PERGAMON

Microelectronics Reliability 40 (2000) 1061–1063

MICROELECTRONICS RELIABILITY

www.elsevier.com/locate/microrel

Research note

Comments on "Reliability and component importance of a consecutive-k-out-of-n system" by Zuo

Frank K. Hwang [a], Lirong Cui [b], Jen-Chun Chang [c], Wen-Dar Lin [c,*]

[a] Department of Applied Mathematics, National Chiao Tung University, Hsinchu 30050, Taiwan, ROC

[b] Laboratories of Reliability and Quality Control, Department of Space, Beijing, People's Republic of China

[c] Department of Computer Science & Information Engineering, National Chiao Tung University, 1001 Ta Hsueh Road, Hsinchu 30050, Taiwan, ROC

Received 19 October 1999; received in revised form 13 December 1999

Abstract

Zuo claimed that the comparison of Birnbaum importance between two components for a consecutive-k-out-of-n:G system is the same as that for the F-system. We show that this is not the case and give a correct relation between the two systems. © 2000 Elsevier Science Ltd. All rights reserved.

Keywords: Consecutive-k-out-of-n system; Reliability; Birnbaum importance

1. Introduction

A consecutive-k-out-of-n:F system, denoted by $L(k, n{:}F)$, is a line of n components such that the system fails iff some k consecutive components all fail. Similarly, we can define a consecutive-k-out-of-n:G system, denoted by $L(k, n{:}G)$ as a line of n components such that the system works iff some k consecutive components all work. The reliability of $L(k, n{:}M), M \in \{F, G\}$, is denoted by $R(k, n{:}M)$.

Zuo [3] claimed that the Birnbaum importance [4] of component i for a consecutive-k-out-of-n:G system is the same as that for the F-system, which means that the comparison of Birnbaum importance between two components is the same for the G-system and the F-system.

In this article, we show that the claim is wrong. We also give a correct relation.

2. The interplay between F-systems and G-systems

The Birnbaum importance of component i in a system S is defined as,

$$I_i(S) = R(S \mid i \text{ working}) - R(S \mid i \text{ failed}).$$

Zuo [3] claimed that

$$I_i(L(k, n{:}F)) = I_i(L(k, n{:}G)), \tag{1}$$

$$I_i(L(k, n{:}F)) \overset{\geq}{\underset{<}{=}} I_j(L(k, n{:}F)) \iff I_i(L(k, n{:}G)) \overset{\geq}{\underset{<}{=}} I_j(L(k, n{:}G)) \tag{2}$$

(Note that (1) $\Rightarrow$ (2)).

For the i.i.d. case, we will include p as a parameter in the importance function.

Papastavridis [2] proved

Lemma 1.

$$I_i(L(k, n{:}F, p)) = \frac{R(k, i-1{:}F, p)R(k, n-i{:}F, p) - R(k, n{:}F, p)}{q}.$$

* Corresponding author. Fax: +886-3572-4176.
E-mail address: wdlen@csie.nctu.edu.tw (W.-D. Lin).

PII: S0026-2714(00)00003-2

Notation

n	the number of components in a system
k	the minimum number of consecutive components required to be failed (good) for the system to be failed (good)
p_i, q_i	probability that component i is good, failed; $i = 1, 2, \ldots, n$
p, q	probability that a component is good, failed in the i.i.d. case
$L(k, n{:}F), L(k, n{:}G)$	consecutive-k-out-of-$n{:}F$ linear system, consecutive-k-out-of-$n{:}G$ linear system
$R(k, n{:}F), R(k, n{:}G)$	reliability of $L(k, n{:}F), L(k, n{:}G)$
$R(S \mid E)$	reliability of system S conditional on E
$\overline{R}$	unreliability, where $\overline{R} = 1 - R$
$I_i(S)$	reliability importance of component i in system S (Birnbaum importance)

Kuo et al. [1] proved

Lemma 2.

$$I_i(L(k, n{:}G, p)) = \frac{\overline{R}(k, i-1{:}G, p)\overline{R}(k, n-i{:}G, p) - \overline{R}(k, n{:}G, p)}{p}.$$

By noting that $\overline{R}(k, n{:}G, p) = R(k, n{:}F, q)$ for all n, we have

Corollary 3.

$$I_i(L(k, n{:}G, p)) = \frac{R(k, i-1{:}F, q)R(k, n-i{:}F, q) - R(k, n{:}F, q)}{p} = I_i(L(k, n{:}F, q)).$$

By not including parameter p in the importance function, Zuo [3] misinterpreted Corollary 3 as

$$I_i(L(k, n{:}G)) = I_i(L(k, n{:}F))$$

and made claim (1). Then he used claim (1) to prove claim (2).

Corollary 4.

$$I_i(L(k, n{:}G, p)) - I_j(L(k, n{:}G, p)) = I_i(L(k, n{:}F, q)) - I_j(L(k, n{:}F, q)).$$

Therefore, if the comparison of I_i and I_j depends on p, in particular, if the sign of their difference can vary with p, then claim (2) will not hold. That this, is indeed the case will be illustrated by a specific example (the smallest in terms of k and n) in Section 3.

3. A specific counter-example

Let $n = 7$ and $k = 3$. Then,

$$\begin{aligned}\Delta(p) &\equiv I_2(L(3, 7{:}F, p)) - I_4(L(3, 7{:}F, p)) \\ &= [R(3, 1{:}F, p)R(3, 5{:}F, p) \\ &\quad - R(3, 3{:}F, p)R(3, 3{:}F, p)]/q \\ &= (p - 6p^2 + 13p^3 - 13p^4 + 6p^5 - p^6)/q \\ &= p(1-p)^2(1 - 3p + p^2).\end{aligned}$$

It is easily verifed that between 0 and 1, $\Delta(p)$ changes sign once at around $p = 0.38$. Therefore $\Delta(0.2) > 0 > \Delta(0.8)$. It follows,

$$I_2(L(3, 7{:}F, 0.2)) > I_4(L(3, 7{:}F, 0.2)),$$

$$I_2(L(3, 7{:}F, 0.8)) < I_4(L(3, 7{:}F, 0.8)).$$

By Corollary 4, the first inequality implies

$$I_2(L(3, 7{:}G, 0.8)) > I_4(L(3, 7{:}G, 0.8)).$$

The curve of $\Delta(p)$ is given in Fig. 1. We conjecture that $\Delta(p)$ changes sign at most once between 0 and 1. One would hope that I_i have some good property, such as convexity. But this is not the case. For example,

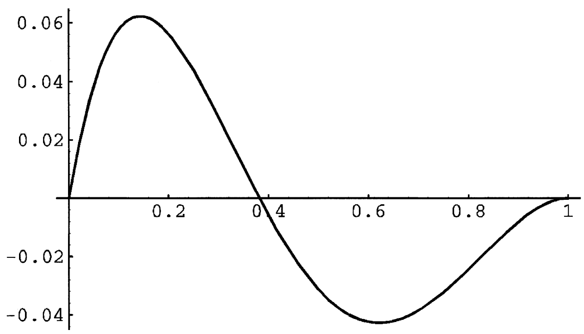

Fig. 1. The curve of $I_2(L(3, 7{:}F, p)) - I_4(L(3, 7{:}F, p))$.

$$I_1(L(k,n{:}F,p)) = \frac{R(k,n-1{:}F,p) - R(k,n{:}F,p)}{q}$$
$$= pq^{k-1}R(k,n-k-1{:}F,p).$$

Thus,

$$I_1(L(3,7{:}F,p)) = pq^2(1-q^3)$$
$$= 3p^2 - 9p^3 + 10p^4 - 5p^5 + p^6.$$

Since $\partial^2 I_1/\partial p^2 = 6 - 54p + 120p^2 - 100p^3 + 30p^4 > 0$ for p small, I_1 is not convex (nor is it concave since the second derivative is negative around $p = 0.38$).

Zuo and Kuo [5] proved that the importance ranking of components in the consecutive-2 *G*-line is same as in the consecutive-2 *F*-line. The counter-example given here showed that this conclusion cannot be extended to general k.

References

[1] Kuo W, Zheng W, Zuo M. A consecutive-k-out-of-n:G system: the mirror image of a consecutive-k-out-of-n:F system. IEEE Trans Reliab 1990;39:244–53.

[2] Papastavridis SG. The most important component in a consecutive-k-out-of-n:F system. IEEE Trans Reliab 1987;36:266–8.

[3] Zuo M. Reliability and component importance of a consecutive-k-out-of-n system. Microelectron Reliab 1993;33:243–58.

[4] Birnbaum ZW. On the importance of different components in a multicomponent system. In: Krishnaiah PR, editor. Multivariate analysis-II. New York: Academic Press, 1969. p. 581–92.

[5] Zuo M, Kuo W. Design and performance analysis of consecutive-k-out-of-n structure. Naval Res Logis 1990;37:203–30.