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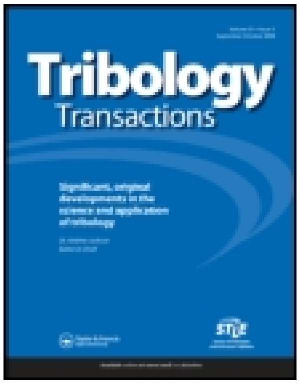

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A Mixed Lubrication Model for Computer Simulation of Extrusion Processes

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A Mixed Lubrication Model for Computer Simulation of Extrusion Processes©

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A mixed lubrication/friction model for extrusion process is developed in the present research. The model combines a rigid-plasticity finite element code to simulate the interface condition between the tooling and workpiece in the extrusion operation. The influence of surface roughness on lubricant flow is treated by using the average Reynolds equation. The active lubrication regime and appropriate friction factor were determined from the current local values of interface variables such as mean lubricant film thickness and workpiece and tooling roughness, in addition to the more traditional external variables such as interface pressure, node sliding velocity and strain rate of the workpiece. Numerical results using the coupled code include friction stress and normal pressure under different lubrication conditions are compared with experimental investigation. The discrepancy is very small and the proposed model proved to be very efficient in predicting interface friction condition in the extrusion processes.

Final manuscript approved February 16, 2000

KEY WORDS

Mixed Lubrication; Finite Element Method; Surface Roughness

INTRODUCTION

Friction and lubrication are of great importance in metal forming operations. In most cases minimizing friction is beneficial since it reduces the force and energy required for a given operation. This will lessen the stresses imposed on tooling and prevent direct metal-to-metal contact which contributes to longer tooling life and better quality control. Too thick a lubricant film will also result in a matt surface due to insufficient constraint by the tool-

NOMENCLATURE

A = fractional contact area
C_a = adhesion coefficient
E = nondimensional strain rate
H = nondimensional film thickness
H_a = nondimensional effective hardness
L_t = average asperity length
S = nondimensional velocity
U = velocity of the workpiece
U_1 = velocity of the workpiece at the contact edge
U_2 = velocity of the workpiece at the outlet edge
U_D = velocity of the tooling
h = lubricant film thickness
h_1 = lubricant film thickness at the edge of contact
k_s = shear strength of the workpiece
p = interface pressure
p_a = contact pressure of the asperity
p_b = lubricant pressure
q = extrusion pressure
x = the distance to the apex of tooling
x_1 = the distance of contact edge to the apex of tooling
x_2 = the distance of outlet edge to the apex of tooling
γ = pressure coefficient of the viscosity
ε = bulk strain rate
θ = semi-die angle
θ_t = mean slope of tooling asperities
μ = lubricant viscosity at pressure p
μ_0 = lubricant viscosity at atmospheric pressure
σ = composite surface roughness
σ_1 = surface roughnesses of sheet (R_q values)
σ_2 = surface roughnesses of tooling (R_q values)
τ_a = adhesive shear stress
τ_f = friction stress
τ_h = hydrodynamic friction stress
τ_p = plowing friction stress
ϕ_f = shear stress factor
ϕ_s = shear flow factors
ϕ_x = pressure flow factors

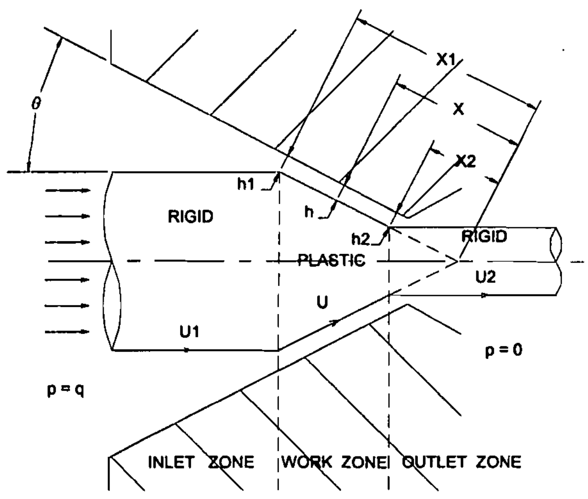

Fig. 1—Schematic view of extrusion process.

ing. Therefore a careful design of a lubrication system to achieve a suitable friction level is essential for a successful metal forming operation *(1)*.

The finite element method has been proved to be the most efficient computational scheme in the computer-based simulation of metal forming processes *(2)*. This method accurately simulates the flow of metal during the forming operation and provides the greatest amount of information about the process *(3)*. Despite the advances possible through finite element modeling of the forming process, one problem which has not been resolved satisfactorily involves modeling of the frictional stresses at the tooling/workpiece interface. One way of defining interface conditions in metal working has relied, to a large extent, on measurements of tool loads or tangential forces from which average friction coefficients could be obtained. Some theoretical simple friction models are also commonly used in describing the interface friction condition between the tooling and workpiece such as the Amontons-Coulomb law and constant friction factor. However, it has been pointed out that these simple friction models are not capable of describing the contact phenomenon between the tooling and workpiece due to large plastic deformation *(4)*. This greatly limits the usefulness of simulation as a design tool. More realistic friction models must take account of the fundamental processes involved in friction and lubrication at the tooling/workpiece interface.

As a first attempt to realistically implement friction in metal forming operations, Hsu and Wilson *(5)* and Wilson, et al. *(6)* developed a friction model in lubricated sheet-metal forming. Friction is expressed in terms of internal interface variables (mean lubricant film thickness, sheet roughness and tooling roughness) in addition to the more traditional external variables (interface pressure, sliding speed and strain rate). The new model was coupled with a finite element code and applied to an axisymmetric stretch forming operation. Numerical results using the coupled codes showed excellent agreement with measured strain distribution over a range of operation conditions. Hsu and Lee *(7)*, *(8)* and Hsu, et al. *(9)* analyzed the simple upsetting operation and rolling process by using the same strategy and the predictions from the model such as the distribution of the friction stress and normal pressure showed good correlation compared with the experimental measurements.

The present paper describes the development of a more realistic friction model for use in the computer simulation of extrusion processes. The friction model spanning the full range of lubrication regimes treats the influence of surface roughness on lubricant flow and on asperity contact. The mean lubricant film thickness is calculated by using the average flow model derived by Patir and Cheng *(10)*, *(11)*. In dealing with the mechanics of asperity contact, the boundary model proposed by Wilson and Sheu *(12)*, *(13)* is adopted in which the joint plastic deformation of surface asperities is considered. The computer simulation of extrusion processes is then conducted by incorporating the realistic friction model to a rigid-plasticity finite element code. Several experimental data are used to check the validity of the proposed model. The distribution of the friction stress and normal pressure between the workpiece and tooling are investigated. The results of computer simulation agree well with the experimental data.

LUBRICATION ANALYSIS

The process wherein a thick film lubrication model is prescribed was derived by Wilson and Walowit *(14)* and is shown in Fig. 1. A cylindrical billet is extruded through a conical die under the action of an extrusion pressure q. The product emerges into a space at zero pressure. No draw force or augmentation force is applied to the workpiece. The interface between the tooling and workpiece may be divided into three regions : the inlet zone, the work zone and the outlet zone. The lubricant is drawn into the space between the rigid workpiece and tooling in the inlet zone. The pressure in the lubricant builds up rapidly until it reaches the yield pressure of the workpiece at the inlet edge of the work zone. The workpiece deforms plastically in the work zone and the lubricant is carried along by the workpiece motion until it reaches the outlet zone. In the outlet zone the workpiece again becomes rigid and the lubricant film pressure falls to zero.

The average Reynolds equation *(10)*, *(11)* in the one dimension can be expressed as

$$\frac{d}{dx}\left(\phi_x \frac{h^3}{12\mu}\frac{dp}{dx}\right)=\frac{U}{2}\frac{dh}{dx}+\frac{U}{2}\sigma\frac{d\phi_s}{dx} \qquad [1]$$

where h is the lubricant film thickness and U is the average velocity of the workpiece and tooling. The hydrodynamic pressure is denoted as p and the composite surface roughness σ is defined as

$$\sigma=\sqrt{\sigma_1^2+\sigma_2^2} \qquad [2]$$

and σ_1 and σ_2 are the surface roughness of the tooling and workpiece, respectively. ϕ_x and ϕ_s are the pressure flow and shear flow factors which compensate for the effect of roughness and may be expressed as functions of h and σ. If the roughness are isotropic

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the authors may use the simple expressions provided by Tripp *(15)*.

$$\phi_x = 1 - \frac{3}{2}\left(\frac{\sigma}{h}\right)^2 \qquad [3]$$

$$\phi_s = \frac{3}{2}\frac{\sigma}{h}\frac{\sigma_1^2 - \sigma_2^2}{\sigma^2} \qquad [4]$$

The viscosity of the lubricant μ is assumed as Newtonian and governed by

$$\mu = \mu_0 e^{\gamma p} \qquad [5]$$

where μ_0 is the viscosity at atmospheric pressure and γ is the pressure coefficient of viscosity.

Since the lubricant film thickness is small compared with the diameter of the workpiece, the film thickness in the inlet zone as shown in Fig. 1 is assumed as

$$h = h_1 + (x - x_1)\tan\theta \qquad [6]$$

where x_1 is the distance from the inlet edge of the work zone to the apex of the tooling and h_1 is the film thickness at the inlet edge of the work zone. Since the pressure gradient in the work zone is so small that its effect on the lubricant flow may be neglected, the pressure gradients in the inlet and outlet zones are assumed to become zero at the edge of the work zone. The boundary conditions are

$$x = x_1,\ h = h_1,\ p = p_1 \text{ and } dp/dx = 0 \qquad [7]$$

$$x = \infty,\ h = \infty \text{ and } p = q \qquad [8]$$

The inlet zone analysis involves the integration of the average Reynolds Eq. [1] with the film thickness giver by Eq. [6]. After applying the boundary conditions given in Eqs. [7] and [8], the governing equation of the inlet film thickness is expressed as

$$\begin{aligned} & e^{-\gamma q} - e^{-\gamma p_1} \\ & = \frac{6\mu_0 U_1 \gamma}{\tan\theta}\left\{\left(\frac{h_1}{2h_c^3} - \frac{1}{h_c}\right) + \frac{1}{\sqrt{6}\sigma}\ln\left[\frac{\left(h_1 - \sqrt{3/2}\,\sigma\right)\left(h_c + \sqrt{3/2}\,\sigma\right)}{\left(h_c - \sqrt{3/2}\,\sigma\right)\left(h_1 + \sqrt{3/2}\,\sigma\right)}\right] + \frac{h_1}{3\sigma^2}\ln\left[\frac{h_1^2\left(h_c^2 - 3/2\,\sigma^2\right)}{h_c^2\left(h_1^2 - 3/2\,\sigma^2\right)}\right]\right\} \\ & + \frac{9\mu_0 U_1 \sigma_0 \gamma}{\tan\theta}\left\{\left(\frac{1}{2h_c^2 h_1} - \frac{1}{3h_c^3}\right) + \frac{2}{3\sigma^2}(h_1 - h_c) + \frac{2}{3\sqrt{6}\sigma}\ln\left[\frac{\left(h_1 - \sqrt{3/2}\,\sigma\right)\left(h_c + \sqrt{3/2}\,\sigma\right)}{\left(h_c - \sqrt{3/2}\,\sigma\right)\left(h_1 + \sqrt{3/2}\,\sigma\right)}\right] + \frac{1}{3\sigma^2 h_1}\ln\left[\frac{h_1^2\left(h_c^2 - 3/2\,\sigma^2\right)}{h_c^2\left(h_1^2 - 3/2\,\sigma^2\right)}\right]\right\} \end{aligned} \qquad [9]$$

where U_1 is the velocity of workpiece at the inlet edge and

$$h_c = 10\,\sigma \qquad [10]$$

and

$$\sigma_0 = \sigma_1^2 - \sigma_2^2 \qquad [11]$$

Since the pressure gradient in the work zone is assumed to be zero, the distribution of the film thickness in the work zone can then be obtained as

$$h = \frac{2U_1 h_1 + \frac{3U_1\sigma_0}{h_1} + \sqrt{\left(2U_1 h_1 + \frac{3U_1\sigma_0}{h_1}\right)^2 - 24U^2\sigma_0}}{4U} \qquad [12]$$

In the full film regime in which the lubricant film thickness is larger than three times of the composite surface roughness, Eqs. [9] and [12] can be used to calculate the film thickness between the tooling and workpiece. However such a condition in which the tooling is completely separated from the workpiece by a thin film of liquid lubricant is relatively rare in practice. The mixed regime in which the load is supported both by the pressurized lubricant film and asperity contact can thus be thought of as "general cases" dealing particularly with the conditions of most practical interest. The load sharing process can be expressed as

$$p_t = p_a A + p_b(1 - A) \qquad [13]$$

in which p_t is the total interface pressure, A is the nondimensional contact area, p_a is the average asperity contact pressure and p_b is the hydrodynamic pressure. An approximation for A is proposed from Christensen *(16)*.

$$A = \frac{35}{32}\left(\frac{16}{35} - z + z^3 - \frac{3}{5}z^5 + \frac{1}{7}z^7\right) \qquad [14]$$

and

$$z = \frac{h}{3\sigma} \qquad [15]$$

Wilson and Sheu *(12)* developed a semi-empirical equation to calculate the effective hardness of the workpiece subjected to plastic deformation and it can be written as

$$H_a = \frac{2}{f_1(A)E + f_2(A)} \qquad [16]$$

and

$$f_1(A) = -0.86A^2 + 0.345A + 0.515 \qquad [17]$$

$$f_2(A) = \frac{1}{2.571 - A - A\ln(1 - A)} \qquad [18]$$

The effective hardness of workpiece H_a can be also expressed as

$$H_a = \frac{(p_a - p_b)}{k_s} \qquad [19]$$

in which k_s is the material shear strength. The nondimensional strain rate E is denoted as

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$$E = \frac{1-A}{S} \qquad [20]$$

and S is the nondimensional velocity which can be written as

$$S = \frac{|U - U_D|\theta_t}{\dot{\varepsilon}L_t} \qquad [21]$$

L_t is the average half spacing of the asperity, θ_t is the average slope of the asperity, U_D is the velocity of the tooling and the strain rate of the workpiece is denoted as $\dot{\varepsilon}$.

The lubricant film thickness in the mixed lubrication regime is calculated by using a numerical scheme. It assumed an initial film thickness which is less than three times of the composite surface roughness first, and the hydrodynamic pressure and contact area can thus be evaluated from Eqs. [9] and [14], respectively. The asperity contact pressure is then determined from Eqs. [16] to [21] from which the nondimensional strain rate E is calculated using pertinent external variables by finite element analysis. The algorithm is repeated by adjusting the film thickness until Eq. [13] is satisfied.

FRICTION MODEL

Friction stress is most commonly characterized by the active lubrication regime at the interface. Generally, it consists of three parts and can be written as

$$\tau_f = \tau_a A + \tau_p A + \tau_h(1-A) \qquad [22]$$

where τ_a, τ_p and τ_h are the adhesion, plowing and viscous friction stress, respectively. The viscous resistance to lubricant shear τ_h is the dominant friction stress in the full film regime since contact area A equals to zero and can be expressed as

$$\tau_f = \tau_h = \phi_f \mu \frac{U_D - U}{h} \qquad [23]$$

The shear stress factor ϕ_f is a correction for surface roughness which is defined by Ref. *(15)* and can be written as

$$\phi_f = 0 \qquad h > 10\sigma \qquad [24]$$

$$\phi_f = \frac{35}{32} z\left\{(1-z^2)^3 \ln\left(\frac{z+1}{z-1}\right) + \frac{z}{15}\left[66 + z^2(30z^2 - 80)\right]\right\} \quad 3\sigma > h > 10\sigma \qquad [25]$$

$$\phi_f = \frac{35}{32} z\left\{(1-z^2)^3 \ln\left(\frac{z+1}{\beta}\right) + \frac{1}{60}\left[-55 + z(132 + z(345 + z(-160 + z(-405 + z(60 + 147z)))))\right]\right\} \qquad [26]$$

and

$$z = \frac{h}{3\sigma} \qquad [27]$$

$$\beta = \frac{1}{300} \qquad [28]$$

In the mixed lubrication regime, where the average film thickness is smaller, some asperity contacts are established and the adhesion and plowing components are given by

$$\tau_a = C_a k_s \qquad [29]$$

$$\tau_p = \theta_t k_s H_a \qquad [30]$$

which C_a is the adhesion coefficient.

RESULTS AND DISCUSSION

A number of test runs have been made using the proposed lubrication/friction model coupled to the rigid plasticity FEM code to investigate the performance of this approach. A simple two-dimensional FEM code, called ALPID (Analysis of Large Plastic Increment Deformation), written for metal-forming simulation by Kobayashi *(3)* is used. It is based on rigid viscoplastic finite-element formulation and also valid for rigid plastic material. One hundred and eighty elements were used in the finite element mesh as shown in Fig. 2. The parameters calculated from the FEM code such as contact pressure and surface velocity were transformed into the lubrication/friction model to determine the local lubricant film thickness. These variables were then combined to evaluate the local friction which was passed back to the FEM code. The basic strategy was outlined in Ref. *(6)*.

The experiments were conducted by Huang and Lu *(17)* and a semi-die angle of 14.036 degree were prepared. The assembly of the die contains the upper mold plate, ram, extrusion ring, cover plate, and the lower mold plate. The ram is attached to the upper mold plate and the workpiece is positioned at the entrance of the die at the beginning of the experiment. The surface profile of the die and workpiece are measured and the geometric quantities such as the half spacing and mean slope of the asperities can then be calculated from the profile by using the stylus instrument. Three different lubricants which range from high viscosity to low viscosity are chosen to apply at the interface between the workpiece and the die. The upper mold plate is then driven by hydraulic power with a constant speed 3.5 mm/sec to push the workpiece through the extrusion ring. Those variables used in the experiments are listed in the Table 1.

Figure 3 compares values of the normal force calculated from the coupled code with the experiments. The comparison of the friction force is shown in Fig. 4. It is evident that increasing the lubricant viscosity tends to reduce the normal force and friction force between the tooling and workpiece. The results of current simulation agree well with the experiments. The discrepancy could be caused by the determination of the experiment parameters such as pressure coefficient of the lubricant and adhesion coefficient.

The distribution of the normal pressure and friction stress in the work zone are shown in Figs. 5 and 6, respectively. The corresponding estimated lubrication regime along the work zone is shown in Fig. 7. All the three lubricants, high viscosity to low viscosity lubricant, produced a single mixed lubrication regime in the work zone. The nondimensional film thickness is between 1.2 to 1.8 which is less than the normal value of three. The magnitude of friction stress depends on the nondimensional film thickness.

TABLE 1—PARAMETERS USED IN THE EXPERIMENT *(17)*

Yielding stress of the workpiece Y (MPa)	21.6
Reduction rate R (%)	41.67
Semi die angle θ (radian)	0.245
σ_1 (μm)	0.2
σ_2 (μm)	(1) 2 (2) 16
μ_0(cSt)	(1) 51.9 (2) 100.9 (3) 763.7
γ (MPa^{-1})	0.01
Ca	0.2

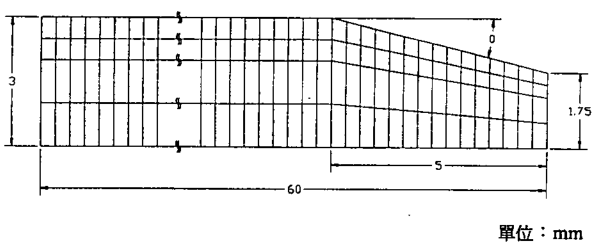

Fig. 2—The finite element mesh of the simulation.

Lower nondimensional film thickness tends to cause higher friction stress and lower normal pressure. The variation caused by the film thickness is much more apparent in the friction stress distribution than in the normal pressure distribution.

Another important parameter which will affect the interface condition is surface roughness. The influence of surface roughness of the workpiece on the distribution of normal pressure and friction stress are shown in Figs. 8 and 9, respectively. It can be expected that rougher surface will produce a higher friction situation. The surface roughness effect on the normal pressure distribution is not as apparent as in the friction distribution. Their corresponding estimated lubrication regime is shown in Fig. 10. The nondimensional film thickness is the ratio between the lubricant film thickness and the composite surface roughness. The mixed lubrication regime which is defined as the nondimensional film thickness less than three is prevailed along the workzone. The angle between the workpiece and tooling is also very crucial in designing the extrusion operation. It is common to select an angle which will require less entry pressure and it can be determined by Fig. 11 for various lubricants.

CONCLUSIONS

The simulation combines a rigid-plasticity finite element code with a lubrication analysis and friction model. The surface roughness effect on lubricant flow is included by using an average Reynolds equation. The local values of roughness and film thickness were then combined to decide the active lubrication regime at each location and increment within the workpiece/tooling interface. A friction model appropriate to the regime then provided an estimate of friction for use in the evolving plasticity solution. The distributions of normal force and friction force predicted by the proposed model can be used to design the tooling and choose the suitable manufacturing parameters.

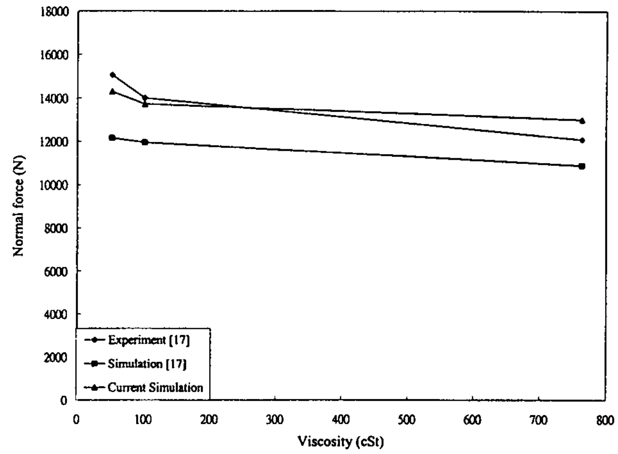

Fig. 3—Variation of the normal force for different lubricant with σ_2 = 2μm.

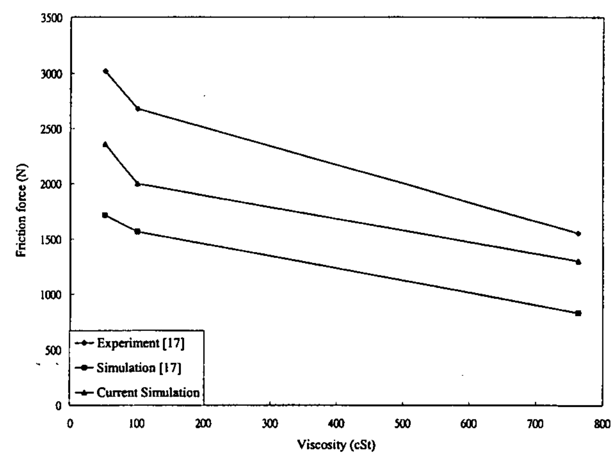

Fig. 4—Variation of the friction force for different lubricant with σ_2 = 2μm.

On the experimental side, it would be valuable to conduct more experiments to help verify and refine the existing model. The inclusion of thermal effect will definitely improve the sensitivity and accuracy of the current code. Work is in progress in each of these areas.

ACKNOWLEDGMENTS

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REFERENCE

(1) Wilson, W. R. D., "Friction and Lubrication in Bulk Metal-Forming Processes," *Jour. Applied Metalworking*, 1, pp 7-19, (1979).
(2) Zienkiewicz, O. C., *The Finite Element Method*, McGraw-Hill Book Co. (UK) Ltd., London, (1997).
(3) Kobayashi, S., Oh, S. I. and Altan, T., "Metal Forming and the Finite Element Method," Oxford University Press, Oxford, 1989.
(4) Wilson, W. R. D., "Mixed Lubrication in Metal Forming," *Advanced Technology of Plasticity*, pp 1667-1675, (1990).

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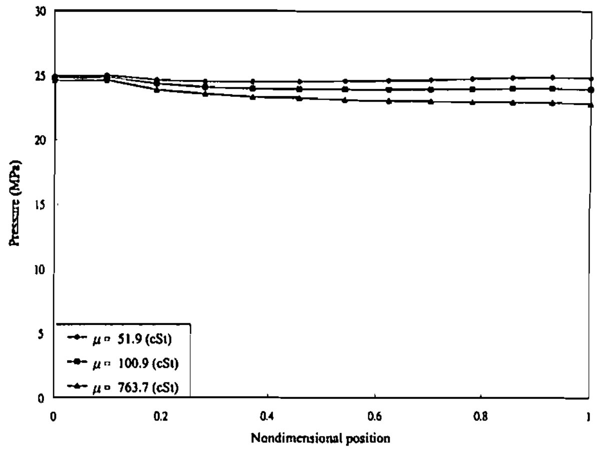

Fig. 5—The distribution of normal pressure in the work zone.

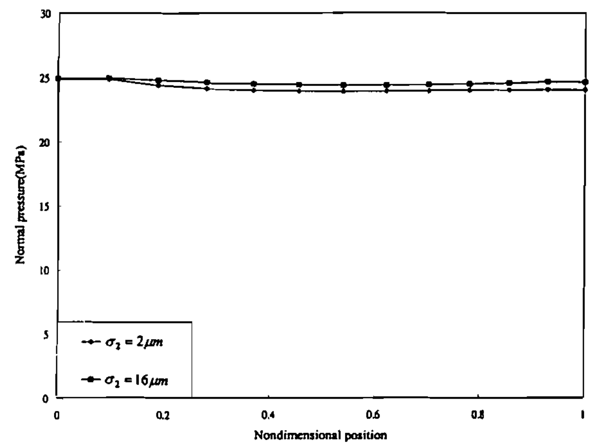

Fig. 8—The distribution of normal pressure in the work zone with μ = 100.9 cSt.

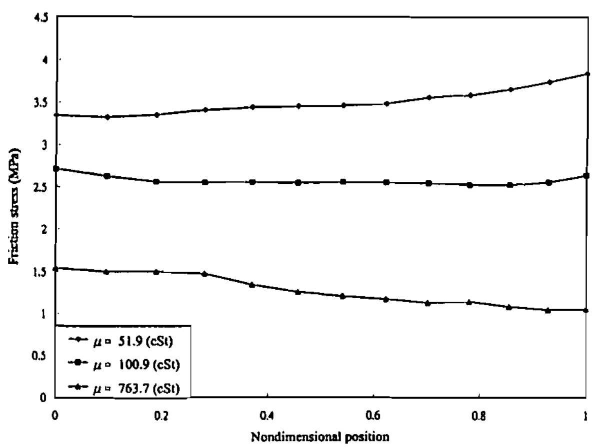

Fig. 6—The distribution of friction stress in the work zone.

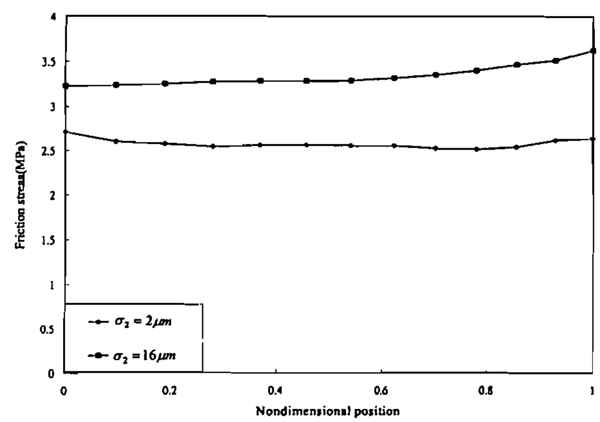

Fig. 9—The distribution of friction stress in the work zone with μ = 100.9 cSt.

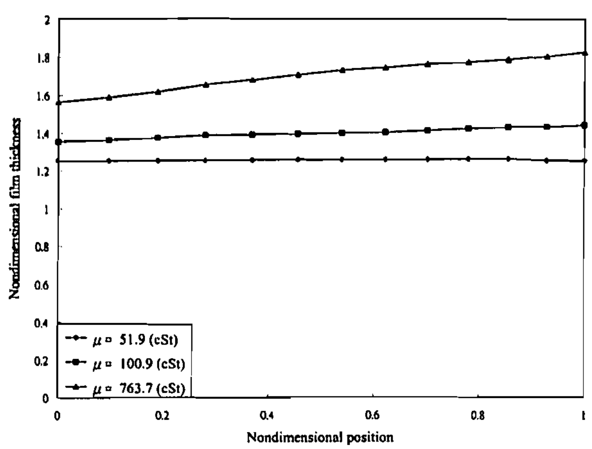

Fig. 7—The distribution of nondimensional film thickness in the work zone.

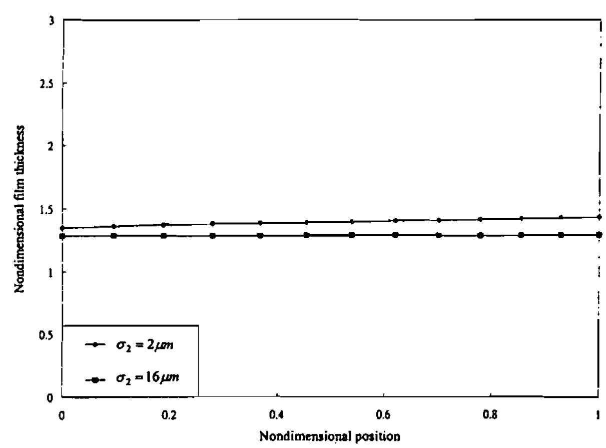

Fig. 10—The distribution of nondimensional film thickness in the work zone.

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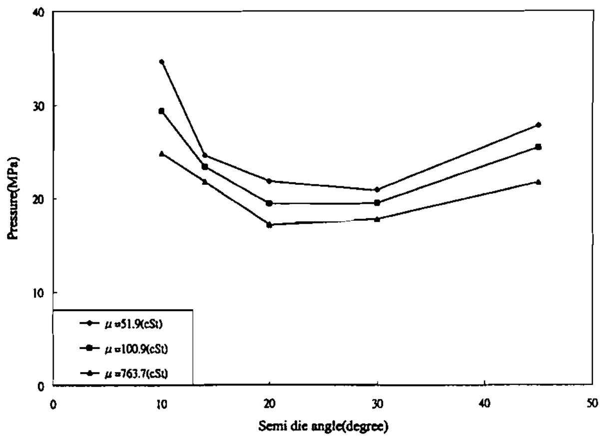

Fig. 11—The entry pressure under different semi die angle.

(5) Hsu, T. C. and Wilson, W. R. D., "Refined Models for Hydrodynamic Lubrication in Axisymmetric Stretch Forming," *ASME, Jour. of Trib.*, **116**, 1, pp 101-110, (1994).

(6) Wilson, W. R. D., Hsu, T. C. and Huang, X. B., "A Realistic Friction Model for Computer Simulation of Sheet Metal Forming Processes," *ASME, Jour. of Eng. for Industry*, **117**, pp 202-209, (1995).

(7) Hsu, T. C. and Lee, C. H., "Realistic Friction Modeling for Simple Upsetting," *ASME, Jour. of STLE*, (1997).

(8) Hsu, T. C. and Lee, C. H., "Refined Friction Modeling for Simple Upsetting," *ASME, Jour. of Manufacturing Science and Eng.*, pp 563-570, (1997).

(9) Hsu, T. C. and Hung-Mo, Wu, "The Computer Simulation of Lubricated Cold Rolling," *ASME, Jour. of STLE*.

(10) Patir, N. and Cheng, H. S., "An Average Flow Model for Determining Effects of Three-Dimensional Roughness on Partial Hydrodynamic Lubrication," *Jour. of Lubr. Tech.*, **100**, 1, pp 12-17, (1978).

(11) Patir, N. and Cheng H. S., "Application of Average Flow Model to Lubrication Between Rough Sliding Surfaces," *Jour. of Lubr. Tech.*, **101**, 4, pp 220-230, (1979).

(12) Wilson, W. R. D. and Sheu, S., "Real Area of Contact and Boundary Friction in Metal Forming," *Int'l. Jour. of Mech. Sci.*, **30**, 7, pp 475-489, (1989).

(13) Sheu, S. and Wilson, W. R. D., "Mixed Lubrication of Strip Rolling," *Trib. Trans.*, **37**, pp 483-493, (1994).

(14) Wilson, W. R. D. and Walowit, J. A., "An Isothermal Hydrodynamic Lubrication

Theory for Hydrostatic Extrusion and Drawing Processes With Conical Dies," *Jour. of Lubr. Tech.*, **93**, 1, pp 69-74, (1971).

(15) Tripp, J. H., "Surface Roughness Effects in Hydrodynamic Lubrication: The Flow Factor Method," *Jour. of Lubr. Tech.*, **105**, 7, pp 458-465, (1983).

(16) Christensen, H., "Stochastic Models for Hydrodynamic Lubrication of Rough Surfaces," *Int'l. Jour. of Mechanical Sciences*, **104**, pp 1022-1033, (1970).

(17) Huang, M. C. and Lu, S. S., "The Experiment of Extrusion Processes," Master Thesis of Department of Mechanical Engineering, Taiwan University, (1988).

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