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A method for measuring two-dimensional refractive index distribution by using Fresnel equations and phase-shifting interferometry

H. C. Hsieh, Z. C. Jian, Y. L. Chen, P. J. Hsieh, and D. C. Su*

Department of Photonics and Institute of Electro-Optical Engineering, National Chiao Tung University, 1001 Ta-Hsueh Road, Hsinchu 300, Taiwan, ROC

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* Corresponding author: e-mail t7503@faculty.nctu.edu.tw, Phone: +886 3 5731951, Fax: +886 3 5716631

Based on Fresnel equations and the phase-shifting interferometry, an alternative method for measuring the two-dimensional refractive index distribution of a material is presented. A linearly polarized light passes through a quarter wave-plate and is incident on the tested material. The reflected light propagates through an analyzer, and then the interference signal can be obtained. The special equations to estimate the phase of the interferometric signal can be derived by using Fresnel equations. Next, the associated two-dimensional phase distribution is measured by the four-step phase-shifting interferometry. Then, the measured data are substituted into the special equations derived previously, and the two-dimensional refractive index distribution of the tested material can be obtained.

1 Introduction Refractive index is an important characteristic constant of optical materials. Although there are some techniques [1-7] that have been proposed for measuring refractive index, almost all of them are used to evaluate the refractive index at one point. To overcome this drawback, an alternative method for measuring the two-dimensional refractive index distribution of a tested material is presented in this paper, based on Fresnel equations [8] and the phase-shifting interferometry [9]. A linearly polarized light passes through a quarter wave-plate and is incident on the tested material. The reflected light propagates through an analyzer, and then the interference signal can be obtained. The special equations to estimate the phase of the interferometric signal can be derived by using Fresnel equations. Next, the two-dimensional phase distribution is measured by the four-step phase-shifting interferometry. An electro-optic modulator driven by a variable modulated voltage acts as a phase-shifter [10]. The CCD camera is used to record the two-dimensional interference signals. The interference signals are sent to a personal computer and they are analyzed with the software "IntelliWave". Then, the estimated data are substituted into the special equations derived previously, and they are calculated with the software "Matlab". Finally, the two-dimensional refractive index distribution of the tested material can be obtained. To show the validity of this method, a mixed liquid of oils and water is tested. Because of its common-path optical configuration, this method has both merits of the common-path interferometry [11] and the phase-shifting interferometry.

2 Principle

2.1 Phase of the interferometric signal resulting from reflection

For convenience, the +z axis is chosen to be along the light propagation direction and the y-axis is along the direction perpendicular to the paper plane, as shown in Fig. 1. A linearly polarized light beam with the direction of vibration at 45° to the x-axis passes through a quarter wave-plate Q. The light beam is then incident on the tested material S at an angle θ_i. The light reflected from the tested material passes through an analyzer A. Let both

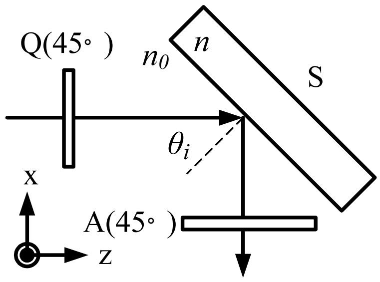

Figure 1 The reflection at the boundary between air and the tested material.

the fast axes and the transmission axes of the Q and the A be also at 45° with respect to the +x axis, then the Jones vector of the light after the A can be written as

$$E_t = \mathrm{A}\left(45^\circ\right)\mathrm{S}\mathrm{Q}\left(45^\circ\right)$$
$$E_{\text{in}} = \frac{1}{2}\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}\begin{pmatrix}r_p & 0\\ 0 & r_s\end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix}1 & -i\\ -i & 1\end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix}1\\ 1\end{pmatrix}$$
$$= \frac{1}{4}\begin{pmatrix}r_{\mathrm{p}}(1-i)+r_{\mathrm{s}}(1-i)\\ r_{\mathrm{p}}(1-i)+r_{\mathrm{s}}(1-i)\end{pmatrix}, \tag{1}$$

where A, S and Q are the Jones matrices of the A, S and Q; rp and rs are the amplitude reflection coefficients of the p- and the s- polarizations. The associated interference signal is

$$I_{\mathrm{t}} = |E_{\mathrm{t}}|^2 = \frac{1}{4}\left[r_p^2 + r_s^2 + 2r_p r_s\right]$$
$$= \frac{1}{4}\left[\left(r_p^2+r_s^2\right)-\left(r_p^2-r_s^2\right)\sin 0^\circ + 2r_p r_s\cos 0^\circ\right]$$
$$= I_0\left[1+\sin(-\phi)\right], \tag{2}$$

where

$$I_0 = \frac{(r_{\mathrm{p}}^2 + r_{\mathrm{s}}^2)}{4}, \tag{3}$$

and

$$\phi = \tan^{-1}\left(\frac{2r_p r_s}{r_{\mathrm{p}}^2 - r_{\mathrm{s}}^2}\right). \tag{4}$$

According to Fresnel equations [8], we have

$$r_p = \frac{n\cos\theta_i - n_0\dfrac{\sqrt{n^2 - n_0^2\sin^2\theta_i}}{n}}{n\cos\theta_i + n_0\dfrac{\sqrt{n^2 - n_0^2\sin^2\theta_i}}{n}}, \tag{5}$$

and

$$r_s = \frac{n_0\cos\theta_i - \sqrt{n^2 - n_0^2\sin^2\theta_i}}{n_0\cos\theta_i + \sqrt{n^2 - n_0^2\sin^2\theta_i}}, \tag{6}$$

respectively, where n_0 and n are the refractive index of air and S. Substituting Eqs. (5) and (6) into Eq. (4), we have

$$\phi = \tan^{-1}\left[\frac{n^2\cos^2\theta_i - n_0^2\sin^2\theta_i}{2n_0\sin^2\theta_i\cos\theta_i\sqrt{n^2-\sin^2\theta_i}}\right], \tag{7}$$

which can be rewritten as

$$n = (\sin^2\theta_i\tan^2\theta_i(n_0^2\csc^2\theta_i + 2(n_0^2\tan^2\phi + \sqrt{n_0^2\tan^2\phi\left(-\cot^2\theta_i + n_0^2\left(\csc^2\theta_i + \tan^2\phi\right)\right)})))^{1/2}. \tag{8}$$

It is obvious from Eq. (8) that n can be calculated with the measurement of phase ϕ under the experimental conditions in which n_0 and θ_i are specified.

2.2 Phase measurements with the phase-shifting interferometry The schematic diagram of this method is shown in Fig. 2. We added an electro-optic modulator EOM to be a phase shifter. Here, the EOM is driven by a voltage power supply VPS.

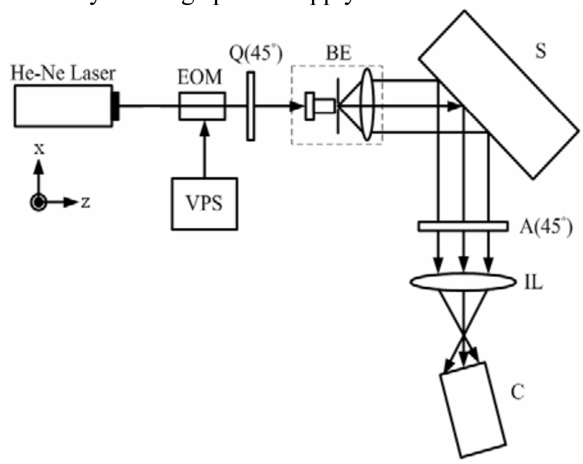

Figure 2 Schematic diagram for measuring the two-dimensional refractive index distribution of a material. EOM, electro-optic modulator; VPS, voltage power supply; Q, quarter wave-plate; BE, beam expander; S, tested material; A, analyzer; IL, imaging lens; C, CCD camera.

The light beam passes after the Q is then collimated by a beam-expander BE and the collimating light is incident on the tested material S at an angle θ_i.The light reflected from the tested material passes through an analyzer A and an imaging lens IL, finally it enters a CCD camera.

Let the fast axes of the EOM be at 0° with respect to the +x axis, then the Jones vector of the light after the Q can be written as

$$E' = \mathrm{Q}\left(45^\circ\right)\mathrm{EOM}\left(\Gamma\right)E_{\mathrm{in}}$$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix}\begin{pmatrix} e^{i\frac{\Gamma}{2}} & 0 \\ 0 & e^{-i\frac{\Gamma}{2}} \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} 1 \\ -i \end{pmatrix}e^{\frac{i\Gamma}{2}} + \frac{1}{2}\begin{pmatrix} -i \\ 1 \end{pmatrix}e^{-\frac{i\Gamma}{2}}, \quad (9)$$

where ***EOM*** are the Jones matrices of the EOM. Γ is the phase retardation induced by the EOM, it can be written as

$$\Gamma = \frac{\pi(V_i - V_0)}{V_{\lambda/2}} = \frac{\pi V_a}{V_{\lambda/2}}, \quad (10)$$

where V_o is the extinction bias voltage and $V_{\lambda/2}$ is the half-wave voltage. The Jones vector of the light arrives at the CCD camera can be expressed as

$$E_{\mathrm{t}}^{'} = \mathrm{A}\left(45^0\right)\mathrm{SE}' = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} r_{\mathrm{p}} & 0 \\ 0 & r_{\mathrm{s}} \end{pmatrix}\frac{1}{2}\begin{pmatrix} e^{i\frac{\Gamma}{2}} - ie^{-i\frac{\Gamma}{2}} \\ -ie^{i\frac{\Gamma}{2}} + e^{-i\frac{\Gamma}{2}} \end{pmatrix}$$

$$= \frac{1}{4}\begin{pmatrix} r_{\mathrm{p}}\left(e^{i\frac{\Gamma}{2}} - ie^{-i\frac{\Gamma}{2}}\right) + r_{\mathrm{s}}\left(-ie^{i\frac{\Gamma}{2}} + e^{-i\frac{\Gamma}{2}}\right) \\ r_{\mathrm{p}}\left(e^{i\frac{\Gamma}{2}} - ie^{-i\frac{\Gamma}{2}}\right) + r_{\mathrm{s}}\left(-ie^{i\frac{\Gamma}{2}} + e^{-i\frac{\Gamma}{2}}\right) \end{pmatrix}. \quad (11)$$

Its associated interference signal is

$$I_t^{'} = \left|E_t^{'}\right|^2 = I_0\left[1 + \sin(\Gamma - \phi)\right]. \quad (12)$$

From the above equation, we can see that Γ is an additional phase introduced by the EOM.

Next, the phase-shifting interferometric technique is applied to measure the two-dimensional phase distribution ϕ. The CCD camera takes four interferograms as Γ changing by the voltage applied to the EOM. An extra phase difference $\pi/2$ is added between two successive interferograms. So the intensities of these interferograms can be written as

$$I_1(x,y) = I_0(x,y)[1 + \sin(0 - \phi(x,y))], \quad (13a)$$

$$I_2(x,y) = I_0(x,y)[1 + \sin(\pi/2 - \phi(x,y))], \quad (13b)$$

$$I_3(x,y) = I_0(x,y)[1 + \sin(\pi - \phi(x,y))], \quad (13c)$$

and

$$I_4(x,y) = I_0(x,y)[1 + \sin(3\pi/2 - \phi(x,y))]. \quad (13d)$$

By solving the simultaneous equations, we get

$$\phi(x,y) = \tan^{-1}\left(\frac{I_1(x,y) - I_3(x,y)}{I_4(x,y) - I_2(x,y)}\right). \quad (14)$$

Substituting the measured data $\phi(x,y)$ into Eq. (8), then the two-dimensional refractive index distribution of the tested material can be estimated.

3 Experiments and results In order to show the feasibility of this method, we tested a mixed liquid of ricinus oil, olive oil, baby oil and water. Their refractive indices are 1.513, 1.474, 1.463 and 1.33, respectively. An He-Ne laser with a 632.8 nm wavelength, an electro-optic modulator (Model 4002, New Focus) with a 148 V half-wave voltage, and a CCD camera (TM-545, PULNiX Inc.) with 510x492 pixels and 8-bit gray levels were used in this test. Four interferograms were taken as $\Gamma = 0, \pi/2, \pi$ and $3\pi/2$. The interferograms were sent to a personal computer, and they were analyzed with the software Intelli-Wave™ (Engineering Synthesis Design Inc.). The results were depicted as shown in Figs. 3 and 4 by using the software "Matlab" (MathWorks Inc.). They are two-dimensional phase distribution $\phi(x,y)$ in waves and the associated two-dimensional refractive index distribution $n(x,y)$ of the tested material, respectively.

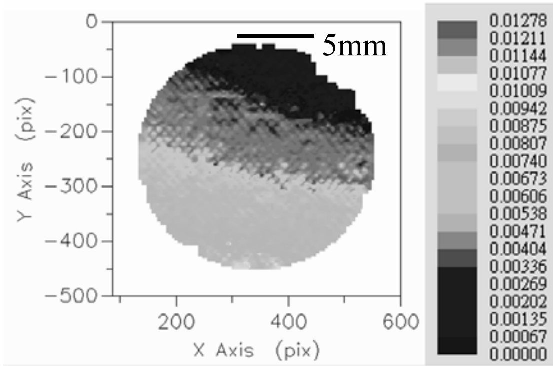

Figure 3 The two-dimensional phase variation distribution $\phi(x,y)$ of the tested material. (the original picture is in color).

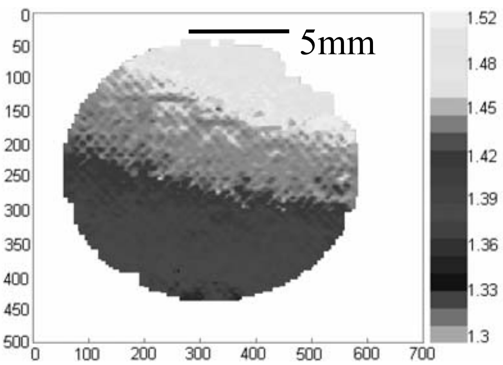

Figure 4 The two-dimensional refractive index distribution $n(x,y)$ of the tested material. (the original picture is in color).

4 Discussion From Eq. (8) we get

$$\triangle n=\left|\frac{\partial n}{\partial \phi}\right|\times\triangle\phi=\frac{\sin^2\theta_B\tan^2\theta_B\left(\tan\phi\sqrt{n_0^4\sec^2\phi\tan^2\phi}+\csc\phi\sec\phi\sqrt{n_0^4\sec^2\phi\tan^2\phi}-2n_0^2\sec^2\phi\tan\phi\right)}{\sqrt{\tan^2\theta_B\left(n_0^2+2\sin^2\theta_B\left(\sqrt{n_0^4\sec^2\phi\tan^2\phi}-n_0^2\tan^2\phi\right)\right)}}\triangle\phi. \tag{15}$$

where $\triangle n$ and $\triangle\phi$ are the errors in n and ϕ, respectively. The error $\triangle\phi$ may be influenced by the phase-resolution of the phase-shifting interferometry and the polarization-mixing error [12]. The gray levels of minima and maxima of the interferograms are 0 and 255, respectively, as the phase-shifting interferometry is fully utilized. Thus, the theoretical resolution of the phase-shifting interferometry is about $\triangle\phi=360^\circ/256\cong 1.406^\circ$. In our experiments, the extinction ratio of the polarizer (Newport Inc.) is 1×10^{-3}. So the polarization-mixing error is about 0.028° [12, 13]. Hence, the total error of $\triangle\phi$ is 1.434°. Substituting the experimental conditions $\theta_i=56.65^\circ$, $n_0=1$ and $\triangle\phi=1.434^\circ$ into Eq. (15), we can obtain the relation curve of $\triangle n$ versus n, which is the upper solid curve in Fig. 5. Because $\theta_i=56.65^\circ$ is the Brewster's angle of ricinus oil, we have $\triangle n\approx 0.02603$ as $n\approx 1.513$. If θ_i is changed to the Brewster's angle of the water (53.12°), another relative curve can be depicted as the lower dash curve in Fig.5. According to the dashed curve, it can be seen that $\triangle n\approx 0.0213$ as $n\approx 1.33$. Hence, we know that if the light beam is incident on the tested material at Brewster angle, the measured error near the corresponding refractive index becomes small.

In addition, this method is suitable only for non-absorbing material. We also presented a method for measuring two-dimensional refractive index distribution with the total internal reflection and the phase-shifting interferometry in our previous work [14]. Its optical configuration is one of two-beam interferometers [15], and it measures the phase difference between the s- and the p- polarizations due to the total internal reflection. Therefore, this method as some merits such as simple optical configuration, high stability and wider measurable range.

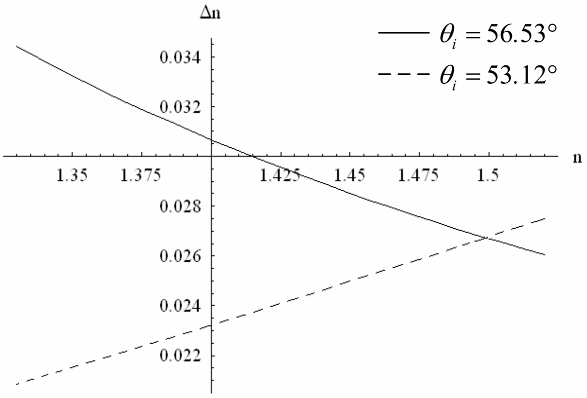

Figure 5 The relation curves of Δn versus n as $\theta_i=56.53°$ (solid curve) and $\theta_i=53.12°$ (dashed curve).

5 Conclusion An alternative method for measuring the two-dimensional refractive index distribution of a material is proposed. A linearly polarized light passes through a quarter wave-plate and is incident on the tested material. The reflected light propagates through an analyzer, then the interference signal can be obtained. The special equations to estimate the phase of the interferometric signal can be derived by using Fresnel equations. Next, the four-step phase-shifting interferometry is used to measure the two-dimensional phase distribution. Then, the measured data are substituted into the special equations derived previously, and the two-dimensional refractive index distribution of the tested material can be obtained. Its validity has been demonstrated, and it has both merits of the common-path interferometry and the phase-shifting interferometry, that is, high stability and high resolution.

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