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The anomalous Hall effect viewed from the time-dependent Ginzburg–Landau equations for d-wave superconductors

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Abstract

From time-dependent Ginzburg–Landau theory of d-wave superconductors, the rate of increase of the total free energy plus the rate of dissipation are demonstrated to be equal to the inflow of energy current. The equation of motion of a single vortex for $h \ll H_{c2}$ in the presence of an applied current is derived. The imaginary parts of the relaxation time for s- and d-wave order parameters play a key role in the sign change of Hall effect. The concentration of non-magnetic impurities affecting the sign change of Hall effect is discussed. © 2000 Elsevier Science B.V. All rights reserved.

Keywords: d-wave superconductivity; Hall effect; Time-dependent Ginzburg–Landau equations

The anomalous behavior of the Hall effect in the high-temperature superconductors (HTSC) is still unsolved problem. There are three different approaches (macroscopic, intermediate, microscopic) to attempt to explain such anomalous phenomenon. In this short report, the complex relaxation time [1–3] in the time-dependent Ginzburg–Landau equation (TDGL) is used to study the sign change of the Hall effect in a mixed s- and d-wave superconductors. In a recent year, Alvarez et al. [4] did the numerical simulation under external driving current with angle φ with respect to the b crystal axis to solve TDGL equations and get the intrinsic Hall effect depending on $\sin(4\varphi)$ for the mixed s- and d-wave superconductors. Very recently Zhu et al. [5,6] microscopically derived the TDGL equations within a weak coupling theory for mixed s- and d-wave superconductors. Their results are cumbersome and are hard to solve them. Now we follow Dorsey [1] and Kopnin et al. [2,3] approach to get equation of motion for a single vortex in the low-field regime. Our results show that imaginary parts of two complex relaxation time and the mixed gradient terms play an important role to change the sign of the Hall effect.

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We follow the procedure made in Refs. [2,3] to derive energy theorem for mixed s- and d-wave superconductors. The total free energy of this system consists of the energy of the metal in the normal state F_n the energy of the electromagnetic field F_{em}, and the free energy F_{sn} for the transition to the superconducting state and for the interaction of the superfluid current with the electromagnetic field. The continuity equation $\nabla \cdot j = 0$ is needed. $j = j_n + j_s = j_s + \hat{\sigma}_n \cdot \boldsymbol{E}$ is the total current density. j_n is the normal current density and $j_s = -\delta F_{sn}/\delta \boldsymbol{A}$ is the supercurrent density. $\hat{\sigma}_n$ is conductivity tensor. The TDGL equations for the s- and d-wave order parameters are also required and expressed as

$$\eta_s(\hbar\partial_t + 2ie\phi)s = -\delta F_{sn}/\delta s^*, \tag{1}$$

$$\eta_s(\hbar\partial_t + 2ie\phi)d = -\delta F_{sn}/\delta d^* \tag{2}$$

and complex-conjugate equations for s^* and d^*. Here ϕ is a scalar potential, η_d and η_s are the dimensionless order parameter complex relaxation times: $\eta_d \equiv \eta_{d1} + i\eta_{d2}$, $\eta_s \equiv \eta_{s1} + i\eta_{s2}$. Then Maxwell's equations are joined together with the total time derivative of the total free energy $dF/dt = dF_n/dt + dF_{em}/dt + dF_{sn}/dt$. The first term in dF/dt is $dF_n/dt \cong \int \nabla \cdot (\mu j_n/e)\,d\boldsymbol{r}$, where μ is the chemical potential and generally small enough to be neglected. The second term $dF_{em}/dt = -\int(\nabla \cdot \boldsymbol{S} + j \cdot \boldsymbol{E})\,d\boldsymbol{r}$, where $\boldsymbol{S}$ is Poynting vector. The third term

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dF_{sn}/dt includes the partial differentiation s, s^*, d, d^*, and vector potential $\boldsymbol{A}$ with respect to time. Then TDGL equations are substituted into this term. Finally, we have

$$dF/dt = \int d\boldsymbol{r}[-w(\boldsymbol{r}) - \nabla \cdot \boldsymbol{j}_E]. \tag{3}$$

This equation demonstrates that the rate of increase of the total free energy plus the rate of dissipation w equal the inflow of energy current $\boldsymbol{j}_E$.

To derive the equation of motion of single vortex in the low-field regime, the complex order parameters d and s in TDGL equations are expressed as $d(\boldsymbol{r},t) = f(\boldsymbol{r},t)\exp(i\theta_d)$ and $s(\boldsymbol{r},t) = g(\boldsymbol{r},t)\exp(i\theta_s(\boldsymbol{r},t))$. Then three essential steps are made. (i) The vortex translates uniformly. (ii) All the quantities characterizing the vortex system are expanded in power of v_L and replace all the time derivatives by $-\boldsymbol{v}_L \cdot \nabla$. The terms of O(1) and O(v_L) correspond to the equilibrium GL equations and a set of inhomogeneous differential equations, respectively. (iii) Final step is to derive a solvability condition for $\boldsymbol{v}_L$. Then the equation of motion for moving vortex is obtained. For a specific situation, the results are briefly stated here. The magnetic field is set in the z-direction, the applied transport current $\boldsymbol{j}_t$ made along x-direction, and the direction of motion of the vortex set at an angle θ_H with respect to the $-y$ direction. The origin of the cylindrical coordinate system (r, φ, z) is at the center of a vortex. The displacement vector $\boldsymbol{l}$ makes an angle χ with respect to the x-axis. The solvability condition may be obtained finally. The equation of motion of a single vortex is $\boldsymbol{j}_E \times \hat{\boldsymbol{z}} = (w_1\kappa/2)\boldsymbol{v}_L + (w_2\kappa/2)\boldsymbol{v}_L \times \hat{\boldsymbol{z}}$. The corresponding Hall angle is expressed as $\tan\theta_H = w_2/w_1$. When w_1 and w_2 have the opposite sign, $\tan\theta_H$ will change sign. The alternative expression of $\tan\theta_H$ is written as $\alpha_1 + \alpha_2 \cdot \alpha_1$ is positive for the large κ limit. α_2 includes the imaginary part of complex relaxation time. If $\alpha_2 < 0$, and $|\alpha_2| > \alpha_1$, then $\tan\theta_H$ will change sign. From the equation of motion of vortex and Faraday's law, we have the longitudinal conductivity and the transverse or Hall conductivity. Thus $\tan\theta_H$ is independent of magnetic field near H_{c1}. Xu et al. [5,6] show that the transition temperature for s-wave can only be affected by magnetic impurities, while the transition temperature for d-wave is affected by non-magnetic impurities. In the high scattering strength, one may find that the range of the sign change of the Hall angle depends on the concentration of non-magnetic impurities.

In conclusion, the energy balance theorem is verified in the mixed d- and s-wave superconductivity. The imaginary part of complex relaxation time and the non-magnetic impurities may affect the sign change of the Hall angle.

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