

Elastic properties of the Abrikosov lattice near the upper critical field and peak effect in the critical current

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Abstract

Abrikosov vortices in type II superconductors under magnetic field form highly correlated configurations with well defined elasticity, like regular atomic solids. While in the London limit vortices can be considered as line-like objects, when the field approaches the upper critical field $H_{c2}(T)$ the core structure becomes important for elastic properties. Taking into account the deformation of the shape of the current distribution leads to significant increase in the shear modulus of the vortex lattice compared to previous estimates. The elastic properties determine the response to vortex matter pinning. The modified value of the elastic modulus is used to support a recent theory of the discontinuous “peak effect” in the critical current. It is argued that the critical current of the vortex lattice decreases monotonously as a function of the magnetic field, jumping to a high value in the homogeneous amorphous phase.

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Physics of the Abrikosov vortex systems in type II superconductors under magnetic field has attracted great attention over the years [1]. In addition to the general significance of this system, it offers a unique testing ground for a great variety of theoretical concepts, ranging from elastic manifolds to glass dynamics. Repulsive interactions between the flux lines result in stable flux line lattice configurations, considered as an elastic medium. Elastic properties of the vortex matter are crucial in understanding such important phenomena as flux pinning and thermodynamics of the vortex matter [2–4]. For example, elastic moduli of the vortex matter determine the critical current of superconductors in magnetic field. A detailed knowledge of the elasticity of the vortex lattice is required for recent experiments [5–7] in *Nb* on the “peak effect,” generally observed in the region close to the mean field transition line. Recently, interest focused on a particular case of

strongly type II superconductors for which the ratio $\kappa = \lambda/\xi$ of the magnetic penetration depth and coherence length is very large, $\kappa \gg 1$ (for high T_c cuprates and most of the widely used and studied low T_c type II superconductors κ ranges between 10 and 100). To describe the superconductors on the mesoscopic scale, one generally uses the phenomenological Ginzburg–Landau (GL) description rather than a microscopic theory.

Using the GL theory under the assumption that the system always stays on the lowest Landau level (LLL), Brandt derived [8] the following expressions for the softest modulus, the shear, of the vortex lattice. At large κ and near the mean field line, c_{66} is proportional to a_h^2

$$c_{66} \simeq 0.24\beta_A^{-2}\kappa^{-2}a_h^2, \quad (1)$$

where $a_h = (1 - t - h)/2$ is a small parameter and $\beta_A = 1.16$ for the Abrikosov hexagonal lattice. Temperature is in units of T_c , $t = T/T_c$, and the magnetic field $\mathbf{h} = (0, 0, h)$ in units of the upper critical field $H_{c2}(T=0) = \frac{\Phi_0}{2\pi\xi^2(T=0)}$, $\Phi_0 = \frac{\pi\hbar c}{e}$, whereas the moduli are given in units of

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$\frac{H_{c2}^2(0)}{\pi}$. The modulus is consistent with the thermodynamic derivation in which lattice energies of different symmetries were compared. The expressions for shear modulus and other moduli are used in numerous theoretical descriptions of phenomena as diverse as vortex lattice melting [9] and critical current [1,2] of the pinned lattice.

Superconductivity is generally lost in magnetic field due to flux flow leading to dissipation. However, if the vortices are pinned, one recovers the basic property of a superconductor: finite net current in the absence of electric field. The pinning force \mathbf{F}_{pin} due to impurities or artificial defects balances the Lorentz force $c^{-1} \int \mathbf{B} \times \mathbf{J}$. The assumption that perturbed vortex matter near $H_{c2}(T)$ is confined to LLL looks reasonable due to the energy gap at high fields; however there is a general argument that in any pinned vortex configuration carrying a net supercurrent, the system is necessarily excited to higher Landau levels. Indeed, for an arbitrary LLL configuration of the order parameter field $\psi(\mathbf{r})$, the local supercurrent, $\mathbf{J}(\mathbf{r})$, is proportional to the curl of the superfluid density $\rho(\mathbf{r}) = |\psi(\mathbf{r})|^2$:

$$\mathbf{J}(\mathbf{r}) \propto [\hat{\mathbf{z}} \times \nabla] \rho(\mathbf{r}), \quad (2)$$

shown for the Abrikosov lattice configuration in symmetric gauge $A_x = \frac{1}{2} \varepsilon_{\alpha\beta} r^\beta$,

$$\begin{aligned} \phi_0 = 3^{1/8} h^{-1/2} \sum_{l=-\infty}^{\infty} \exp [ihxy \\ + i\pi l \left(\frac{l}{2} + 2h^{1/2}x/a \right) - \frac{1}{2} (h^{1/2}y - 2\pi l/a)^2], \end{aligned} \quad (3)$$

in Fig. 1a (normalized to unit superfluid density). $a = 2\pi^{1/2}/3^{1/4}$ is the lattice spacing at H_{c2} in units of the coherence length. Therefore, the total supercurrent is equal to zero, despite the existence of strong circular diamagnetic currents. For higher LLs, relation Eq. (2) is violated, leading to the coexistence of effective pinning and electric current. An example of such a configuration is shown in Fig. 1b, in which an admixture of the first Landau level, $0.1 \phi_1$, is added to ϕ_0 . In Fig. 1c the diamagnetic currents are subtracted, demonstrating the overall persistent current component. Due to this fact it is unlikely that the elasticity of the vortex lattice responsible for pinning is mainly due to the LLL excitations near $H_{c2}(T)$; see quantitative discussion below.

We therefore revisit the calculation of elastic moduli near $H_{c2}(T)$ and extend the theory beyond LLL. Expansion in both κ^{-2} and a_h is explicitly performed using a field theoretical method. Our main result is that the shear modulus is much larger than that in Eq. (1), namely,

$$c_{66} \simeq \frac{1}{2} h \beta_A^{-1} \kappa^{-2} a_h, \quad (4)$$

which originates from rerouting of supercurrents by external force in the strained state correlated with the shape change of distorted vortices. Mathematically, it is due to the higher Landau level excitations. Then we point out what went wrong with the ‘‘thermodynamic’’ argument

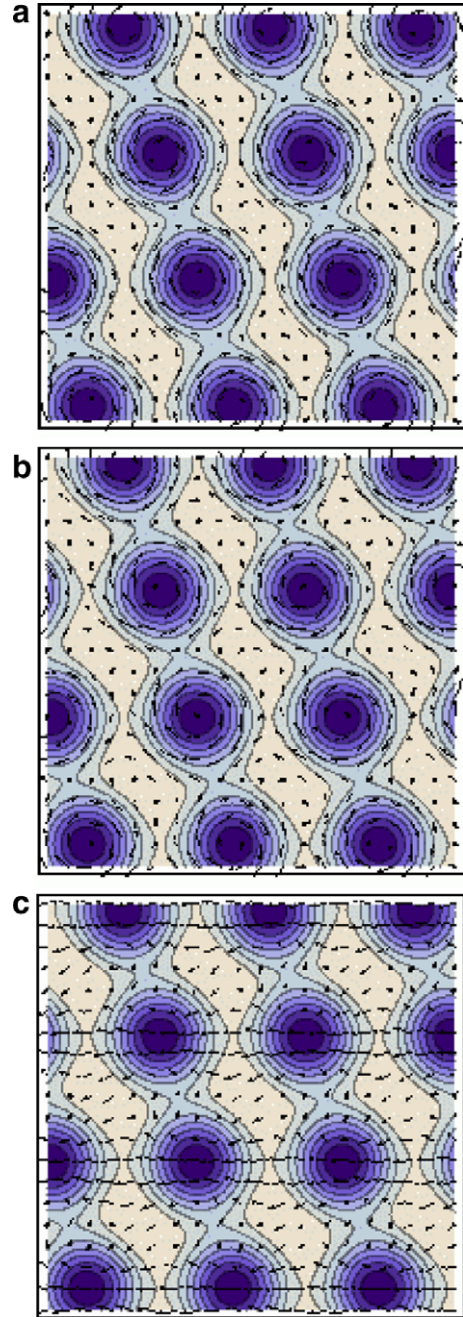


Fig. 1. Superfluid density and current distribution for LLL hexagonal lattice (a) and for the state with the first LL added (b). In (c) the diamagnetic component of the current is subtracted.

and why pinning generally leads to supercurrents on the mesoscopic scale.

The GL Gibbs energy density of a superconductor is

$$\begin{aligned} g = \frac{1}{\text{vol}} \frac{1}{\kappa^2} \\ \times \int_r \left[\frac{1}{2} |D_i \psi|^2 - \left(a_h - \frac{\hbar}{2} \right) \rho + \frac{1}{2} \rho^2 + \frac{\kappa^2}{4} (\mathbf{b} - \mathbf{h})^2 \right], \end{aligned} \quad (5)$$

where $D_i = \partial_i - iA_i$, $i = 1, 2, 3$ are covariant derivatives with respect to coordinates $\mathbf{r} = (r^\alpha, z)$, $\alpha = 1, 2$, with vector

potential A_i generating the local induction $\mathbf{b}(\mathbf{r}) = \mathbf{rot}(\mathbf{A})$. For strongly type II superconductors and not very far from the normal-superconductor transition, the mean field configuration minimizing the functional Eq. (5) is given by a well defined expansion in two small parameters κ^{-2} and a_h [10]:

$$\begin{aligned}\psi_{mf}(r) &\simeq \sqrt{\frac{a_h}{\beta_A}}[\phi_0 + a_h\phi_c + \mathcal{O}(a_h^2)] + \mathcal{O}(\kappa^{-2}), \\ \mathbf{b}_{mf}(r) &\simeq \mathbf{h} + \kappa^{-2}\mathbf{b}_c + \mathcal{O}(\kappa^{-4}).\end{aligned}\quad (6)$$

The correction $\phi_c = c_0\phi_0 + c_6\phi_6 + \dots$ with $c_0 = 0.0134/h$ is composed of the LLL contribution, ϕ_0 , and the higher Landau level ones (the next to the lowest level contribution being the sixth due to hexagonal symmetry), while $\mathbf{b}_c = -\mathbf{h}\frac{a_h}{h\beta_A}|\phi_0|^2 + \mathcal{O}(a_h^3)$.

Generally, elastic moduli describe the properties of the body (in our case magnetic and the order parameter fields) under local translations with a displacement of the fields in the plane perpendicular to the external magnetic field, $\mathbf{u}(\mathbf{r}) = (u^x, 0)$:

$$\begin{aligned}\psi'(\mathbf{r}) &= \psi(\mathbf{r} + \mathbf{u}), \\ A'_i(\mathbf{r}) &= A_i(\mathbf{r} + \mathbf{u}) + \partial_i u^\beta A_\beta(\mathbf{r} + \mathbf{u}).\end{aligned}\quad (7)$$

Expanding to second order in displacement,

$$\begin{aligned}g &= g_0 + \Phi^{(1)} + \frac{1}{2}\Phi^{(2)}, \\ \Phi^{(1)} &= \frac{1}{\text{vol}} \int_r g'_A \delta A_i + g'_\psi \delta \psi + g'_{\psi^*} \delta \psi^*, \\ \Phi^{(2)} &= \frac{1}{\text{vol}} \int_{r,r'} g''_{AA} \delta A_k \delta A_l + g''_{\psi\psi^*} \delta \psi \delta \psi^* + 2g''_{A\psi^*} \delta A_k \delta \psi^* \\ &\quad + g''_{\psi^*\psi} \delta \psi^* \delta \psi + c.c.\end{aligned}\quad (8)$$

where

$$\begin{aligned}g'_A &= \frac{\delta g}{\delta A_\alpha(r)}, \quad g'_\psi = \frac{\delta g}{\delta \psi(r)}, \quad g'_{\psi^*} = \frac{\delta g}{\delta \psi^*(r)} \\ g''_{AA} &= \frac{\delta^2 g}{\delta A_\beta(r') \delta A_\alpha(r)}, \quad g''_{\psi\psi^*} = \frac{\delta^2 g}{\delta \psi(r') \delta \psi^*(r)}, \\ g''_{A\psi^*} &= \frac{\delta^2 g}{\delta A_\beta(r') \delta \psi^*(r)}, \quad g''_{\psi^*\psi} = \frac{\delta^2 g}{\delta \psi^*(r') \delta \psi(r)}\end{aligned}$$

are functional derivatives. Since the mean field configuration is determined by the order parameter and magnetic field satisfying the equations of motion, $g'_\psi = g'_A = 0$, $\Phi^{(1)}$ vanishes, while $\Phi^{(2)}$ is quadratic in $\delta\psi$ and δA_i and, therefore, one needs variations of fields up to the first order in displacement,

$$\begin{aligned}A'_\alpha(r) &= A_\alpha(r) + \delta A_\alpha; \quad \delta A_\alpha = u^\beta A_{\alpha,\beta} + u^\beta_{,\alpha} A_\beta, \\ A'_z(r) &= A_z(r) + \delta A_z; \quad \delta A_z = u^\beta_{,z} A_\beta.\end{aligned}\quad (9)$$

Defining a displacement wave-vector by $\partial_\beta u^\alpha = v^z k_\beta$ and expanding $\Phi^{(2)}$ in powers of k , one observes that the order k^0 contributions cancel. This is just the Goldstone theorem,

which asserts that a continuous symmetry (global translations in the present case) is spontaneously broken and there appears a “soft” mode. Terms linear in k vanish due to reflection symmetry of the Abrikosov lattice configuration, while the terms quadratic in k determine the elastic moduli.

Calculating explicitly the second functional derivatives, one obtains a simple expression:

$$\begin{aligned}\Phi^{(2)} &= \frac{1}{\text{vol}} v^\alpha v^\beta k^i k^j \int_r [\kappa^2 (\delta_{ij} \partial_\alpha A_l \partial_\beta A_l \\ &\quad + \partial_j A_\alpha \partial_\beta A_i) + \delta_{ij} (D_\beta^* \psi^* D_\alpha \psi + D_\beta \psi D_\alpha^* \psi^*)],\end{aligned}\quad (10)$$

which should be compared with the equation that defines the elastic moduli of continuous media:

$$\Phi^{(2)} = c_{11}(k_x v_x + k_y v_y)^2 + c_{66}(k_x v_y + k_y v_x)^2 + c_{44} k_z^2 (v_x^2 + v_y^2).$$

The contribution to the leading order in κ^{-2} comes solely from the magnetic energy term. Substituting the mean field solution, Eq. (6), into Eq. (10), one notes that the κ^0 – contributions to the compression and the tilt moduli are

$$c_{11}^0 = c_{44}^0 = \frac{h^2}{4},\quad (11)$$

while the shear modulus in this order is equal to zero. The contribution to the integral in $\Phi^{(2)}$ of order κ^{-2} comes both from magnetic terms in Eq. (10), which to order a_h^2 is

$$\begin{aligned}\frac{1}{\text{vol}} \int_r (\delta_{ij} \partial_\alpha A_i^0 \partial_\beta A_j^c + \partial_j A_\alpha^c \partial_\beta A_i^0 + (\alpha \leftrightarrow \beta)) \\ = -\frac{h a_h}{2\beta_A} (1 + 2a_h c_0) \delta_{\alpha i} \delta_{\beta j},\end{aligned}\quad (12)$$

with $A_{\alpha,\beta}^c = -\frac{1}{2} \varepsilon_{\alpha\beta} \frac{a_h}{\beta_A} |\phi_0|^2 (1 + 2a_h c_0) + \mathcal{O}(a_h^3)$, and from the order parameter terms in Eq. (10):

$$\frac{1}{\text{vol}} \int_r (D_\beta^* \psi^* D_\alpha \psi + D_\beta \psi D_\alpha^* \psi^*) = \delta_{\alpha\beta} \frac{h a_h}{2\beta_A} (1 + 2a_h c_0).\quad (13)$$

Therefore, one has the following contributions to the elastic moduli:

$$\begin{aligned}c_{11}^1 &= c_0 \frac{a_h^2 h}{\kappa^2 \beta_A}, \\ c_{44}^1 &= c_{66}^1 = \frac{h a_h}{2\kappa^2 \beta_A} (1 + 2a_h c_0).\end{aligned}\quad (14)$$

This has an order a_h correction to the tilt modulus, reproducing the value $c_{44} = \frac{hb}{4}$, in accordance with the “thermodynamic” derivation [1].

The leading term in the shear modulus is larger by a factor h/a_h compared to that obtained thermodynamically in Eq. (1), so let us discuss why, while the thermodynamic argument certainly works well in the London limit, it breaks down near $H_{c2}(T)$. If one considers, for example a system of interacting point-like particles forming a hexagonal lattice, a (global) shear deformation transforms it uniquely into an oblique lattice with opening angle θ . Then the second derivative of the lowest energy state with respect

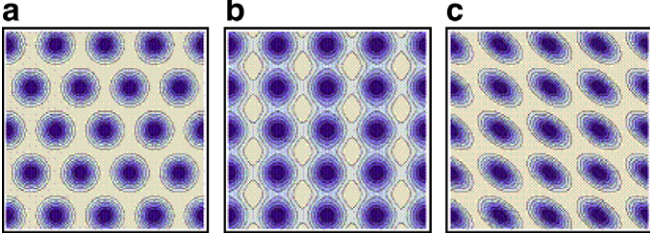


Fig. 2. The superfluid density of the hexagonal lattice (a) sheared by moving zeros (b) within LLL according to Eq. (18) and by transformation, Eq. (9) (c).

to θ is related to the shear modulus. If however the particles have an internal degrees of freedom, the deformation generally leads to excitation of internal degrees of freedom. The deformed state is no longer the ground state of the system with given symmetry. In the London limit the vortex structure is ignored and as a result the deformed state is still unique. However, far from $H_{c1}(T)$ the core structure becomes important and there are many states with the same oblique symmetry and various vortex shapes. This is illustrated in Fig. 2a, where the superfluid density of the hexagonal vortex lattice is subjected to global shear. A similar phenomenon was noted long ago in the density functional theory [11] of atomic lattice. Our method is quite analogous to theirs.

Next, we explicitly construct a transformation of the order parameter $\psi(x)$ under linear local translations in the coordinate space. Uniform magnetic field is assumed and is described in arbitrary linear gauge by the vector potential $A_x = \mathcal{B}_{\alpha\beta} r^\beta$. The matrix,

$$\mathcal{B}_{\alpha\beta} = \frac{\hbar}{2} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix},$$

in the Landau gauge and $\mathcal{B}_{\alpha\beta} = \frac{\hbar}{2} \varepsilon_{\alpha\beta}$ in the symmetric gauge. Writing down a global linear transformation in the coordinate space as

$$r^\alpha \rightarrow r^\alpha + u^\alpha(\mathbf{r}), \quad (15)$$

$$u_x = (u_r L_r^{\alpha\beta} + u_{sq} L_{sq}^{\alpha\beta} + u_{sh} L_{sh}^{\alpha\beta}) r_\beta,$$

where $u_i = (u_r, u_{sq}, u_{sh})$ and $L_i^{\alpha\beta} = (L_r^{\alpha\beta}, L_{sq}^{\alpha\beta}, L_{sh}^{\alpha\beta})$ are parameters and generators of rotation, squash, and shear transformations correspondingly:

$$L_r = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad L_{sq} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad L_{sh} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad (16)$$

one obtains

$$\begin{aligned} \phi_0(\mathbf{r}) &\rightarrow e^{-\frac{i}{2} u_i (L_i^{\alpha\gamma} \mathcal{B}_{\gamma\beta} + \mathcal{B}_{\alpha\gamma} L_i^{\beta\gamma}) r_\alpha r_\beta} \phi_0(\mathbf{r} + \mathbf{u}) \\ &= \exp(i u_i \hat{L}_i) \phi_0(\mathbf{r}). \end{aligned} \quad (17)$$

The infinitesimal operators:

$$\hat{L}_i = -i r^\alpha L_i^{\alpha\beta} \partial_\beta - \frac{1}{2} (L_i^{\alpha\gamma} \mathcal{B}_{\gamma\beta} + \mathcal{B}_{\alpha\gamma} L_i^{\beta\gamma}) r_\alpha r_\beta,$$

form a representation of the group defined by Eq. (15). It can be easily checked that the operator \hat{L}_r commutes with the kinetic part,

$$g_H = \frac{1}{\text{vol}} \frac{1}{\kappa^2} \int_r \left(\frac{1}{2} |D_i \psi|^2 + \frac{\hbar}{2} \rho \right),$$

of the GL functional, Eq. (5), in agreement with the invariance of the system under global rotations. In contrast to rotations, the operator of global shears has a non vanishing commutator due to relations:

$$\begin{aligned} [\hat{L}_{sh}, D_\beta] &= i L_{sh}^{\beta\alpha} D_\alpha \\ [\hat{L}_{sh}, D^2] &= -2i L_{sh}^{\alpha\beta} D_\alpha D_\beta \end{aligned}$$

The calculation of the energy, g_H , variation under shear transformation yields the same shear modulus, Eq. (14), in the leading order in κ^{-2} . In symmetric gauge there is no additional (regauging) phase factor, which generally accompanies geometrical transformation in a gauge theory.

It is important to note that the sheared state is not a ground state of the Abrikosov lattice with the same symmetry shown in Fig. 2b. The later is explicitly constructed in the symmetric gauge by Brandt [8]:

$$\phi_0(\mathbf{r}) \rightarrow \phi_0(\mathbf{r}) \prod_a \frac{x - x_a - i(y - y^a) - u_x^a - i u_y^a}{x - x^a - i(y - y^a)}, \quad (18)$$

where \mathbf{r}^a are positions of zeros of the order parameter in the hexagonal lattice. Restricting the shear transformations to LLL, he effectively retained the notion of a single state for a given lattice symmetry. Physically, it is equivalent to the assumption that the degrees of freedom related to the shape of the vortices can “relax” to their positions with minimal energy. It would mean that the system returns to an LLL state upon this relaxation. There is a popular belief that all degrees of freedom can be divided into two sets: “slow” and “fast”. “Slow” variables are the locations of vortices determined, for example, by the vortex center positions (where $\psi = 0$), and “fast” variables that contain all the other degrees of freedom related to the shape of the vortices. While near H_{c1} one can argue that the internal degrees of freedom are energetically very costly, near H_{c2} this is not correct. To our knowledge, there are no works which establish in what field range the separation between two set of degrees of freedom becomes possible. The GL energy does not contain an evident small parameter or “energy gap” that allows such a separation. Correspondingly, in dynamics based on the time-dependent GL equation, there is no separation of time relaxation of different degrees of freedom.

Mathematically, the shear transformation Eqs. (15) and (17) takes the ground state ϕ_0 out of the LLL sector since it does not commute with the “Hamiltonian” $-\frac{1}{2} D^2 + \frac{\hbar}{2}$. Although in our calculation magnetic induction deviates slightly from the external field h , the reason for a significant increase of the shear modulus compared to earlier estimates is not related to this. One can fix the field and calculate the modulus from the energy variation due to the shear

distortion, Eq. (17), and obtain as it was pointed out the same result, Eq. (4), to the leading order in κ^{-2} .

Physically, external forces leading to deformation of the vortex matter (“stress”) originate due to pinning by inhomogeneities. Let us consider pinning due to inhomogeneity of the critical temperature T_c , described by an “electric” pinning potential U that varies on a scale larger than the intervortex spacing a : $g_{\text{pin}} = \frac{1}{\kappa^2} \int_{\mathbf{r}} U(\mathbf{r}) \rho(\mathbf{r})$. To first order in U the correction to the wave function can be expanded in LL basis: $\psi = \sum_N \psi_N$ with

$$\psi_1 = -b^{-1} a_h^{1/2} \beta_A^{-1/2} U_{10} \phi_1, \quad U_{10} = \int_{\mathbf{r}} \phi_1 U(\mathbf{r}) \phi_0.$$

The correction to the current is $\delta \mathbf{I} = \int_{\mathbf{r}} \delta \mathbf{J}(\mathbf{r})$, where

$$\begin{aligned} \delta J_x &= \frac{i}{2} \sqrt{\frac{a_h}{\beta_A}} i (2b)^{-1/2} (\psi_1^* \phi_1 + \psi_1 \phi_1^*) + cc \\ &= \frac{a_h}{\beta_A b} (2b)^{-1/2} \sum_k [(\phi_{1k}^* U \phi_0) + cc], \\ \delta J_y &= \frac{i}{2} \sqrt{\frac{a_h}{\beta_A}} (2b)^{-1/2} (\psi_1^* \phi_1 - \psi_1 \phi_1^*) + cc \\ &= \frac{i a_h}{\beta_A b} (2b)^{-1/2} \sum_k [(\phi_{1k}^* U \phi_0)^* - cc]. \end{aligned}$$

The Lorentz force, $\mathbf{F}_L = c^{-1} [\delta \mathbf{I} \times \mathbf{b}]$, is

$$-F_{Ly} + iF_{Lx} = a_h \beta_A^{-1} (b/2)^{-1/2} U_{10}.$$

Only ψ_1 contributes, since the covariant derivatives in $\mathbf{J}(\mathbf{r})$ contain one “raising” operator. The Lorentz force is balanced by the “electric” pinning force, $\mathbf{F}_{\text{pin}} = \int_{\mathbf{r}} \rho \nabla U$, which, after integrating by parts and using $-iD_x \phi_0 = D_y \phi_0 = (2b)^{-1/2} \phi_1$, is given by the same expression. The above consideration demonstrates that an equilibrium state in the presence of the pinning force inevitably has mesoscopic supercurrents and in view of Eq. (2) cannot be treated in the framework of LLL only.

The shear modulus softening just below $H_{c2}(T)$ may play a crucial role in explaining the “peak effect” in the critical current [12]. The peak generally appears just before the “melting” of the Abrikosov lattice due to thermal fluctuations. Within the collective pinning theory [1], the critical current is estimated from the balance of the pinning force on Larkin domain $f_p = f_0 n R_c^2 L_c$ and the Lorentz force $J_c B$, where n is pinning density, f_0 – pinning strength proportional to a_h near H_{c2} and L_c , R_c are Larkin lengths along the field and in perpendicular direction, respectively. Larkin lengths in turn can be expressed via relevant elastic moduli leading to

$$J_c \propto (f_0 n)^2 / (bc_{44} c_{66}^2). \quad (19)$$

Using c_{66} of Eq. (1) one obtains a gradual increase in J_c as a_h^{-2} approaching $H_{c2}(T)$ since “softening” of the vortex lattice overcomes decrease of the pinning force. This corresponds to an “old” view of the “peak effect,” when this increase was thought to be followed by an abrupt jumps of the critical current to zero at the melting point (in prac-

tice might be smeared out by sample inhomogeneities). The recent view, supported by experiments in which Corbino geometry or width dependence were used to minimize or subtract the edge effects [5,6], attributes the peak to the amorphous homogeneous state. The critical current actually decreases monotonically with the field and then jumps from a relatively low value in the crystalline state to a very high value in the vortex glass (this was already noticed in [13]). Qualitatively, this is due to the fact that it is easier to pin a disordered homogeneous state than a rigid crystalline one. The continuous rise of the critical current observed in numerous earlier experiments was caused by poor resolution due to overheating of the solid and overcooling of the homogeneous states. The critical current in the amorphous phase rapidly drops as $\sqrt{T - T^g}$ when temperature approaches the glass temperature [14] T^g . Thus, the traditional picture predicts a gradual increase with subsequent drop of the critical current, while a modern view predicts a sudden increase followed by a fast but continuous decrease. If one uses the larger value of the shear modulus obtained here, Eq. (4), one indeed obtains a monotonic decrease to a constant value, since both pinning force and softening drop with similar rate in Eq. (19).

To summarize, we considered elastic response of the vortex lattice near the second critical field $H_{c2}(T)$ and showed that in the pinned state the system is necessarily excited to states outside of the lowest Landau level. This reflects the deformation of the current distribution profile under stress. As a result, the shear modulus is much larger (of order $1 - T/T_c - H/H_{c2}$) than that found by considering minimal energies of configurations with symmetries corresponding to sheared lattice, leading to $(1 - T/T_c - H/H_{c2})^2$. The obtained shear modulus leads to a monotonic decrease of the bulk contribution to the critical current in the crystalline phase before it discontinuously jumps to a much higher value in the vortex glass. Such behavior was recently obtained experimentally when edge contributions were minimized.

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References

- [1] G. Blatter, M.V. Feigel'man, V.B. Geshkenbein, A.I. Larkin, V.M. Vinokur, Rev. Mod. Phys. 66 (1994) 1125.
- [2] E.H. Brandt, Rep. Prog. Phys. 58 (1995) 1465.
- [3] T. Nattermann, S. Scheindl, Adv. Phys. 49 (2000) 607.
- [4] T. Giamarchi, P. Le Doussal, Phys. Rev. B52 (1995) 1242; T. Giamarchi, P. Le Doussal, Phys. Rev. B55 (1997) 6577.

- [5] Y. Paltiel et al., *Phys. Rev. Lett.* 85 (2000) 3712;
Y. Paltiel et al., *Nature* 403 (2000) 398.
- [6] Z.L. Xiao, E.Y. Andrei, M.J. Higgins, *Phys. Rev. Lett.* 83 (1999) 1664;
Z.L. Xiao et al., *Phys. Rev. B* 65 (2002) 094511.
- [7] N. Kokubo, R. Besseling, P.H. Kes, *Phys. Rev. B* 69 (2004) 064504;
N. Kokubo, K. Kadowaki, K. Takita, *Phys. Rev. Lett.* 95 (2005) 177005;
N. Kokubo, T. Asada, K. Kadowaki, K. Takita, T.G. Sorop, P.H. Kes, *Phys. Rev. B* 75 (2007) 1.
- [8] E.H. Brandt, *J. Low Temp. Phys.* 26 (1977) 709, 735.
- [9] A. Houghton, R.A. Pelcovits, A. Sudbo, *Phys. Rev. B* 40 (1989) 6763;
J. Kierfeld, V. Vinokur, *Phys. Rev. B* 61 (2000) R14928;
G.P. Mikitik, E.H. Brandt, *Phys. Rev. B* 64 (2001) 184514;
G.P. Mikitik, E.H. Brandt, *Phys. Rev. B* 68 (2003) 054509.
- [10] G. Lascher, *Phys. Rev. A* 140 (1965) 523;
D. Li, B. Rosenstein, *Phys. Rev. B* 60 (1999) 10460.
- [11] F.H. Stillinger, *Phys. Rev.* 142 (1966) 237;
M.D. Lipkin, S.A. Rice, U. Mohanti, *J. Chem. Phys.* 82 (1986) 472.
- [12] A.I. Larkin, M.C. Marchetti, V.M. Vinokur, *Phys. Rev. Lett.* 75 (1995) 2992;
C. Tang, X. Ling, S. Bhattacharya, P.M. Chaikin, *Europhys. Lett.* 35 (1996) 597.
- [13] R. Wordenweber, P.H. Kes, *Phys. Rev. B* 34 (1986) 494.
- [14] G. Bel et al., *Physica C* 460–462 (2007) 1213;
B. Rosenstein, V. Zhuravlev, *Phys. Rev. B* 76 (2007) 014707.