1, $=1$, or <1. The Malmquist productivity index (Färe et al., 1992) measures the productivity change of a particular DMU_0 in period t and $(t+1)$:

$$M_0^{t+1} = \left[\frac{D^t(x_0^{t+1}, y_0^{t+1})}{D^t(x_0^t, y_0^t)} \frac{D^{t+1}(x_0^{t+1}, y_0^{t+1})}{D^{t+1}(x_0^t, y_0^t)} \right]^{1/2}. \qquad (1)$$

When $M_0^{t+1} > 1$, this signifies a productivity gain; when $M_0^{t+1} < 1$, this signifies a productivity loss; and when $M_0^{t+1} = 1$, there is no change in productivity.

The above measure is actually the geometric mean of two Malmquist productivity indices: technical efficiency change (TEC_0) and frontier shift (FS_0) (Caves et al., 1982; Färe et al. 1992).

$$M_0^{t+1} = \left[\frac{D^t(x_0^{t+1}, y_0^{t+1})}{D^t(x_0^t, y_0^t)} \frac{D^{t+1}(x_0^{t+1}, y_0^{t+1})}{D^{t+1}(x_0^t, y_0^t)} \right]^{1/2} = TEC_0 * FS_0, \qquad (2)$$

$$TEC_0 = \frac{D^{t+1}(x_0^{t+1}, y_0^{t+1})}{D^t(x_0^t, y_0^t)} = R3, \qquad (3)$$

$$FS_0 = \left[\frac{D^t(x_0^{t+1}, y_0^{t+1})}{D^{t+1}(x_0^{t+1}, y_0^{t+1})} \frac{D^t(x_0^t, y_0^t)}{D^{t+1}(x_0^t, y_0^t)} \right]^{1/2} = (R_1 * R_2)^{1/2}. \qquad (4)$$

TEC_0 is used to measure the change in technical efficiency; on the other hand, it is also a measure of how much closer to the boundary the company is in period $(t+1)$ compared with period t. If TEC_0 is 1.0, the particular DMU_0 (maybe a company) has the same distance in periods $(t+1)$ and t from the respective efficient boundaries. If TEC_0 is over 1.0, the company has moved closer to the period $(t+1)$ boundary than it was to the period t boundary; the converse is the case if the TEC_0 is under 1.0. As for FS_0, it is used to measure the technology frontier shift between time periods t and $(t+1)$. Färe et al. (1992, 1994a) point out that a value of FS_0 less than 1.0 indicates negative shift of frontier or technical regress; FS_0 greater than 1.0 indicates positive shift of frontier or technical progress; FS_0 equal to 1.0 indicates no shift in technology frontier.

3. Insights from the Malmquist productivity approach

Chen and Ali (2004) further analyzed the properties of two ratios of FS_0, $D^t(x_0^{t+1}, y_0^{t+1})/D^{t+1}(x_0^{t+1}, y_0^{t+1})$ and $D^t(x_0^t, y_0^t)/D^{t+1}(x_0^t, y_0^t)$, the backward and forward frontier shifts, respectively. They are the performance of DMU_0 in periods $(t+1)$ and t against the frontiers of period t and $(t+1)$.

As depicted in Fig. 1, a company's performance in period t could be the six possible locations, A_1^t–A_6^t. The oblique line that connects the origin and the intersection of the two frontiers is the tradeoff on the strategy changes. A_1^t, A_2^t, and A_3^t locate on the upper part and inside the t-frontier, between the two frontiers, and outside the $(t+1)$-frontier, respectively. The distances of A_2^t and A_3^t to the t- and $(t+1)$-frontiers, respectively, are the measurement of super-efficiencies. Similarly, A_4^t, A_5^t, and A_6^t locate on the lower part and inside the $(t+1)$-frontier, between the two frontiers, and outside the t-frontier, respectively. The distances of A_6^t and A_5^t to the t- and $(t+1)$-frontiers, respectively, are the measurement of super-efficiencies. It is noticeable that the locations of the six points A_1^{t+1}–A_6^{t+1} have similar occasions.

For convenience of illustration, we temporarily employ a radial model such as CCR to express the efficiency measurement of each point by the ratio of distances; for instance, by drawing a line that connects the origin and point A_1^{t+1}. The line intersects with the t-frontier and $(t+1)$-frontier at points α_1 and β_1, respectively. The ratio of $D^t(x_0^{t+1}, y_0^{t+1})$ to $D^{t+1}(x_0^{t+1}, y_0^{t+1})$ could be expressed as $\overline{O\alpha_1}/\overline{OA_1^{t+1}}$ and $\overline{O\beta_1}/\overline{OA_1^{t+1}}$, respectively. Thus, $D^t(x_0^{t+1}, y_0^{t+1})/D^{t+1}(x_0^{t+1}, y_0^{t+1}) = \overline{O\alpha_1}/\overline{O\beta_1}$. Similarly, drawing a line connects the origin and point A_1^t. The line intersects with the t-frontier and $(t+1)$-frontier at points γ_1 and δ_1, respectively. Tables 1 and 2 depict the models employed to measure the two distances. The signs of R_1 and R_2 in the last columns are visible from Fig. 1.

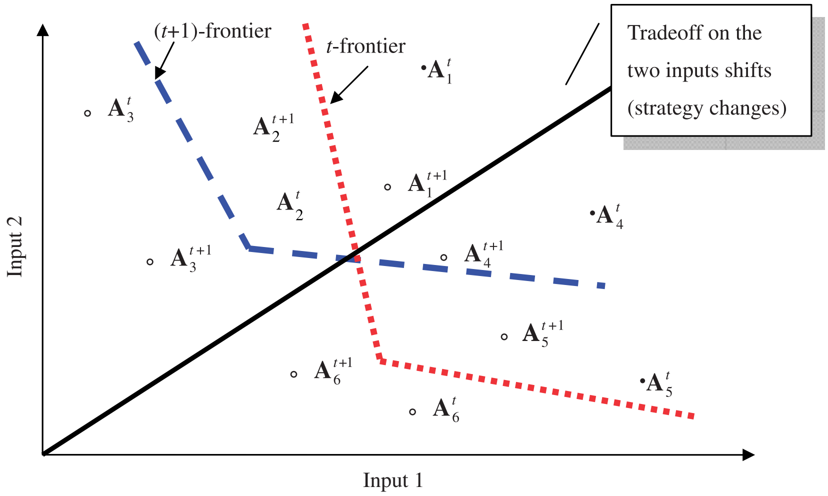

Fig. 1. Frontier shift.

Table 1
The computation of ratio R_1

$t+1$	$D^t(x_0^{t+1}, y_0^{t+1})$	$D^{t+1}(x_0^{t+1}, y_0^{t+1})$	$R_1 = \frac{D^t(x_0^{t+1}, y_0^{t+1})}{D^{t+1}(x_0^{t+1}, y_0^{t+1})}$	R_1
A_1^{t+1}	Use M3 ($\rho_{30}^* < 1$)	Use M1 ($\rho_{40}^* < 1$)	$\frac{\overline{O\alpha_1}/\overline{OA_1^{t+1}}}{\overline{O\beta_1}/\overline{OA_1^{t+1}}} = \frac{\overline{O\alpha_1}}{\overline{O\beta_1}}$	>1
A_2^{t+1}	Use M4 ($\pi_{30}^* > 1$)	Use M1 ($\rho_{40}^* < 1$)	$\frac{\overline{O\alpha_2}/\overline{OA_2^{t+1}}}{\overline{O\beta_2}/\overline{OA_2^{t+1}}} = \frac{\overline{O\alpha_2}}{\overline{O\beta_2}}$	>1
A_3^{t+1}	Use M4 ($\pi_{30}^* > 1$)	Use M2 ($\pi_{40}^* > 1$)	$\frac{\overline{O\alpha_3}/\overline{OA_3^{t+1}}}{\overline{O\beta_3}/\overline{OA_3^{t+1}}} = \frac{\overline{O\alpha_3}}{\overline{O\beta_3}}$	>1
A_4^{t+1}	Use M3 ($\rho_{30}^* < 1$)	Use M1 ($\rho_{40}^* < 1$)	$\frac{\overline{O\alpha_4}/\overline{OA_4^{t+1}}}{\overline{O\beta_4}/\overline{OA_4^{t+1}}} = \frac{\overline{O\alpha_4}}{\overline{O\beta_4}}$	<1
A_5^{t+1}	Use M3 ($\rho_{30}^* < 1$)	Use M2 ($\pi_{40}^* > 1$)	$\frac{\overline{O\alpha_5}/\overline{OA_5^{t+1}}}{\overline{O\beta_5}/\overline{OA_5^{t+1}}} = \frac{\overline{O\alpha_5}}{\overline{O\beta_5}}$	<1
A_6^{t+1}	Use M4 ($\pi_{30}^* > 1$)	Use M2 ($\pi_{40}^* > 1$)	$\frac{\overline{O\alpha_6}/\overline{OA_6^{t+1}}}{\overline{O\beta_6}/\overline{OA_6^{t+1}}} = \frac{\overline{O\alpha_6}}{\overline{O\beta_6}}$	<1

In Fig. 1, a downward frontier shift (towards the origin) from period t to $(t+1)$ represents a positive shift. The converse situation (away from the origin) represents a negative shift. For a company, from period t to $(t+1)$, the four possible frontier shifts are as follows in (a)–(d). The 36 possible movements are depicted in Table 3.

(a) If $R_2>1$ and $R_1>1$, then the FS_0 must be larger than 1.0, indicating the DMU_0 has a positive shift and the technology of DMU_0 progresses. As shown in Fig. 1, the points of period t, A_1^t, A_2^t, and A_3^t in the *upper* part could be one of the points at period $(t+1)$ in the *upper* part, A_1^{t+1}, A_2^{t+1}, and A_3^{t+1}.

Table 2
The computation of ratio R2

t	$D^t(x_0^t, y_0^t)$	$D^{t+1}(x_0^t, y_0^t)$	$R_2 = \dfrac{D^t(x_0^t, y_0^t)}{D^{t+1}(x_0^t, y_0^t)}$	R_2
A_1^t	Use M1 ($\rho_{10}^* < 1$)	Use M3 ($\rho_{20}^* < 1$)	$\dfrac{\overline{O\gamma_1}/\overline{OA_1^t}}{\overline{O\delta_1}/\overline{OA_1^t}} = \dfrac{\overline{O\gamma_1}}{\overline{O\delta_1}}$	>1
A_2^t	Use M2 ($\pi_{10}^* > 1$)	Use M3 ($\rho_{20}^* < 1$)	$\dfrac{\overline{O\gamma_2}/\overline{OA_2^t}}{\overline{O\delta_2}/\overline{OA_2^t}} = \dfrac{\overline{O\gamma_2}}{\overline{O\delta_2}}$	>1
A_3^t	Use M2 ($\pi_{10}^* > 1$)	Use M4 ($\pi_{20}^* > 1$)	$\dfrac{\overline{O\gamma_3}/\overline{OA_3^t}}{\overline{O\delta_3}/\overline{OA_3^t}} = \dfrac{\overline{O\gamma_3}}{\overline{O\delta_3}}$	>1
A_4^t	Use M1 ($\rho_{10}^* < 1$)	Use M3 ($\pi_{20}^* > 1$)	$\dfrac{\overline{O\gamma_4}/\overline{OA_4^t}}{\overline{O\delta_4}/\overline{OA_4^t}} = \dfrac{\overline{O\gamma_4}}{\overline{O\delta_4}}$	<1
A_5^t	Use M1 ($\rho_{10}^* < 1$)	Use M4 ($\pi_{20}^* > 1$)	$\dfrac{\overline{O\gamma_5}/\overline{OA_5^t}}{\overline{O\delta_5}/\overline{OA_5^t}} = \dfrac{\overline{O\gamma_5}}{\overline{O\delta_5}}$	<1
A_6^t	Use M2 ($\pi_{10}^* > 1$)	Use M4 ($\pi_{20}^* > 1$)	$\dfrac{\overline{O\gamma_6}/\overline{OA_6^t}}{\overline{O\delta_6}/\overline{OA_6^t}} = \dfrac{\overline{O\gamma_6}}{\overline{O\delta_6}}$	<1

Table 3
The four possible frontier shifts for a company between two periods

From period t	To period $(t+1)$					
	A_1^{t+1}	A_2^{t+1}	A_3^{t+1}	A_4^{t+1}	A_5^{t+1}	A_6^{t+1}
A_1^t A_2^t A_3^t	(a) $R_2>1$ and $R_1>1$			(d) $R_2>1$ and $R_1<1$		
A_4^t A_5^t A_6^t	(c) $R_2<1$ and $R_1>1$			(b) $R_2<1$ and $R_1<1$		

(b) If $R_2<1$ and $R_1<1$, then the FS_0 must be less than 1.0, indicating the DMU_0 has a negative shift and the technology of DMU_0 declines. As shown in Fig. 1, the points of period t, A_4^t, A_5^t, and A_6^t in the *lower* part could be one of the points at period $(t+1)$ in the *lower* part, A_4^{t+1}, A_5^{t+1}, and A_6^{t+1}.

(c) If $R_2<1$ and $R_1>1$, then FS_0 may be larger or less than 1.0. But, certainly we can conclude DMU_0 moves from a negative shift facet towards a positive shift facet. Also, there is a change in the tradeoff between the two inputs. Furthermore, $FS_0<1$ indicates that the change resulting from the positive shift facet is less than that of the negative shift facet; and, on average, the technology of DMU_0 declines. In contrast, $FS_0>1$ indicates that the change resulting from the positive shift facet is lager than that of the negative shift facet; and, on average, the technology of DMU_0 progresses. $FS_0=1$ indicates that, on average, the technology of DMU_0 remains the same. As shown in Fig. 1, the points of period t, A_4^t, A_5^t, and A_6^t in the *lower* part could be one of the points at period $(t+1)$ in the *upper* part, A_1^{t+1}, A_2^{t+1}, and A_3^{t+1}.

(d) If $R_2>1$ and $R_1<1$, then FS_0 may be greater or less than 1.0. But, we can certainly conclude DMU_0 moves from a positive shift facet towards a negative shift facet. Also, there is a change in the tradeoff between the two inputs. Furthermore, $FS_0<1$ indicates that the change resulting from the positive shift facet is less than that of the negative shift facet; and, on average, the technology of DMU_0 declines. In contrast, $FS_0>1$ indicates that the change resulting from the positive shift facet is lager than that of the negative shift facet; and, on average, the technology of DMU_0 progresses. $FS_0=1$ indicates that on average the technology of DMU_0 remains the same. As shown in Fig. 1, the points of period t, A_1^t, A_2^t, and A_3^t, in the *upper* part could be one of the points at period $(t+1)$ in the *lower* part, A_4^{t+1}, A_5^{t+1}, and A_6^{t+1}.

3.1. *Definition of TEC_0*

Note that $M_0^{t+1} = TEC_0 \times FS_0$ and $TEC_0 = D^{t+1}(x_0^{t+1}, y_0^{t+1})/D^t(x_0^t, y_0^t)$ if (i) $TEC_0>1$, indicating $D^{t+1}(x_0^{t+1}, y_0^{t+1}) > D^t(x_0^t, y_0^t)$. This implies that DMU_0 in time $(t+1)$ is closer to the frontier in time t, (ii) $TEC_0<1$ implies DMU_0 in time $(t+1)$ is further away from the frontier in $(t+1)$ than DMU_0 in time t to the frontier in t, and (iii) $TEC_0=1$ implies DMU_0 in time $(t+1)$ is as close to the $(t+1)$-frontier as DMU_0 in time t to the t-frontier.

4. An application

We employ the proposed approach to analyze the performance changes in semiconductor packaging and testing firms in Taiwan between the years 2000 and 2003. There are 15 companies in this category. The calculations are based upon one input, Liability ratio, and four outputs: (i) growth rate (%), (ii) net profit after tax ($100 million NT dollars), (iii) profitability ratio (%), and (iv) output value by employee ($million/people). Let us examine the technical efficiency changes. Table 5 reports the basic data of each company. Tables 6 and 7 report the DEA technical efficiency and the associated technical changes from 2000 to 2003.

4.1. Data collection and index description

In recent years, many semiconductor packaging and testing firms have been founded and their sales value has increased rapidly. This study uses the data published in the popular business magazine *Common Wealth* (2004) to analyze the relative performance of these companies between 2000 and 2003. The profile of the firms over these four years is listed in Tables 4 and 5.

Table 5 shows five indices: (i) Y_1 = Growth Rate (%), (ii) Y_2 = Net profit after tax ($100 million NT dollars), (iii) Y_3 = Profitability ratio (%), (iv) Y_4 = Output value by employee ($ million/people), and (v) X_1 = Liability ratio (%).

The measured efficiencies are depicted in Tables 6 and 7.

Tables 6 and 7 report the DEA technical efficiency and the associated technical efficiency changes from 2000 to 2003. *Hi-Sincerity* is the only company to improve its performance year after year. Table 6 shows its technical efficiency in 2000 to be less than 1.0 but larger than 1.0 afterwards. However, the technical change for *Hi-Sincerity* shown in Table 7 is larger than 1.0 only between 2000 and 2001, but less than 1.0 in the remaining years, indicating an exact definition of technical efficiency progress still needs to be investigated; all technical changes larger than or equal to 1.0 would be perfect, generally. Note that, in Table 7, only *KingPak* and *OSE* do not show technical efficiency progress from 2000 to 2003; on the other hand, we can conclude that other companies show improvement and decline in technical efficiency change. For the industry average, technical efficiency declines 6.3% from 2000 to 2001, improves 9.5% from 2001 to 2002, and improves 7.3% from 2002 to 2003.

Table 4
Profile of the firms, 2000–2003

	2000	2001	2002	2003
Revenue ($100 million US dollars)	33.19	25.38	31.52	38.21
Total assets ($100 million US dollars)	76.13	74.12	74.20	82.00
Capital ($100 million US dollars)	27.17	32.23	32.55	34.62
Liability ($100 million US dollars)	14.08	13.55	14.33	15.39
Number of employees	34,106	31,055	34,149	42,228

Table 8 reports the Malmquist frontier shift component, FS_0. It can be seen that on average, the industry technology frontier declines 31.3% from 2000 to 2001, improves 23.8% from 2001 to 2002, and improves 2.3% from 2002 to 2003.

As indicated by FS_0, we can see all companies show negative shift in technology frontier from 2000 to 2001. From 2001 to 2002, only *Sigurd* and *OSE* show a negative shift in technology frontier, indicating the period 2001–2002 has changed drastically compared with the previous period. Regarding the period 2002–2003, four companies show a negative shift, while eleven show a positive shift.

In the previous section, FS_0 is known as a product of two ratios, $D^t(x_0^{t+1}, y_0^{t+1})/D^{t+1}(x_0^{t+1}, y_0^{t+1})$ and $D^t(x_0^t, y_0^t)/D^{t+1}(x_0^t, y_0^t)$. Moreover, the value of each ratio represents a different implication; thus, we still need to discuss the two components of FS_0.

Note that $R_1 = D^t(x_0^{t+1}, y_0^{t+1})/D^{t+1}(x_0^{t+1}, y_0^{t+1})$, $R_2 = D^t(x_0^t, y_0^t)/D^{t+1}(x_0^t, y_0^t)$ in Table 9.

Table 9 reports the component shifts in technical frontier. We can see that no companies show a cross-frontier shift from 2000 to 2001, corresponding with the fact that no one shows a positive frontier shift in Table 8. To take *OSE*, *UTC*, and *Hi-Sincerity* between 2001 and 2002 as an example, their $R_1 < 1$ and $R_2 > 1$ indicates frontier moves from a positive shift facet towards a negative shift facet. In terms of management, this situation should be avoided. However, other companies all show the pure positive shift ($R_1 > 1$, $R_2 > 1$), indicating they stand for consistent operation strategies. From 2002 to 2003, only *KYEC*, *Hi-Sincerity*, *UTC*, and *KingPak* do not show a pure positive frontier shift. For the industry average, it is worth noting there is a negative frontier shift from 2000 to 2001, but that it moves to a desirable shift from 2001 to 2003. Commonly, only a minority of the companies showed a frontier shift from a good shift facet to a bad shift facet ($R_1 < 1$, $R_2 > 1$).

Table 10 reports the Malmquist productivity index M_0^{t+1}. It can be seen, on industry average, that there is about a 37.6% productivity loss from 2000 to 2001, while from 2001 to 2002 there is about a 23.1% productivity gain and from 2002 to 2003 there is about a 9.8% productivity gain.

However, the Malmquist productivity index is a combined product of TEC_0 and FS_0; that is, $M_0^{t+1} = TEC_0 \times FS_0$. In order to analyze the per-

Table 5
Basic data

DMU	Firms	Index				
		Y_1	Y_2	Y_3	Y_4	X_1
Year 2000						
1	ASE	145.86	98.37	122.87	3.50	38.26
2	SIPIN	158.16	72.21	117.09	3.56	32.84
3	OSE	146.85	41.04	100.73	2.19	31.12
4	ChipMos	128.82	55.39	118.71	4.11	33.80
5	KYEC	239.66	51.78	128.17	1.41	43.37
6	ASE Chung Li	284.76	55.90	121.02	3.47	50.90
7	Sharp in Taiwan	157.53	58.19	135.43	3.31	28.55
8	Greatek	154.48	45.25	114.15	2.68	44.83
9	Lingsen	153.12	43.38	110.27	2.07	26.09
10	PowerTech	344.42	42.50	118.85	1.46	56.07
11	UTC	136.54	49.02	125.65	4.49	23.01
12	KingPak	200.28	38.75	98.05	22.27	53.41
13	Hi-Sincerity	100.75	40.25	101.68	12.37	38.89
14	Formosa	143.13	41.77	110.24	2.37	58.83
15	Sigurd	135.29	41.50	114.49	1.98	32.05
Year 2001						
1	ASE	80.35	18.57	89.55	3.40	41.46
2	SIPIN	87.71	28.17	92.84	2.50	38.11
3	OSE	75.04	8.10	70.14	1.98	56.19
4	ChipMos	65.79	24.91	72.58	3.24	31.91
5	KYEC	92.71	32.08	79.57	1.44	53.48
6	ASE Chung Li	64.80	40.57	101.16	2.66	38.12
7	Sharp in Taiwan	78.55	37.60	94.05	2.75	25.01
8	Greatek	89.43	42.48	107.48	2.74	41.80
9	Lingsen	71.17	41.25	105.34	1.87	20.73
10	PowerTech	234.47	41.73	105.56	3.57	43.30
11	UTC	38.25	31.10	33.83	2.43	24.64
12	KingPak	33.53	39.17	96.14	7.68	48.35
13	Hi-Sincerity	70.15	40.21	102.02	11.32	37.24
14	Formosa	59.51	41.22	111.86	1.76	58.27
15	Sigurd	82.70	40.08	100.93	1.91	26.29
Year 2002						
1	ASE	125.00	41.29	100.50	4.20	42.50
2	SIPIN	134.90	44.25	101.91	2.79	43.28
3	OSE	119.56	7.00	74.16	2.65	64.18
4	ChipMos	118.57	27.92	81.49	3.21	44.48
5	KYEC	137.94	36.97	94.33	1.76	49.08
6	ASE Chung Li	105.22	43.66	107.09	2.29	30.66
7	Sharp in Taiwan	118.37	37.99	95.79	2.74	32.12
8	Greatek	134.67	46.34	114.19	3.36	36.48
9	Lingsen	125.40	36.33	87.51	2.13	25.67
10	PowerTech	90.74	41.87	106.63	2.80	34.86
11	UTC	159.26	36.73	84.73	3.17	22.31
12	KingPak	98.79	38.87	94.68	4.38	54.26
13	Hi-Sincerity	96.83	39.64	96.43	11.59	39.12
14	Formosa	162.59	40.92	105.50	2.51	55.16
15	Sigurd	143.22	42.38	120.12	2.31	43.77
Year 2003						
1	ASE	122.85	67.43	108.71	3.11	41.08
2	SIPIN	122.80	68.39	110.37	2.99	45.06
3	OSE	105.91	5.64	74.60	2.72	66.88
4	ChipMos	129.77	48.61	110.17	3.36	39.43

Table 5 (*continued*)

DMU	Firms	Index				
		Y_1	Y_2	Y_3	Y_4	X_1
5	KYEC	126.91	47.73	111.39	2.38	33.89
6	ASE Chung Li	116.65	41.83	103.04	2.08	34.23
7	Sharp in Taiwan	140.26	51.91	117.79	2.68	34.58
8	Greatek	116.10	49.27	117.88	3.42	35.63
9	Lingsen	133.22	43.69	109.43	2.43	30.28
10	PowerTech	155.44	50.40	123.72	3.21	45.67
11	UTC	107.53	39.92	99.63	2.93	19.95
12	KingPak	59.82	40.94	107.40	2.87	44.82
13	Hi-Sincerity	101.98	39.35	93.68	11.05	40.29
14	Formosa	122.77	41.91	109.30	2.90	54.62
15	Sigurd	149.37	44.18	123.66	2.47	34.16

Table 6
DEA technical efficiency from 2000 to 2003

Firms	$D^t(x_0^t, y_0^t)$			
	2000	2001	2002	2003
ASE	1.038	0.414	0.558	0.584
SIPIN	0.762	0.504	0.518	0.530
OSE	0.525	0.174	0.156	0.114
ChipMos	0.665	0.549	0.428	0.591
KYEC	0.349	0.250	0.342	0.618
ASE Chung Li	0.490	0.533	0.642	0.548
Sharp in Taiwan	0.807	0.864	0.638	0.664
Greatek	0.423	0.532	0.668	0.651
Lingsen	0.659	1.163	0.731	0.687
PowerTech	1.009	1.101	0.562	0.546
UTC	1.185	0.479	1.246	1.345
KingPak	1.093	0.418	0.394	0.377
Hi-Sincerity	0.734	1.161	1.150	1.131
Formosa	0.288	0.269	0.401	0.390
Sigurd	0.485	0.756	0.485	0.646
Industry average	0.701	0.611	0.595	0.628

Table 7
Technical efficiency change

Firms	*TEC*		
	2000 vs. 2001	2001 vs. 2002	2002 vs. 2003
ASE	0.399	1.349	1.046
SIPIN	0.662	1.028	1.022
OSE	0.331	0.897	0.728
ChipMos	0.825	0.781	1.380
KYEC	0.715	1.370	1.807
ASE Chung Li	1.088	1.206	0.853
Sharp in Taiwan	1.071	0.739	1.041
Greatek	1.256	1.257	0.975
Lingsen	1.764	0.629	0.939
PowerTech	1.091	0.510	0.972
UTC	0.404	2.601	1.080
KingPak	0.383	0.942	0.957
Hi-Sincerity	1.581	0.990	0.984
Formosa	0.934	1.489	0.972
Sigurd	1.557	0.642	1.331
Industry average	0.937	1.095	1.073

formances of these companies more precisely, the information in Tables 7 and 8 is not only helpful, but essential. Fortunately, M_0^{t+1} is consistent with TEC_0 and FS_0 here. However, if we see that the Malmquist productivity index is larger than 1.0 on average in a certain case, this is maybe a combined effect of an average improvement in technology frontier shown in Table 8 and an average declining technical efficiency shown in Table 7. Such a situation is not met in this case, but it would be absolutely necessary for management to make a detailed investigation to find the real cause of productivity gains or losses.

Therefore, for the conclusion regarding company productivity change, we must refer to Tables 7 and 8. In addition, Table 11 is derived comprehensively as follows.

Next, let us examine the detailed Malmquist change information. Here, we denote R_1 (first component of $FS) = D^t(x_0^{t+1}, y_0^{t+1})/D^{t+1}(x_0^{t+1}, y_0^{t+1})$,

Table 8
Frontier shift

Firms	*FS*		
	2000 vs. 2001	2001 vs. 2002	2002 vs. 2003
ASE	0.853	1.133	1.034
SIPIN	0.709	1.139	1.034
OSE	0.664	0.998	1.137
ChipMos	0.694	1.142	1.052
KYEC	0.521	1.041	0.996
ASE Chung Li	0.590	1.216	1.021
Sharp in Taiwan	0.708	1.214	1.041
Greatek	0.720	1.177	1.031
Lingsen	0.587	1.203	1.046
PowerTech	0.670	1.430	1.028
UTC	0.792	1.060	0.988
KingPak	0.771	1.323	0.997
Hi-Sincerity	0.621	1.001	0.937
Formosa	0.765	1.091	1.038
Sigurd	0.639	0.934	1.025
Industry average	0.687	1.140	1.027

Table 9
Individual shift

Firms	Time					
	2000 vs. 2001		2001 vs. 2002		2002 vs. 2003	
	R_1	R_2	R_1	R_2	R_1	R_2
ASE	0.760	0.957	1.144	1.122	1.022	1.045
SIPIN	0.743	0.677	1.136	1.141	1.021	1.048
OSE	0.790	0.559	0.962	1.036	1.139	1.135
ChipMos	0.734	0.656	1.102	1.185	1.037	1.067
KYEC	0.797	0.340	1.100	0.985	1.029	0.964
ASE Chung Li	0.726	0.480	1.205	1.227	1.018	1.023
Sharp in Taiwan	0.709	0.708	1.203	1.225	1.036	1.045
Greatek	0.755	0.687	1.213	1.143	1.022	1.040
Lingsen	0.566	0.609	1.392	1.039	1.041	1.052
PowerTech	0.459	0.979	1.177	1.736	1.045	1.010
UTC	0.729	0.861	0.928	1.210	0.834	1.169
KingPak	0.631	0.942	1.168	1.499	0.968	1.028
Hi-Sincerity	0.551	0.700	0.892	1.124	0.783	1.122
Formosa	0.765	0.765	1.106	1.077	1.038	1.039
Sigurd	0.696	0.587	1.158	0.754	1.041	1.009
Industry average	0.694	0.700	1.126	1.167	1.005	1.053

R_2 (second component of $FS) = D^t(x_0^t, y_0^t)/D^{t+1}(x_0^t, y_0^t)$, $R_3(TEC) = D^{t+1}(x_0^{t+1}, y_0^{t+1})/D^t(x_0^t, y_0^t)$, R_4

$$(M_0^{t+1}) = \left[\frac{D^t(x_0^{t+1}, y_0^{t+1})}{D^t(x_0^t, y_0^t)} \frac{D^{t+1}(x_0^{t+1}, y_0^{t+1})}{D^{t+1}(x_0^t, y_0^t)}\right]^{1/2}.$$

Table 10
Malmquist productivity

Firms	M_0^{t+1}		
	2000 vs. 2001	2001 vs. 2002	2002 vs. 2003
ASE	0.34	1.528	1.081
SIPIN	0.469	1.170	1.057
OSE	0.220	0.895	0.828
ChipMos	0.573	0.892	1.451
KYEC	0.373	1.426	1.799
ASE Chung Li	0.642	1.467	0.871
Sharp in Taiwan	0.759	0.897	1.083
Greatek	0.904	1.480	1.005
Lingsen	1.035	0.756	0.983
PowerTech	0.732	0.729	0.999
UTC	0.320	2.757	1.067
KingPak	0.295	1.247	0.955
Hi-Sincerity	0.982	0.992	0.923
Formosa	0.715	1.625	1.009
Sigurd	0.996	0.600	1.364
Industry average	0.624	1.231	1.098

Table 11 reports the component information associated with productivity change. Contents include results of CCR models constructed by Chen and Ali (2004) and SBM/Super-SBM models. Theoretically, SBM/Super-SBM models have a truly specific interpretation in these 15 firms because we could discover a few differences with the CCR model.

In Table 11, among the 180 comparisons of two measurement methods, 39 (21.7%) are in different signs, a large percentage of total. This proves the current SBM-based approach indeed revises the weak points of the radial-based measure, leading to an appropriate result. It is obvious that applying the current approach leads to a different managerial interpretation. Theoretically, SBM/Super-SBM models have a truly specific interpretation in these 15 firms. One of the major reasons for the difference is that Chen and Ali (2004) do not measure the super-efficiency for DMU_0 in a single period t or $(t+1)$.

We will first expand on the managerial purpose concerning the results of SBM and Super-SBM measures. By analyzing some meaningful cases, we will determine the essential factor of each productivity result. First, in Table 11, the Malmquist productivity indices of the *PowerTech* company are both less than 1.0 ($M_0^{t+1} < 1$) in two periods—from 2000 to 2001 and from 2001 to 2002—yet the contents of R_1, R_2 and R_3 in each period are

Table 11
Detailed Malmquist productivity change information

	R_1		R_2		R_3		R_4	
	CCR	SBM	CCR	SBM	CCR	SBM	CCR	SBM
2000 vs. 2001								
ASE	<1	<1	<1	<1	<1	<1	<1	<1
SIPIN	<1	<1	<1	<1	<1	<1	<1	<1
OSE	<1	<1	<1	<1	<1	<1	<1	<1
ChipMos	<1	<1	<1	<1	<1	<1	<1	<1
KYEC	<1	<1	<1	<1	<1	<1	<1	<1
ASE Chung Li	<1	<1	<1	<1	<1	>1	<1	<1
Sharp in Taiwan	<1	<1	<1	<1	<1	>1	<1	<1
Greatek	<1	<1	<1	<1	>1	>1	<1	<1
Lingsen	<1	<1	<1	<1	>1	>1	<1	>1[a]
PowerTech	<1	<1	<1	<1	1	>1[a]	<1	<1
UTC	<1	<1	<1	<1	<1	<1	<1	<1
KingPak	<1	<1	<1	<1	<1	<1	<1	<1
Hi-Sincerity	<1	<1	<1	<1	>1	>1	<1	<1
Formosa	<1	<1	<1	<1	<1	<1	<1	<1
Sigurd	<1	<1	<1	<1	>1	>1	<1	<1
2001 vs. 2002								
ASE	>1	>1	<1	>1[a]	>1	>1	>1	>1
SIPIN	>1	>1	<1	>1[a]	>1	>1	>1	>1
OSE	>1	<1[a]	<1	>1[a]	<1	<1	>1	<1[a]
ChipMos	>1	>1	<1	>1[a]	<1	<1	<1	<1
KYEC	>1	>1	>1	<1[a]	>1	>1	>1	>1
ASE Chung Li	<1	>1[a]	<1	>1[a]	>1	>1	>1	>1
Sharp in Taiwan	>1	>1	<1	>1[a]	<1	<1	<1	<1
Greatek	>1	>1	<1	>1[a]	>1	>1	>1	>1
Lingsen	>1	>1	<1	>1[a]	<1	<1	<1	<1
PowerTech	<1	>1[a]	>1	>1	<1	<1	<1	<1
UTC	>1	<1[a]	<1	>1[a]	>1	>1	>1	>1
KingPak	>1	>1	<1	>1[a]	<1	<1	<1	>1[a]
Hi-Sincerity	>1	<1[a]	<1	>1[a]	1	<1[a]	>1	<1
Formosa	>1	>1	<1	>1[a]	>1	>1	>1	>1
Sigurd	>1	>1	<1	<1	<1	<1	<1	<1
2002 vs. 2003								
ASE	>1	>1	>1	>1	>1	>1	>1	>1
SIPIN	>1	>1	>1	>1	>1	>1	>1	>1
OSE	<1	>1[a]	<1	>1[a]	<1	<1	<1	<1
ChipMos	>1	>1	<1	>1[a]	>1	>1	>1	>1
KYEC	>1	>1	<1	<1	>1	>1	>1	>1
ASE Chung Li	>1	>1	>1	>1	<1	<1[a]	<1	<1
Sharp in Taiwan	>1	>1	>1	>1	<1	>1[a]	>1	>1
Greatek	>1	>1	>1	>1	<1	<1	>1	>1
Lingsen	>1	>1	<1	>1[a]	<1	<1	<1	<1
PowerTech	>1	>1	>1	>1	<1	<1	<1	<1
UTC	>1	<1[a]	<1	>1[a]	1	>1[a]	<1	>1[a]
KingPak	>1	<1[a]	>1	>1	<1	<1	>1	<1[a]
Hi-Sincerity	<1	<1	<1	>1[a]	1	<1[a]	<1	<1
Formosa	>1	>1	<1	>1[a]	<1	<1	<1	>1[a]
Sigurd	>1	>1	>1	>1	>1	>1	>1	>1

[a]Indicates the difference between the CCR and SBM/Super-SBM models.

contrary. From 2000 to 2001, the components of FS_0 display a pure negative frontier shift, and the only inferior effect on its whole performance is positive technical efficiency change. However, from 2001 to 2002, the only benefit in the performance is the technical efficiency progress, while the components of FS_0 reveal a pure positive frontier shift.

Secondly, *OSE* shows a productivity loss from 2002 to 2003 due to improvement in FS_0 (R_1 and R_2 both >1), and the only decline in technical efficiency, representing the positive frontier shift, cannot overtake the harm from technical efficiency decline. In terms of chasing a good performance, management strategy should focus on this issue.

UTC shows productivity gain with an improvement in technical efficiency from 2001 to 2002. Actually, the company is moving to a negative shift facet because the $R_1<1$ and $R_2>1$. The implication of these two ratios has been discussed previously. Therefore, *UTC* demonstrates an unfavorable strategy in this period.

Hi-Sincerity from 2001 to 2002 shows the least favorable strategy for change under the scenario R_1 and R_2 performs inconsistently, involving $R_1 >1$, $R_2 <1$ or $R_1 <1$, $R_2 >1$. Since its $M_0^{t+1}<1$, $TEC_0<1$, $R_1<1$ and $R_2>1$, we can conclude that it also suffers productivity loss, technical efficiency decline, and has moved from a positive shift facet towards a negative shift facet. This situation must be discussed because every company or industry may encounter such potential danger, and it is easily ignored.

Among the current set of performance assessments of semiconductor packaging and testing firms in Taiwan, *KYEC* is the polar opposite of *Hi-Sincerity*. It is significant to know that the most favorable strategy change under the scenario R_1 and R_2 performs inconsistently occurs if $M_0^{t+1}>1$, $TEC_0>1$, $R_1>1$ and $R_2<1$. In other words, the conditions demonstrate that besides the particular company showing productivity gain and progress in technical efficiency, its strategy moves from a negative shift facet towards a positive shift facet.

The last two simple cases are (i) $M_0^{t+1}>1$, $TEC_0>1$, $R_1>1$ and $R_2>1$, which indicates the best result of all, and (ii) $M_0^{t+1}<1$, $TEC_0<1$, $R_1<1$ and $R_2<1$, which indicates the worst result of all. The above discussion shows that by further analyzing the Malmquist components, more insights into productivity changes can be obtained.

5. Comparisons of CCR and SBM measures

We compare our results and the results obtained by Chen and Ali (2004) employing the CCR model.

As noted earlier in this paper, θ_0^*, ρ_0^*, and π_0^* are the optimal efficiency scores of CCR, SBM, and Super-SBM models, respectively. When measuring the distances $D^t(x_0^t, y_0^t)$ and $D^{t+1}(x_0^{t+1}, y_0^{t+1})$, if the object company is inefficient, the CCR score θ_0^* is greater or equal to the SBM score. If the object company is efficient, we further measure its distance to the frontier constructed by the other companies; the Super-SBM efficiency scores are greater than 1.0 and greater than the CCR scores, 1.0. In the other case, we measure the distances across two periods of $D^{t+1}(x_0^t, y_0^t)$ and $D^t(x_0^{t+1}, y_0^{t+1})$; if the object company is inefficient, the CCR score θ_0^* is greater or equal to the SBM score. If the object company is efficient, we further measure its distance to the frontier constructed by all the companies in other periods; the Super-SBM efficiency scores are greater than 1.0 and greater than the CCR scores, 1.0.

Chen and Ali (2004) do not measure the Super-CCR efficiency score (Andersen and Petersen, 1993) of DMU_0 in a single period t or $(t+1)$; therefore, $\pi_0^* \geqslant 1$, $\theta_0^* \leqslant 1$ and verified that $\pi_0^* \geqslant \theta_0^*$. As a result, the changes in optimal efficiency score for the three models might affect the ratios R_1, R_2, R_3, and R_4.

Measuring the ratio R_1 of DMU_0, $R_1 = D^t(x_0^{t+1}, y_0^{t+1})/D^{t+1}(x_0^{t+1}, y_0^{t+1})$ by our proposed SBM/Super-SBM models and the CCR model could be inefficient or efficient. Their values are depicted in Table 12. The ratio R_1 could be obtained by the three possible combinations as shown in Table 13,

Table 12
Values of $D^t(x_0^{t+1}, y_0^{t+1})$ and $D^{t+1}(x_0^{t+1}, y_0^{t+1})$

	SBM/Super-SBM		CCR	
	Inefficient	Efficient	Inefficient	Efficient
$D^t(x_0^{t+1}, y_0^{t+1})$	□1	□1	□1	□1
$D^{t+1}(x_0^{t+1}, y_0^{t+1})$	□1	□1	□1	1

Table 13
Values of $[D^t(x_0^{t+1}, y_0^{t+1})/D^{t+1}(x_0^{t+1}, y_0^{t+1})]$

No.	Combination	$R_{1,\text{SBM/Super-SBM}} \leqslant 1$	$R_{1,\text{CCR}}$
1	I/I	$\leqq 1$	$\leqq 1$ or $\geqq 1$
2	E/E	$\leqq 1$	$\geqq 1$
3	I/E	$\leqq 1$	$\leqq 1$

where I and E denote inefficient and efficient, respectively. Given the ratio R_1 is less than 1.0 for the SBM/Super-SBM models, the ratio R_1 for the CCR model could be inferred. The first and second combinations have different outcomes in two models. One could perform similar analysis for the ratios R_2, R_3, and R_4 under the two models. The current paper provides measurement different from the CCR measure proposed by Chen and Ali (2004).

6. Conclusions

We benefited from use of the DEA Malmquist productivity approach employed by Chen and Ali (2004) to discover that in-depth information could be obtained by analyzing each individual component of the Malmquist productivity index. However, the result is more precise using the slacks-based measures. In fact, among these 15 firms in Taiwan, atop firms may have huge influence on their own country, or even, global market. Therefore, the current approach that involves the super efficiency on Malmquist productivity measure is more helpful to analysts who are highly curious about atop firms having DEA efficient performances. According to the comparison with CCR, there are number of differences at the end. Such analyses not only help revise the weak points in the CCR model but also match the reality of Taiwan semiconductor companies. Moreover, it is sometimes very critical to capture a firm's performance through an analysis of the components of the Malmquist productivity index to reveal the managerial implications of each component and limit misleading information. As a result, a firm will be aware of what kind of weaknesses they should watch out for and remedy. Furthermore, in terms of industrial management, this method allows judgments to be made concerning whether or not the strategic shift is favorable and promising.

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