

New Unified Evolution Equation

Jyh-Liong Lim¹ and Hsiang-nan Li²

¹*Department of Electrophysics, National Chiao-Tung University,
Hsinchu, Taiwan 300, R.O.C.*

²*Department of Physics, National Cheng-Kung University,
Tainan, Taiwan 701, R.O.C.*

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We propose a new unified evolution equation for parton distribution functions appropriate for both large and small Bjorken variables x , which is an improved version of the Ciafaloni-Catani-Fiorani-Marchesini equation. In this new equation the cancellation of soft divergences between virtual and real gluon emissions is explicit without introducing infrared cutoffs and next-to-leading contributions to the Sudakov resummation can be included systematically. It is shown that the new equation reduces to the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi equation at large x , to the Balitsky-Fadin-Kuraev-Lipatov (BFKL) equation at small x , and to the modified BFKL equation with a dependence on momentum transfer Q , if hard virtual gluon contributions are retained.

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I. Introduction

It is known that the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equation [1] sums the large logarithmic corrections $\ln Q$ to a parton distribution function at large x , with Q being the momentum transfer and x the Bjorken variable, and that the Balitsky-Fadin-Kuraev-Lipatov (BFKL) equation [2] sums $\ln(1/x)$ at small x . The Ciafaloni-Catani-Fiorani-Marchesini (CCFM) equation [3], which is appropriate for both large and small x , has been proposed to unify the above two equations. The conventional derivation of the evolution equations involves an all-order summation of ladder diagrams with rung gluons obeying specific kinematic orderings. For the DGLAP, BFKL, and CCFM equations, they are transverse momentum ordering, rapidity ordering, and angular ordering, respectively.

Recently, we have applied the Collins-Soper (CS) resummation technique [4] to the all-order summation of various large logarithms [5]. This technique was developed originally to organize double logarithms $\ln^2 Q$. We have demonstrated that it is also applicable to single-logarithm cases, *i.e.*, the derivation of the evolution equations mentioned above [5]. In this approach all the evolution kernels are obtained from the same one-loop diagrams, which are evaluated under appropriate soft approximations that correspond to specific kinematic orderings. Therefore, our method is simpler, and provides a unified viewpoint to the evolution equations. It is easy to modify the approximation employed in the evaluation of the BFKL kernel, such that a Q dependence can be introduced into the BFKL equation [6], which is important for the explanation of the experimental data.

As reproducing the CCFM equation in the framework of resummation, we notice that real and virtual gluon emissions must be treated in a different way: the former are computed to lowest order, generating a splitting function, while the latter are resummed to all orders, giving a Sudakov form factor. This observation is consistent with that in the conventional derivation [3]. However, infrared divergences must cancel between real and virtual gluon emissions order by order. Hence, in the conventional derivation infrared cutoffs are necessary for regularizing the individual divergences in real and virtual corrections. These cutoffs may cause an ambiguity, when one intends to include next-to-leading logarithmic summation into the evolution equation.

In Ref. [7] we have proposed a new unified evolution equation, which results from one of the applications of the CS formalism to logarithmic summations. In this paper we shall present a detailed derivation of the new equation, explain the ingredients that are added compared to the CCFM equation, and discuss features of the gluon distribution function solved from this new equation. In our approach real gluon correction is separated into two pieces: the piece containing an infrared pole is combined with the corresponding virtual correction, both of which are resummed into a Sudakov form factor. The other piece, being infrared finite, is evaluated to lowest order, giving a splitting function. With such an arrangement, both the Sudakov form factor and the splitting function are rendered infrared finite. Using the CS formalism, the running of coupling constants appearing in the Sudakov form factor and in the splitting function, which contributes to next-to-leading logarithmic summation, can be taken into account. It will be shown that hard virtual gluon contributions introduce a dependence on the kinematic variable Q to the rise of the gluon distribution function at small x . This Q dependence is similar to that of the CCFM equation, which arises from the $\ln Q$ summation.

The CCFM equation is reproduced using the CS resummation technique in Sec. 2. The new unified evolution equation is derived in Sec. 3. We demonstrate in Sec. 4 that the new equation reduces to the DGLAP equation, to the BFKL equation, and to the modified BFKL equation with a Q dependence. Section 5 is the conclusion.

II. CCFM equation

We study the unintegrated gluon distribution function defined in the axial gauge $n \cdot A = 0$, n being a gauge vector:

$$F(x, k_T, p^+) = \frac{1}{p^+} \int \frac{dy^-}{2\pi} \int \frac{d^2 y_T}{4\pi} e^{-i(xp^+ y^- - \mathbf{k}_T \cdot \mathbf{y}_T)} \times \frac{1}{2} \sum_{\sigma} \langle p, \sigma | F_{\mu}^+(y^-, y_T) F^{\mu+}(0) | p, \sigma \rangle, \quad (1)$$

where $|p, \sigma\rangle$ denotes the incoming hadron with light-like momentum $p^{\mu} = p^+ \delta^{\mu+}$ and spin σ , and F_{μ}^+ is the field tensor. An average over color is understood. $F(x, k_T, p^+)$ describes the probability of a gluon carrying a longitudinal momentum fraction x and transverse momenta k_T .

For $n^{\mu} = \delta^{\mu-}$ lying on the light cone, Eq. (1) is gauge invariant. This can be understood by considering a variation of n on the light cone, under which Eq. (1) is still defined for $A^+ = 0$ and remains the same. To derive an evolution equation of a parton distribution function using the CS formalism, we must allow n to vary arbitrarily away from the light cone ($n^2 \neq 0$), and a parton distribution function becomes gauge dependent. Parton distribution functions defined for different

n possess the same infrared structure, but different ultraviolet structure [8]. It has been shown that the gauge dependence of a parton distribution function is cancelled by that from nonfactorizable soft gluon exchanges in a physical process, rendering cross section gauge invariant. Hence, the vector n acts as an auxiliary tool.

The key step of the CS formalism is to obtain the derivative $p^+ dF/dp^+$. Because of the scale invariance of F in the vector n as indicated by the gluon propagator, $-iN^{\mu\nu}(l)/l^2$, with

$$N^{\mu\nu} = g^{\mu\nu} - \frac{n^\mu l^\nu + n^\nu l^\mu}{n \cdot l} + n^2 \frac{l^\mu l^\nu}{(n \cdot l)^2}, \quad (2)$$

F must depend on p^+ via the ratio $(p \cdot n)^2/n^2$. Hence, we have the chain rule relating $p^+ d/dp^+$ to d/dn [4, 9]:

$$p^+ \frac{d}{dp^+} F = -\frac{n^2}{v \cdot n} v_\alpha \frac{d}{dn_\alpha} F, \quad (3)$$

with $v^\alpha = \delta^{\alpha+}$ a dimensionless vector along p . The operator d/dn_α applies to a gluon propagator, giving [4]

$$\frac{d}{dn_\alpha} N^{\mu\nu} = -\frac{1}{n \cdot l} (l^\mu N^{\alpha\nu} + l^\nu N^{\mu\alpha}). \quad (4)$$

The loop momentum l^μ (l^ν) in the above expression contracts with a vertex the differentiated gluon attaches, which is then replaced by a special vertex [4,9]

$$\hat{v}_\alpha = \frac{n^2 v_\alpha}{v \cdot n n \cdot l}. \quad (5)$$

This special vertex can be easily read off from the combination of Eqs. (3) and (4).

Employing the Ward identities for the contraction of l^μ (l^ν) [5], the sum of diagrams with different differentiated gluons reduces to a new diagram, in which the special vertex moves to the outer end of a parton line [4,9]. We obtain the formula

$$p^+ \frac{d}{dp^+} F(x, k_T, p^+) = 2\bar{F}(x, k_T, p^+), \quad (6)$$

shown in Fig. 1(a), where \bar{F} denotes the new diagram, and the square represents the special vertex. The coefficient 2 comes from the equality of the new functions with the special vertex on either of the two parton lines.

The leading regions of the loop momentum flowing through the special vertex are soft and hard, since the vector n does not lie on the light cone, and collinear enhancements are suppressed. If following the standard resummation procedure, we should factorize out subdiagrams containing the special vertex order by order in the leading regions (see Figs. 1(b) and 1(c)). This procedure will lead to a new unified evolution equation in the next section. To reproduce the CCFM equation, however, the factorization must be performed in a different way described by Fig. 2(a), where the two jet functions J group all-order virtual corrections, and the lowest-order real gluon emission between them is soft. This factorization picture is motivated by the conventional derivation in [3].

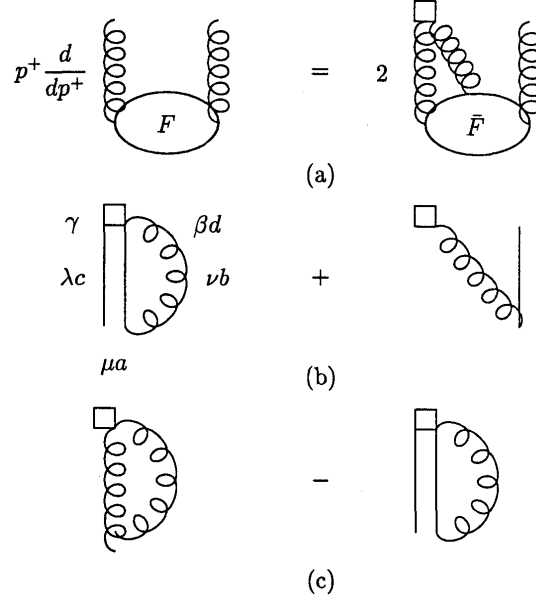


FIG. 1. (a) The derivative $p^+ dF/dp^+$ in the axial gauge. (b) The lowest-order subdiagrams for K . (c) The lowest-order subdiagrams for G .

We first resum the double logarithms contained in J by considering its derivative

$$p^+ \frac{d}{dp^+} J(p_T, p^+) = \bar{J}(k_T, p^+) = [K_J(k_T/\mu, \alpha_s(\mu)) + G_J(p^+/\mu, \alpha_s(\mu))] J(k_T, p^+), \quad (7)$$

which is similar to Eq. (6). In the second expression subdiagrams containing the special vertex have been factorized into the functions K_J and G_J in the leading soft and hard regions, respectively. At lowest order, K_J comes from Fig. 2(b), where the replacement of the gluon line by an eikonal line will be explained below, and G_J comes from Fig. 2(c). The second diagram in Fig. 2(c) serves as a soft subtraction, which guarantees a hard momentum flow in G_J . We have set the infrared cutoff of K_J to k_T , as indicated by its argument, in order to regularize its soft pole. This cutoff is necessary here due to the lack of the corresponding real gluon emission diagram. At this step, an ambiguity in the summation of single logarithms is introduced.

We evaluate the contribution from Fig. 2(b). The color factor is extracted from the relation $f_{abc} f_{bdc} = -N_c \delta_{ad}$, where the indices a, b, \dots are referred to Fig. 1(b), and $N_c = 3$ is the number of colors. The one-loop K_J is written as

$$K_J = iN_c g^2 \int \frac{d^4 l}{(2\pi)^4} N_{\nu\beta}(l) \frac{\hat{v}^\beta \Gamma^{\mu\nu\lambda}}{-2xp \cdot ll^2} - \delta K_J, \quad (8)$$

with δK_J an additive counterterm. The triple-gluon vertex is

$$\Gamma^{\mu\nu\lambda} = g^{\mu\nu}(xp + l)^\lambda + g^{\nu\lambda}(xp - 2l)^\mu + g^{\lambda\mu}(l - 2xp)^\nu, \quad (9)$$

$$\approx xp^+(g^{\mu\nu}v^\lambda + g^{\nu\lambda}v^\mu - 2g^{\lambda\mu}v^\nu), \quad (10)$$

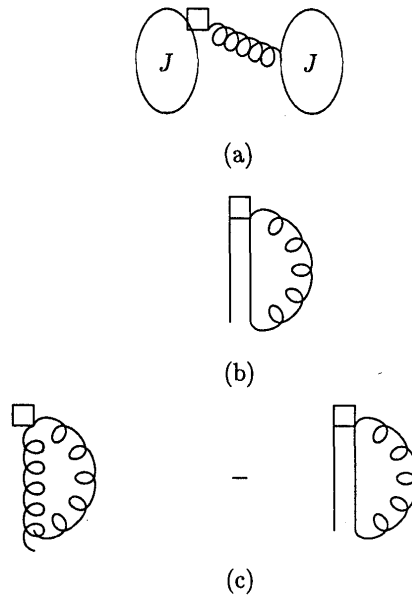


FIG. 2. (a) The subdiagram containing the special vertex for the CCFM equation. (b) The lowest-order subdiagram for K_J . (c) The lowest-order subdiagrams for G_J .

where Eq. (10) is the soft loop momentum ($l \rightarrow 0$) limit of Eq. (9). The factor $1/(-2xp \cdot l)$ is the soft approximation of the gluon propagator $1/(l - xp)^2$.

It has been shown that the terms v^λ and v^μ in Eq. (10) result in contributions suppressed by a power $1/s$, $s = (p + q)^2$, compared to the contribution from the last term v^ν [5,6]. Absorbing the metric tensor $g^{\lambda\mu}$ into F , Eq. (8) is simplified into

$$K_J = iN_c g^2 \int \frac{d^4 l}{(2\pi)^4} N_{\nu\beta}(l) \frac{\hat{v}^\beta v^\nu}{v \cdot ll^2} - \delta K_J. \tag{11}$$

The factor $v^\nu/(v \cdot l)$ is represented by an eikonal line in Fig. 2(b), with the Feynman rules v^ν for an eikonal vertex and $1/(v \cdot l)$ for an eikonal propagator.

Since ultraviolet poles exist in Fig. 2(b) and in the second diagram of Fig. 2(c), the sum $K_J + G_J$ is independent of μ , *i.e.*, renormalization-group (RG) invariant. The standard RG analysis leads to

$$\mu \frac{d}{d\mu} K_J(k_T/\mu, \alpha_s(\mu)) = -\gamma_J(\alpha_s(\mu)) = -\mu \frac{d}{d\mu} G_J(p^+/\mu, \alpha_s(\mu)), \tag{12}$$

whose solution is written as

$$K_J(k_T/\mu, \alpha_s(\mu)) + G_J(p^+/\mu, \alpha_s(\mu)) = -\gamma_J(\alpha_s) \int_{k_T}^{p^+} \frac{d\bar{\mu}}{\bar{\mu}}. \tag{13}$$

The anomalous dimension γ_J is found to be [5]

$$\gamma_J = \mu \frac{d}{d\mu} \delta K_J = \bar{\alpha}_s, \quad (14)$$

with the coupling constant $\bar{\alpha}_s = N_c \alpha_s / \pi$. To the accuracy of leading (double) logarithms [3], we have neglected the initial conditions $K_J(1, \alpha_s(k_T))$ and $G_J(1, \alpha_s(p^+))$ of the RG evolution, the higher-order terms in γ_J , and the running of $\bar{\alpha}_s$, which contribute to next-to-leading (single) logarithmic summation. In the derivation of the new unified evolution equation these ingredients can be included.

Substituting Eq. (13) into Eq. (7), we solve for

$$J(k_T, Q) = \Delta^{1/2}(k_T, Q) J^{(0)}, \quad (15)$$

with the double-logarithm exponential

$$\Delta(k_T, Q) = \exp \left[-2\bar{\alpha}_s \int_{k_T}^Q \frac{dp^+}{p^+} \int_{k_T}^{p^+} \frac{d\bar{\mu}}{\bar{\mu}} \right]. \quad (16)$$

where the upper bound of p^+ has been chosen as Q . The initial condition $J^{(0)}$ can be regarded as a tree-level gluon propagator, which will be eikonalized in the evaluation of the lowest-order soft real gluon emission below.

We split the above exponential into

$$\Delta(k_T, Q) = \Delta_S(zq, Q) \Delta_{NS}(z, q, k_T), \quad (17)$$

with $z = x/\xi$ and $q = l_T/(1-z)$, where ξ is the momentum fraction entering J from the bottom, and l_T is the transverse loop momentum carried by the soft real gluon. The so-called ‘‘Sudakov’’ exponential Δ_S and the ‘‘non-Sudakov’’ exponential Δ_{NS} are given by

$$\begin{aligned} \Delta_S(zq, Q) &= \exp \left[-2\bar{\alpha}_s \int_{zq}^Q \frac{dp^+}{p^+} \int_{k_T}^{p^+} \frac{d\bar{\mu}}{\bar{\mu}} \right], \\ &= \exp \left[-\bar{\alpha}_s \int_{(zq)^2}^{Q^2} \frac{dp^2}{p^2} \int_0^{1-k_T/p} \frac{dz'}{1-z'} \right], \end{aligned} \quad (18)$$

$$\begin{aligned} \Delta_{NS}(z, q, k_T) &= \exp \left[-2\bar{\alpha}_s \int_{k_T}^{zq} \frac{dp^+}{p^+} \int_{k_T}^{p^+} \frac{d\bar{\mu}}{\bar{\mu}} \right], \\ &= \exp \left[-\bar{\alpha}_s \int_z^{k_T/q} \frac{dz'}{z'} \int_{(z'q)^2}^{k_T^2} \frac{dp^2}{p^2} \right], \end{aligned} \quad (19)$$

where the variable changes $\bar{\mu} = (1-z')p$ and $p^+ = p$ for Δ_S , and $\bar{\mu} = p$ and $p^+ = z'q$ for Δ_{NS} have been adopted to obtain the second expressions.

Using Eq. (15) for the two jet functions, \bar{F} from Fig. 2(a) is written as

$$\begin{aligned} \bar{F}(x, k_T, p^+) &= iN_c g^2 \int \frac{d^4 l}{(2\pi)^4} N_{\nu\beta}(l) \frac{\hat{v}^\beta v^\nu}{v \cdot l} 2\pi i \delta(l^2) \Delta(k_T, Q) \\ &\quad \times \theta(Q - zq) F(x + l^+/p^+, |\mathbf{k}_T + \mathbf{l}_T|, p^+), \end{aligned} \quad (20)$$

where the tree-level gluon propagator $J^{(0)}$ on the right-hand side has been eikonized to give the factor $1/v \cdot l$, and that on the left-hand side has been absorbed into F . The extra θ function, requiring $Q > zq$, renders the Sudakov exponential Δ_S meaningful. This relation comes from the angular ordering of radiative gluons, $Q/(xp^+) > l_T/[(\xi - x)p^+]$. Compared with the transverse momentum ordering for the DGLAP equation, Q (l_T) has been divided by the longitudinal momentum xp^+ ($(\xi - x)p^+$). Therefore, the inserted scale zq reflects the specific kinematic ordering for the CCFM equation. Those radiative gluons, which do not obey angular ordering, contribute to the non-Sudakov exponential.

Adopting the variable change $\xi = x + l^+/p^+$ and performing the integration over l^- , we obtain

$$\begin{aligned} \bar{F}(x, k_T, p^+) &= \frac{\bar{\alpha}_s}{2} \int_x^1 d\xi \int \frac{d^2 l_T}{\pi} \frac{2n^2(\xi - x)p^{+2}}{[n^+ l_T^2 + 2n^-(\xi - x)^2 p^{+2}]^2} \Delta(k_T, Q) \\ &\quad \times \theta(Q - zq) F(\xi, |\mathbf{k}_T + \mathbf{l}_T|, p^+), \end{aligned} \quad (21)$$

where $n = (n^+, n^-, \mathbf{0})$ has been chosen for convenience. The above expression is then substituted into Eq. (6) to find a solution of F . Integrating Eq. (21) from $p^+ = 0$ to Q , and applying the variable changes $\xi = x/z$ and $\mathbf{l}_T = (1 - z)\mathbf{q}$, we have

$$\begin{aligned} F(x, k_T, Q) &= F^{(0)} + \bar{\alpha}_s \int_x^1 dz \int \frac{d^2 q}{\pi q^2} \theta(Q - zq) \Delta_S(Q, zq) \Delta_{NS}(z, q, k_T) \\ &\quad \times \frac{1}{z(1 - z)} F(x/z, |\mathbf{k}_T + (1 - z)\mathbf{q}|, q), \end{aligned} \quad (22)$$

where the initial condition $F^{(0)}$ corresponds to the lower bound of p^+ . To work out the p^+ integration, $F(x/z, |\mathbf{k}_T + \mathbf{l}_T|, p^+)$ in Eq. (21) has been approximated by $F(x/z, |\mathbf{p}_T + \mathbf{l}_T|, q)$. Note that the n dependence has disappeared, and the CCFM kernel is gauge invariant.

Equation (22) can be reexpressed as

$$\begin{aligned} F(x, k_T, Q) &= F^{(0)} + \bar{\alpha}_s \int_x^1 dz \int \frac{d^2 q}{\pi q^2} \theta(Q - zq) \Delta_S(zq, Q) \tilde{P}(z, q, k_T) \\ &\quad \times F(x/z, |\mathbf{k}_T + (1 - z)\mathbf{q}|, q), \end{aligned} \quad (23)$$

with the function

$$\tilde{P} = \left[\frac{1}{(1 - z)_+} + \Delta_{NS}(z, q, k_T) \frac{1}{z} - 2 + z(1 - z) \right], \quad (24)$$

which is close to the splitting function

$$P_{gg} = \left[\frac{1}{(1 - z)_+} + \frac{1}{z} - 2 + z(1 - z) \right]. \quad (25)$$

Equation (23) is the CCFM equation [3]. To arrive at Eq. (24), we have employed the identity $1/[z(1 - z)] \equiv 1/(1 - z) + 1/z$, and put in by hand the last term $-2 + z(1 - z)$. This term,

finite at $z \rightarrow 0$ and at $z \rightarrow 1$, can not be obtained in [3] either due to a soft approximation. Note that Eq. (22) contains an infrared singularity from $z \rightarrow 1$. Hence, we have replaced the factor $1/(1-z)$ by the plus distribution $1/(1-z)_+$ in Eq. (23), and dropped the soft pole corresponding to $z \rightarrow 1$. On the other hand, only the non-Sudakov form factor Δ_{NS} in front of $1/z$ is retained. Because Δ_{NS} vanishes when the upper bound zq of p^+ approaches zero, as indicated by Eq. (19), it smears the $z \rightarrow 0$ pole in \tilde{P} [3].

As shown above, the double-logarithm exponential Δ and the function \tilde{P} are infrared divergent individually. To have a well-defined evolution equation, the infrared regularizations for Δ and \tilde{P} must be implemented. The lower bound of the variable z is x , and never goes down to zero. Therefore, the smearing factor Δ_{NS} and the splitting of the double-logarithm exponential in Eq. (17) are not essential. The new unified evolution equation derived in the next section will take into account these observations.

III. New unified equation

In this section we shall derive a new unified evolution equation following the standard procedure of the CS formalism. We start with Eq. (6) for the unintegrated gluon distribution function $F(x, k_T, p^+)$. We Fourier transform Eq. (6) into b space with b being the conjugate variable of k_T . In the leading soft and hard regions of the loop momentum flowing through the special vertex, \bar{F} can be factorized into the convolution of subdiagrams containing the special vertex with the original distribution function F . To lowest order, the soft contribution from the subdiagrams in Fig. 1(b) is written as

$$\bar{F}_s(x, b, p^+) = \bar{F}_{sv}(x, b, p^+) + \bar{F}_{sr}(x, b, p^+), \quad (26)$$

with

$$\bar{F}_{sv} = iN_c g^2 \int \frac{d^4 l}{(2\pi)^4} N_{\nu\beta}(l) \frac{\hat{v}^\beta v^\nu}{v \cdot l l^2} F(x, b, p^+), \quad (27)$$

$$\bar{F}_{sr} = iN_c g^2 \int \frac{d^4 l}{(2\pi)^4} N_{\nu\beta}(l) \frac{\hat{v}^\beta v^\nu}{v \cdot l} 2\pi i \delta(l^2) e^{i l_T \cdot \mathbf{b}} \times F(x + l^+/p^+, b, p^+), \quad (28)$$

corresponding to virtual and real gluon emissions, respectively. Equation (27) is the same as the integral in Eq. (11). The factor $\exp(i l_T \cdot \mathbf{b})$ in Eq. (28) comes from Fourier transformation of the real gluon contribution.

We reexpress the function F in the integrand of \bar{F}_{sr} as

$$F(x + l^+/p^+, b, p^+) = F(x, b, p^+) + [F(x + l^+/p^+, b, p^+) - F(x, b, p^+)]. \quad (29)$$

The contribution from the first term on the right-hand side of Eq. (29) is combined with \bar{F}_{sv} , leading to $K(b\mu, \alpha_s(\mu))F(x, b, p^+)$ with the function

$$K = iN_c g^2 \int \frac{d^4 l}{(2\pi)^4} N_{\nu\beta}(l) \frac{\hat{v}^\beta v^\nu}{v \cdot l} \times \left[\frac{1}{l^2} + 2\pi i \delta(l^2) e^{i l_T \cdot \mathbf{b}} \right] - \delta K, \quad (30)$$

where δK is an additive counterterm. The contribution from the second term in Eq. (29) is denoted as $\bar{F}'_s(x, b, p^+)$,

$$\bar{F}'_s = iN_c g^2 \int \frac{d^4 l}{(2\pi)^4} N_{\nu\beta}(l) \frac{\hat{v}^\beta v^\nu}{v \cdot l} 2\pi i \delta(l^2) \times [F(x + l^+/p^+, b, p^+) - F(x, b, p^+)], \quad (31)$$

where the exponential $e^{i l_T \cdot b}$ has been neglected as a soft approximation. Therefore, we have separated the real gluon contribution into two pieces: the piece with an infrared pole, along with the virtual gluon contribution, goes into the function K . Compared with K_J in Eq. (11), K is infrared finite due to the inclusion of part of the real correction. The other piece \bar{F}'_s , being also infrared finite, will lead to a splitting function below.

Fig. 1(c) gives $G(p^+/\mu, \alpha_s(\mu))F(x, b, p^+)$ with the function

$$G = iN_c g^2 \int \frac{d^4 l}{(2\pi)^4} N_{\nu\beta}(l) \frac{\hat{v}^\beta}{l^2} \left[\frac{(l - 2xp^+v)^\nu}{(l - xp^+v)^2} - \frac{v^\nu}{v \cdot l} \right] - \delta G, \quad (32)$$

where xp^+ is the parton momentum and δG an additive counterterm. To obtain the above expression, Eq. (9) for the triple-gluon vertex has been inserted, which generates the first term in the brackets. At intermediate x , G is characterized by a large scale p^+ , whose logarithms $\ln p^+$ imply that collinear divergences are present in F , and that the Sudakov resummation of double logarithms from the overlap of collinear and soft enhancements is necessary. Through this function G , the dependence on a large scale is introduced into the new evolution equation as stated before.

The functions K and G contain ultraviolet divergences individually as indicated by the renormalization scale μ . However, their ultraviolet divergences, both from the virtual contribution \bar{F}_{sv} , cancel each other. This is obvious from the counterterms

$$\delta K = -\delta G = \bar{\alpha}_s \left(\frac{1}{\epsilon} + \frac{1}{2} \ln 4\pi - \frac{\gamma_E}{2} \right), \quad (33)$$

with the Euler constant γ_E . Therefore, the sum $K + G$ is RG invariant, the same as $K_J + G_J$ in Eq. (13). The results of K and G from Eqs. (30) and (32) are

$$\begin{aligned} K(b\mu, \alpha_s(\mu)) &= -\bar{\alpha}_s(\mu) \left[\ln(b\mu) + \ln \frac{e^{\gamma_E}}{2} \right], \\ G(p^+/\mu, \alpha_s(\mu)) &= -\bar{\alpha}_s(\mu) \left[\ln \frac{xp^+}{\mu} + \ln(2\nu) \right], \end{aligned} \quad (34)$$

with the gauge factor $\nu = \sqrt{(v \cdot n)^2 / |n^2|}$.

The RG solution of $K + G$ is given by

$$K(b\mu, \alpha_s(\mu)) + G(p^+/\mu, \alpha_s(\mu)) = K(1, \alpha_s(p^+)) + G(1, \alpha_s(p^+)) - s(b, p^+), \quad (35)$$

with

$$K(1, \alpha_s(p^+)) = -\bar{\alpha}_s(p^+) \ln \frac{e^{\gamma_E}}{2}, \quad (36)$$

$$G(1, \alpha_s(p^+)) = -\bar{\alpha}_s(p^+) \ln(2x\nu), \quad (37)$$

$$s(b, p^+) = \int_{1/b}^{p^+} \frac{d\bar{\mu}}{\bar{\mu}} \left[\gamma_K(\alpha_s(\bar{\mu})) + \beta(g) \frac{\partial}{\partial g} K(1, \alpha_s(\bar{\mu})) \right], \quad (38)$$

$\gamma_K = \mu d\delta K/d\mu$ being the anomalous dimension of K and $\beta(g)$ the beta function.

The one-loop expression of γ_K in Eq. (38) gives leading (double) logarithmic summation, and the two-loop expression gives next-to-leading (single) logarithmic summation. Another term proportional to $\beta\partial K/\partial g$ and the initial conditions $K(1, \alpha_s)$ and $G(1, \alpha_s)$ also give next-to-leading contributions. Hence, the Sudakov resummation can be performed up to the accuracy of single logarithms, and the running of the coupling constant must be taken into account in our formalism. Note that it is equivalent to choose Cp^+ as the upper bound of $\bar{\mu}$ in Eq. (38) with C a constant of order unity. The ambiguity from this arbitrary constant C is cancelled by the ambiguity from the initial condition $G(1/C, \alpha_s)$ at the single-logarithm level. The terms involving the gauge factor $\ln \nu$ in the initial condition of G will be cancelled by the gauge factor from nonfactorizable soft gluon exchanges in a physical process [9].

For \bar{F}'_s , we employ the variable change $\xi = x + l^+/p^+$, and perform the integration straightforwardly, obtaining

$$\bar{F}'_s = \bar{\alpha}_s(p^+) \int_x^1 d\xi \frac{F(\xi, b, p^+) - F(x, b, p^+)}{\xi - x}. \quad (39)$$

The argument of $\bar{\alpha}_s$ in the above expression has been chosen as p^+ , the same as that in Eq. (35). Further applying the variable change $\xi = x/z$ and using the identity $1/[z(1-z)] \equiv 1/(1-z) + 1/z$, Eq. (39) is rewritten as

$$\bar{F}'_s = \bar{\alpha}_s(p^+) \int_x^1 dz \left(\frac{1}{z} + \frac{1}{1-z} \right) [F(x/z, b, p^+) - F(x, b, p^+)]. \quad (40)$$

The term

$$- \int_x^1 \frac{dz}{z} F(x, b, p^+) = \ln x F(x, b, p^+), \quad (41)$$

in the above expression cancels $-\ln x$ in Eq. (37). Combining Eqs. (35) and (40), the evolution equation becomes

$$p^+ \frac{d}{dp^+} F(x, b, p^+) = -2s(b, p^+) F(x, b, p^+) + 2\bar{\alpha}_s(p^+) \int_x^1 dz P_{gg}(z) F(x/z, b, p^+), \quad (42)$$

with P_{gg} defined by Eq. (25). The terms finite at $z \rightarrow 0$ and at $z \rightarrow 1$ in P_{gg} are put in by hand as in derivation of the CCFM equation [3]. In Eq. (42) we have concentrated on the leading-logarithm resummation for simplicity.

The Sudakov form factor Δ can be extracted by assuming

$$F(x, b, p^+) = \Delta(b, p^+, Q_0) F^{(s)}(x, b, p^+), \quad (43)$$

with the exponential

$$\Delta(b, p^+, Q_0) = \exp \left[-2 \int_{Q_0}^{p^+} \frac{dp}{p} s(b, p) \right], \quad (44)$$

where Q_0 is an arbitrary low energy scale, satisfying $Q_0 \geq 1/b$. As $Q_0 < 1/b$, implying the vanishing of collinear enhancements, the Sudakov form factor should be set to unity. Equation (44) indicates that the Sudakov form factor exhibits strong suppression at large b .

Substituting Eq. (43) into Eq. (42), we have the differential equation of $F^{(s)}$,

$$p^+ \frac{d}{dp^+} F^{(s)}(x, b, p^+) = \frac{2\bar{\alpha}_s(p^+)}{\Delta(b, p^+, Q_0)} \int_x^1 dz P_{gg}(z) F(x/z, b, p^+). \quad (45)$$

The above equation is rewritten as

$$F^{(s)}(x, b, Q) = F^{(s)}(x, b, Q_0) + \int_x^1 dz \int_{Q_0^2}^{Q^2} \frac{d\mu^2}{\mu^2} \frac{\bar{\alpha}_s(\mu) P_{gg}(z)}{\Delta(b, \mu, Q_0)} F(x/z, b, \mu), \quad (46)$$

where $F^{(s)}(x, b, Q_0) = F(x, b, Q_0)$ is the initial condition of F at the low scale $p^+ = Q_0$. We have chosen the upper bound of p^+ as the measured kinematic variable Q . Finally, using Eq. (43), the unintegrated gluon distribution function in b space is given by

$$F(x, b, Q) = \Delta(b, Q, Q_0) F(x, b, Q_0) + \int_x^1 dz \int_{Q_0^2}^{Q^2} \frac{d\mu^2}{\mu^2} \bar{\alpha}_s(\mu) \Delta(b, Q, \mu) P_{gg}(z) F(x/z, b, \mu). \quad (47)$$

Equation (47) is the new unified evolution equation.

IV. Features of the new equation

In this section we shall explore features of the new unified evolution equation. We first demonstrate that the new equation approaches the DGLAP equation at large x . The strong k_T ordering of radiative gluons corresponds to small transverse separation b among these gluons. In the region with small b we have

$$\lim_{b \rightarrow 1/p^+ \sim 0} s(b, p^+) = 0, \quad (48)$$

$$\lim_{b \rightarrow 0} F(x, b, Q) = xG(x, Q). \quad (49)$$

Equation (48) corresponds to the fact that the effect of Sudakov resummation is weak at small b as explained in Sec. 3. Equation (49) comes from the definition of the gluon density $G(x, Q)$ as the integral of $F(x, k_T, Q)$ in Eq. (1) over k_T . Differentiating Eq. (47) with respect to Q^2 under Eqs. (48) and (49), we obtain

$$Q^2 \frac{d}{dQ^2} G(x, Q) = \bar{\alpha}_s(Q) \int_x^1 \frac{dz}{z} P_{gg}(z) G(x/z, Q), \quad (50)$$

which is the DGLAP equation for $G(x, Q)$. The initial condition $F(x, b, Q_0)$ does not depend on Q , and thus its derivative vanishes.

We then show that the new evolution equation reduces to the BFKL equation in the small- x limit. For convenience, we shall adopt the one-loop expression $\gamma_K = \bar{\alpha}_s$, which is the same as γ_J for the CCFM equation, and ignore the running of $\bar{\alpha}_s$ below. Strong rapidity ordering corresponds to the assumption that the z integral in the second term of Eq. (47) is dominated by the behavior of $F(x/z, b, \mu)$ at small x/z , *i.e.*, at $z \gg x$ [5]. Substituting $F(x, b, Q)$ for $F(x/z, b, Q)$, Eq. (47) becomes

$$F(x, b, Q) = \Delta(b, Q, Q_0)F(x, b, Q_0) + \bar{\alpha}_s \int_x^1 \frac{dz}{z} \int_{Q_0^2}^{Q^2} \frac{d\mu^2}{\mu^2} \Delta(b, Q, \mu) F(x, b, \mu). \quad (51)$$

In the above expression the term $-2 + z(1 - z)$ in the splitting function P_{gg} , which is finite as $z \rightarrow x \rightarrow 0$, has been neglected.

Differentiating F with respect to x , Eq. (51) leads to

$$\begin{aligned} -x \frac{d}{dx} F(x, b, Q) &= \bar{\alpha}_s \int_{Q_0^2}^{Q^2} \frac{d\mu^2}{\mu^2} \Delta(b, Q, \mu) F(x, b, \mu), \\ &\approx \bar{\alpha}_s \ln \frac{Q^2}{Q_0^2} F(x, b, Q). \end{aligned} \quad (52)$$

The derivative of the first term on the right-hand side of Eq. (51) has been neglected, since $F(x, b, Q_0)$ is assumed to be ‘‘flat’’ [10]. The derivative of $F(x, b, \mu)$ in the integrand of the second term, giving next-to-leading-order contributions, has been also neglected. To derive the second expression, we have made the approximation $\Delta(b, Q, \mu) F(x, b, \mu) \approx F(x, b, Q)$, which maintains the feature of the evolution in x . Equation (52) is the modified BFKL equation with a Q dependence derived in [6].

If we drop hard virtual gluon contributions, the Q dependence will not appear in the evolution kernel. Replacing Q by $1/b$, which is the only scale, Eq. (52) reduces to

$$-x \frac{d}{dx} F(x, b) = -\bar{\alpha}_s \ln(Q_0^2 b^2) F(x, b). \quad (53)$$

Since the evolution kernel does not depend on Q , the argument Q of F has been suppressed. It can be shown that the Fourier transformation of the BFKL equation adopted in [6,10],

$$-x \frac{d}{dx} F(x, k_T) = \bar{\alpha}_s \int \frac{d^2 l_T}{\pi l_T^2} [F(x, |\mathbf{k}_T + \mathbf{l}_T|) - \theta(Q_0 - l_T) F(x, k_T)], \quad (54)$$

coincides with Eq. (53). The conventional BFKL equation predicts that the small- x rises of the gluon distribution function and of structure functions involved in deep inelastic scattering are almost Q -independent as indicated by Eq. (54). However, recent HERA data of the structure function $F_2(x, Q^2)$ [11] exhibit a more sensitive Q dependence: the rise is rapider at larger Q . It is easy to observe that the evolution kernel in Eq. (52) enhances the rise of the gluon distribution function when Q is large. It has been shown that predictions from Eq. (52) are well consistent with the data [6].

Compared with the CCFM equation, the new evolution equation bears other similarities and differences. They both contain the Sudakov exponentials, but the forms are different. The upper bounds of the evolution variables are the same (Q), but the lower bounds are zq in the CCFM equation, and μ in the new equation. The longitudinal variable μ controls the magnitude of the Sudakov form factor by changing the partition of collinear enhancements between $\Delta(b, Q, \mu)$ and $F(x/z, b, \mu)$. While zq from angular ordering changes the partition of collinear enhancements between $\Delta_S(zq, Q)$ and $\Delta_{NS}(z, q, k_T)$. There is no separation of Δ_{NS} from Δ_S in the new equation. As explained in Sec. 2, this smearing factor Δ_{NS} is not necessary, since the $z \rightarrow 0$ pole does not exist.

V. Conclusion

In this paper we have derived a new unified evolution equation for the gluon distribution function using the CS resummation technique, which is appropriate for both the large and small x . The features of this equation are summarized below. The infrared cancellation between real and virtual gluon emissions is explicit in both the Sudakov form factor and the splitting function. The Sudakov form factor can take into account next-to-leading logarithmic summation systematically, including the running of the coupling constant. Hard virtual gluon emissions introduce a desired Q dependence into the evolution kernel. The splitting function does not contain the non-Sudakov form factor compared to that in the CCFM equation. Hence, the new unified evolution equation can be regarded as an improved version of the CCFM equation.

In the future studies we shall explore the properties of this new equation in details, and results will be published elsewhere.

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