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Primitive spatial relations based on SKIZ ☆

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Abstract

Spatial relations between objects or regions in an image play important roles in scene understanding. However, it is difficult to define these relations for automation. Most existing methods for defining spatial relations depend on angle measurements between points of the two objects of interest. In this study, we propose a new approach based on the concept of skeleton by zones of influence (SKIZ) for recognizing the primitive spatial relations between objects or regions in a segmented image. The approach is compared with the aggregation and compatibility methods. The experimental result shows the proposed approach is effective and fast.

Keywords: Spatial relations; Aggregation method; Compatibility method; Skeleton by zones of influence; Watershed

1. Introduction

In computer vision, both object recognition and scene analysis are important tasks. They can be greatly enhanced when the spatial organization information in an image, such as "A is to the right of B", is available. However, it is not a simple task to define spatial relations between objects within an image [1,2].

Many studies [3–16] have pointed out that the notion of fuzzy sets is a reasonable representation of spatial relations. Koczy [7] defines the relative positions of two fuzzy patterns by projections. Keller and Sztandera [8] also use projections, however, they further analyse the α-level sets of the projections to capture the approximate relationships. Other methods utilize information of angles between two fuzzy regions. The simplest method [2,9] is to use the angle between the centroids of the regions under consideration, however, this "all-or-none" definition leads to unsatisfactory results in several situations. The aggregation method [9,10] and compatibility method [12,14] both calculate all the angles between the x-axis and the lines determined by pairs of points of the two regions. One common drawback of these methods is that they are time-consuming when the sizes of the two regions under consideration are large. Matsakis et al. [15] define the forces histogram instead of the angles histogram to avoid the computation of angles. However, their method is not easy to implement.

Recently, Bloch [3–5] evaluates the membership values of the points in the image. It corresponds to the degree of satisfaction the spatial relation under examination with respect to the reference object. Then a fuzzy pattern matching approach is used to evaluate the relative position. It should be noted that the fuzzy pattern matching method produces two membership values for each relationship, one is "optimistic" value and the other is "pessimistic" value. However, it needs more computation to find nearest points for concave objects [5]. Gader [6] uses the concept of fuzzy morphology to define spatial relationships. His approach always produces "optimistic" values. However, these "optimistic" values cannot conform to human intuition in some situations.

In this paper, we propose a new approach employing the concept of skeleton by zones of influence (SKIZ) to provide alternative angle measurements between the objects. The concept of SKIZ is introduced in [17], and derived from the notion of influence zones. It can be imaged as a subset of the medial axis of the background of a binary image. The proposed approach includes three steps. First, we use the watershed algorithm to find SKIZ between objects. Then the angles between the horizontal axis and the tangent lines determined by the watershed points are computed. Finally, spatial relationships are obtained by an aggregation operator or a compatibility operator. The aggregation method and the compatibility method will be briefly reviewed in Section 2. The proposed method will be

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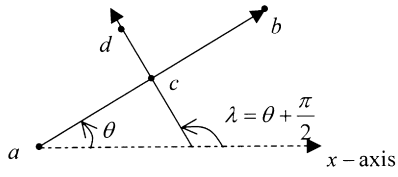


(a)

Fig. 1. Spatial relation between two points in *xy* coordinate system.

described in Section 3. The concept of watershed and a modified algorithm will also be presented in that section. Some experimental results will be shown in Section 4. Finally, our conclusions will be given in Section 5.

2. Aggregation and compatibility methods

From the classical mathematical considerations, the relative position between two points can be expressed accurately by the angle formed by the line passing through these two points and the *x*-axis. An angle opened toward the positive *x*-axis indicates a "right of" relation. Observe that the sharper the angle the more the relation holds. A similar argument holds for other directions. Both the aggregation method and the compatibility method are developed based on this observation.

In the representation of "A REL R", the region A is referred to as the argument object and the region R the reference object [13]. The relationship "is right of", "is left of", "above of" and "below of" will replace the operator "REL" in the relationship computation.

2.1. Aggregation method

An aggregation method [9,10] uses all pairs of points from both objects. For any pair of points (a, b), a in the reference object R and b in the argument object A, the angle $\theta(a, b)$ is computed. Memberships in the spatial relations, "left of", "above", "right of" and "below", are then defined between pairs of points. For example, the membership function "right of" may be defined as:

$$\mu_{\text{right}}(\theta) = \begin{cases} 1 & |\theta| < a\pi/2 \\ \dfrac{\pi/2 - |\theta|}{\pi/2(1-a)} & a\pi/2 \le |\theta| \le \pi/2 \\ 0 & |\theta| > \pi/2 \end{cases}. \quad (1)$$

Note that the parameter a is used to adjust the result of the relationship. A large value for a trends to produce an optimistic result while a small value would produce a pessimistic result. These point-pair memberships are then aggregated to produce a membership value of a spatial relationship between R and A. The generalized mean, defined by

$$g(\mu_1, \ldots, \mu_n; p, \omega_1, \ldots, \omega_n) = \left(\sum_{i=1}^{n} \omega_i \mu_i^p \right)^{1/p}, \qquad \sum_{i=1}^{n} \omega_i = 1, \quad (2)$$

is suggested [10] as an operator for aggregating the point-pair memberships. The weights, $\omega_i \ge 0$, express the relative importance of the corresponding membership values μ_i, and they may all be chosen to be equal. The parameter p may used to adjust the required degree of optimism or pessimism. By varying the value of p between $-\infty$ and $+\infty$, we can obtain all values between minimum and maximum of $\mu_1, \mu_2, \ldots, \mu_n$.

2.2. Compatibility method

The compatibility method has been proposed by Miyajima et al. [12–14] where $\cos^2 \theta$ and $\sin^2 \theta$ are chosen to indicate the degrees of "lateral" and "vertical" spatial relations. For instance, the membership function "right of" is defined as:

$$\mu_{\text{right}}(\theta) = \begin{cases} \cos^2 \theta & \text{if } -\pi/2 \le \theta \le \pi/2, \\ 0 & \text{otherwise.} \end{cases} \quad (3)$$

First, same as in the aggregation method, the angles $\theta(a, b)$ are computed for all point-pairs. All of these angles form a multi-set Θ. For each $\theta \in \Theta$, one computes the frequency f_θ of θ to define the histogram (θ, f_θ). The normalized version of this histogram can be treated as a fuzzy set H. Then the compatibility set between H and the membership function of each spatial relation is computed using the extension principle. For example, $\mu_{CP(\mu_{\text{right}};H)}$ is defined as:

$$\mu_{CP(\mu_{\text{right}};H)}(v) = \sup_{s:v=\mu_{\text{right}}(s)} \mu_H(s) \quad (4)$$

The final degree to which a spatial relation holds is obtained as the center of gravity of the compatibility fuzzy set.

255	255	255	255
0	255	255	0
255	255	255	255

(a)

1	1	2	2
1	1	2	2
1	1	2	2

(b)

1	1	0	2
1	1	0	2
1	1	0	2

(c)

Fig. 2. (a) Two objects in the example image each having gray value 0, (b) the result of the original watershed algorithm, (c) the result of the modified watershed algorithm. The watershed points are marked by 0.

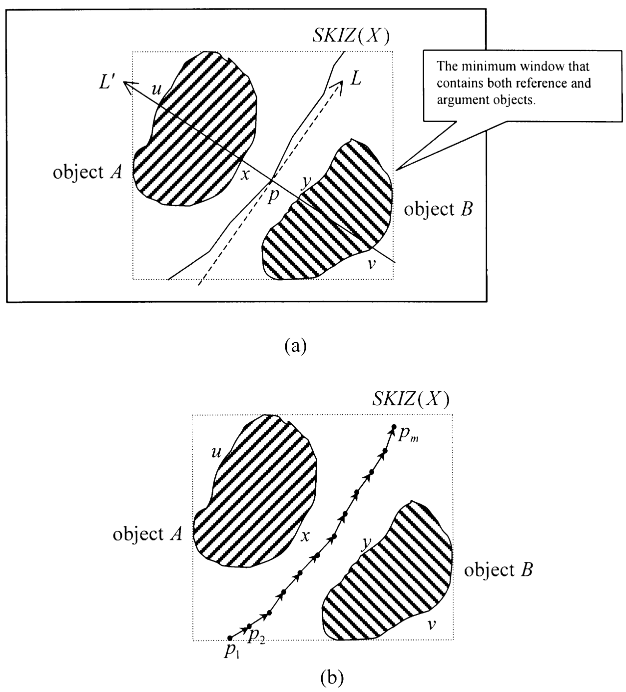


Fig. 3. (a) Points on SKIZ are equidistant to both objects, (b) using neighbor watershed points to determine the tangent lines in practical implementation.

3. SKIZ and the proposed method

Let $a, b \in Z^2$ be two distinct points. The spatial relations between a and b can be expressed by the angle θ of the vector $\overrightarrow{ab}$ (see Fig. 1). Alternately, they can also be expressed by the direction of the vector $\overrightarrow{cd}$, i.e. the angle $\theta + (\pi/2)$, where c is the midpoint of the segment $\overline{ab}$. The line determined by the vector $\overrightarrow{cd}$ is the bisector of a and b which has the property that every point on the bisector has equal distance to a and b.

For two non-overlapped region objects A and B, we can find points that are equidistant to A and B (see Fig. 3(a)). Such points form a curve, called the SKIZ of the image consisted of A and B. Then, the tangent of the SKIZ can be aggregated to describe the spatial relations between A and B.

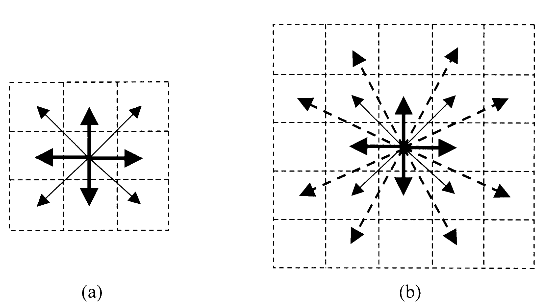


Fig. 4. The possible directions for 4- and 8-connected watershed: (a) tangent lines determined by p_i and p_{i+1}; and (b) tangent lines determined by p_i and p_{i+2}.

3.1. SKIZ

Let $X = \bigcup_{p \in N} X_p$ be a countable union of disjoint compact subsets of R^n. The influence zone of X_p, denoted by $\mathrm{IZ}(X_p)$, is the set of points x of R^n that are closer to X_p than to any other set X_q :

$$\mathrm{IZ}(X_p) = \{x \in R^n | \forall q \in N, q \neq p, d(x, X_p) < d(x, X_q)\}. \quad (5)$$

The SKIZ of X, denoted by SKIZ(X), is the set of points that belong to no influence zones. That is

$$\mathrm{SKIZ}(X) = \left[\bigcup_{p \in N} \mathrm{IZ}(X_p)\right]^c, \quad (6)$$

or, equivalently,

$$\mathrm{SKIZ}(X) = \partial\left[\bigcup_{p \in N} \mathrm{IZ}(X_p)\right], \quad (7)$$

where $\partial(\cdot)$ denotes the boundary [17]. For each point $x \in$ SKIZ(X), there exist two compact subsets such that the distances from x to them are equal.

An intuitive method to find the SKIZ is to compute the skeleton of the image background. However, this method sometimes may produce surplus skeletons. One other method is to use the watershed algorithm. In mathematical morphology [18,19], a gray scale images are often considered as topographic reliefs. When a drop of water placed on the relief surface, it will fall with certainty to a regional minimum. The point set of which a drop of water will fall to the regional minimum is called the catchment basin. Usually, the drop of water will fall to a single minimum. However, if the drop of water is placed on the regional maximum point then it may fall equally to more than a single minimum. Those points at which a drop of water may fall equally to more than a regional minimum form crest lines on the topographical surface called watershed lines. Extracting watersheds from digital images is not an easy task. Many efforts have been devoted to develop a fast watershed algorithm [18–22]. In this study, the watershed algorithm proposed by Vincent et al. [19] is chosen for our experiment. It is based on immersion simulations, i.e. on the recursive detection and fast labeling of the different catchment basins using queues. The algorithm consists of two steps: sorting and flooding. At the first step, the image pixels are sorted in increasing order according to their intensities. At the flooding step, the pixels are quickly accessed in increasing intensity order and labels are assigned to catchment basins. The label propagation is based on queues constructed using neighborhoods. In the flooding step, only those pixels that are exactly half way between two catchment basins are marked as watershed points. However, this is a defect in the current application. When the distance between two objects is even, watershed points cannot be detected normally. Fig. 2 illustrates a typical

Table 1
Result of degree of location between reference and argument objects for Fig. 5 using watershed-aggregation method with 4- and 8-connected watershed points

	4-connected watershed				8-connected watershed			
	(a)	(b)	(c)	(d)	(a)	(b)	(c)	(d)
Right	0.000000	0.219100	0.852985	0.949359	0.000000	0.043325	0.912940	0.949588
Below	0.000000	0.018084	0.083866	0.028157	0.007503	0.082124	0.083044	0.037225
Above	0.000000	0.000000	0.032916	0.011876	0.004502	0.002587	0.005428	0.003862
Left	1.000000	0.763165	0.032916	0.011530	0.988475	0.874085	0.000000	0.010137
Time used (s)	0.21	0.48	0.52	0.52	0.21	0.41	0.44	0.44

situation. For our experiment, we modify[1] the flooding step of their algorithm to mark the points whose neighbors belong to different catchment basins as watershed points.

3.2. The proposed method for crisp sets

For a SKIZ point p, let L be the tangent line passing through p. The angle formed by the x-axis and the direction of the line L reveals information of the position relation between objects, see Fig. 3(a). They trend to above–below relation if the angle trends to 0 or $\pm\pi$. Their relation trends to left–right relation if the angle trends to $\pm\pi/2$. Before the next step is proceeded, the angle must be rotated ninety-degree clockwise or counterclockwise depending on whether the argument object is in the right side of line L or left. If the argument object is in the right side of L, then the angle is rotated clockwise. Otherwise it is rotated counterclockwise. We evaluate the membership value of relationship at each angle using the membership function same as in the aggregation method and then aggregate these values by an aggregation operator, for instance, the generalized mean. Another way to obtain the final spatial relation is using the angle information to form a fuzzy histogram and then making a compatibility operation between the fuzzy histogram and the corresponding membership function. In this situation, the membership function is defined exactly the same as in the compatibility method. Finally, the spatial relation is obtained from this compatibility fuzzy set.

In the previous description, the proposed method does not take into account the shape information of the objects. An idea similar to the method proposed by Matsakis et al. [15] can be added to the proposed method. We assign a weight w_i to each angle θ_i. The weight w_i is determined by the number of points that belong to the objects and are on the line passing through point p and perpendicular to L, i.e. determined by the points on the segments $\overline{ux}$ and $\overline{vy}$ (see Fig. 3(a)). Then we aggregate these membership values by an aggregation operator. Such method will be called the *watershed-aggregation* method.

The collection of all pairs (θ_i, w_i) also forms a histogram. The horizontal axis of the normalized histogram denotes the angle and the vertical axis denotes the weight. Then, using the compatibility notion we can obtain the spatial relations between two objects. Such method will be called the *watershed-compatibility* method.

3.3. Extend to fuzzy sets

The input data of our approach is a segmented image that is the result of image processing. It can be a crisp or fuzzy set. If it is a fuzzy set, we will use α-cuts to translate it to crisp sets. The watershed algorithm is then applied to the window that contains the objects in question. As mentioned in the previous section, the angles made by the x-axis and normal vectors of tangent lines passing through SKIZ points are calculated. We collect all these rotated angles to form a multi-set. Then we can evaluate relationship at each angle by using the generalized mean. That is, the spatial relations μ^{α_i} for each level α_i, $i = 1, 2, \ldots, m$, can be computed by

$$\mu^{\alpha_i} = g(\mu_{\alpha_{i1}}, \ldots, \mu_{\alpha_{in}}; p, \omega_{\alpha_{i1}}, \ldots, \omega_{\alpha_{in}}).$$

After the spatial relations μ^{α_i} of each α-level set have been computed, a fuzzy aggregation operator can be applied to obtain the final degree of relationship. For instance, the generalized mean can also be used for this propose to yield the final degree of relationship $\mu = g(\mu^{\alpha_1}, \ldots, \mu^{\alpha_m}; p, \omega^{\alpha_1}, \ldots, \omega^{\alpha_m})$.

As mentioned in the previous section, the spatial relations can also be obtained by compatibility. That is, after the spatial relation μ^{α_i} of each α-level set has been computed, a fuzzy aggregation operator can be applied to obtain the final degree of relationship.

4. Experimental results and discussions

In the present study, we do not need to use all SKIZ points around the object. Only those SKIZ points that between the reference and argument objects are required. Thus the watershed algorithm just applies to the smallest window that contains the reference and argument objects (see Fig. 3(a)).

The SKIZ detected by the watershed algorithm used in our experiment may be a 4-connected or 8-connected chain that depends on which distance measurement is used in the flooding step [19]. If the 8-connectivity is used, the resulting watersheds are only 4-connected. Examples of 4-connectivity and 8-connectivity watersheds are shown in Fig. 8.

[1] The source code of the modified algorithm can be obtained from ftp://140.113.88.136.

Table 2
Result of degree of location between reference and argument objects for Fig. 5 using watershed-compatibility method with 4- and 8-connected watershed points

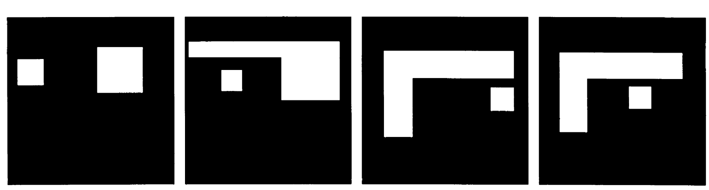
	4-connected watershed				8-connected watershed			
	(a)	(b)	(c)	(d)	(a)	(b)	(c)	(d)
Right	0.000000	0.924379	0.911048	0.975892	0.000000	0.364130	0.916667	0.972276
Below	0.000000	0.022681	0.088952	0.024455	0.007418	0.075581	0.083333	0.031371
Above	0.000000	0.000000	0.036517	0.007789	0.004478	0.002915	0.005988	0.003996
Left	1.000000	0.987889	0.282609	0.232558	0.992582	0.957521	0.000000	0.244186
Time used (s)	0.21	0.49	0.52	0.53	0.21	0.42	0.45	0.45

Table 3
Result of degree of location between reference and argument objects for Fig. 5 using aggregation method and watershed-aggregation method

	Aggregation method				Watershed-aggregation method			
	(a)	(b)	(c)	(d)	(a)	(b)	(c)	(d)
Right	0.000000	0.052131	0.672039	0.515711	0.000000	0.054293	0.880616	0.887376
Below	0.056886	0.266388	0.314997	0.383995	0.019650	0.109875	0.113219	0.042690
Above	0.042192	0.022746	0.023568	0.033341	0.005447	0.004043	0.008734	0.063492
Left	0.920329	0.678570	0.009041	0.086829	0.976574	0.834985	0.000000	0.011712
Time used (s)	3.63	5.62	7.56	6.08	0.21	0.41	0.44	0.44

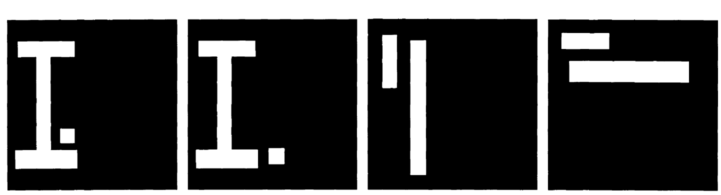
Table 4
Result of degree of location between reference and argument objects for Fig. 5 using compatibility method and watershed-compatibility method

	Compatibility method				Watershed-compatibility method			
	(a)	(b)	(c)	(d)	(a)	(b)	(c)	(d)
Right	0.000000	0.333660	0.693125	0.627291	0.000000	0.359934	0.880006	0.927948
Below	0.034790	0.232274	0.309531	0.385455	0.013648	0.096136	0.117790	0.043269
Above	0.027421	0.030331	0.040047	0.081430	0.003918	0.003294	0.006743	0.044104
Left	0.965210	0.775246	0.073328	0.319108	0.982927	0.940894	0.000000	0.249021
Time used (s)	3.61	5.57	7.47	6.02	0.21	0.41	0.44	0.44



(a) (b) (c) (d)

Fig. 5. Experimental data applied to relative spatial relationship (I).



(a) (b) (c) (d)

Fig. 6. Experimental data applied to relative spatial relationship (II).

Table 5
Result of degree of location between reference and argument objects for Fig. 6 using aggregation method and watershed-aggregation method

	Aggregation method				Watershed-aggregation method			
	(a)	(b)	(c)	(d)	(a)	(b)	(c)	(d)
Right	0.381583	0.608652	0.000000	0.036623	0.767371	0.878712	0.000000	0.000000
Below	0.424333	0.389670	0.050313	0.000000	0.191330	0.126938	0.000000	0.000000
Above	0.204080	0.021267	0.537849	0.469199	0.054948	0.002724	0.139714	0.827211
Left	0.009816	0.000000	0.432064	0.514381	0.000721	0.000000	0.867821	0.182437
Time used (s)	1.84	2.15	4.18	4.82	0.28	0.36	0.25	0.25

Table 6
Result of degree of location between reference and argument objects for Fig. 6 using compatibility method and watershed-compatibility method

	Compatibility method				Watershed-compatibility method			
	(a)	(b)	(c)	(d)	(a)	(b)	(c)	(d)
Right	0.351884	0.521577	0.000000	0.191522	0.822604	0.909577	0.000000	0.000000
Below	0.730552	0.498824	0.252447	0.000000	0.145309	0.089474	0.000000	0.000000
Above	0.438019	0.036201	0.673022	0.379258	0.126280	0.002966	0.114536	0.847590
Left	0.066631	0.000000	0.326978	0.621647	0.016014	0.000000	0.885464	0.152410
Time used (s)	1.84	2.12	4.19	4.85	0.28	0.36	0.25	0.25

In practical implementation, we may use the line passing through the watershed point p and its adjacency watershed points to replace the tangent line at point p. Let $p_1, p_2, \ldots, p_m$ be watershed points which form a chain. Fig. 4(a) presents all possible directions of the line formed by two adjacency points. If the watersheds are 4-connected, then we may obtain the 4 directions indicated by four black arrows. Conversely, if the watersheds are 8-connected, then the line passing through two adjacency points have eight possible directions indicated by four black arrows and four thin arrows. Then we have lines $\overrightarrow{p_1p_2}, \overrightarrow{p_2p_3}, \ldots, \overrightarrow{p_{m-1}p_m}$. Each line $\overrightarrow{p_ip_{i+1}}$, $1 \le i < m$, provide part of information of the relationships, i.e. the angle determined by line $\overrightarrow{p_ip_{i+1}}$ and x-axis (see Fig. 3(b)). Therefore, the spatial relationships can be determined by collect all such information. Table 1 shows the results of watershed-aggregation method for test images in Fig. 5 using the 4-connected and 8-connected watershed points, respectively. Table 2 shows the results of watershed-compatibility method for the same test images.

In this study, we used some sampled angles to replace angles of all pairs of points. However, there are only 4 or 8 sampled angles appear in the proposed method. For more conformable to the human intuition, we need more sampled angles. If we use the line passing through the p_i and p_{i+2} instead of using the tangent line at p_i, the number of sampled angles is increased. If 4-connected watersheds are used, there are 8 possible directions. Up to 16 possible directions can be used when watersheds are 8-connected. Fig. 4(b) shows all possible directions for this case. Hence, 8-connected watersheds are chosen for our experiments.

To demonstrate the proposed methods, we apply them to some geometrical figures. We want to recognize the spatial relation "A is left of R" for the region A and R in each of Fig. 5(a)–(d). In each figure, the small square region denotes the argument region A and the other is the reference region R. For the evaluation of results we compare them with those obtained by the compatibility method and aggregation method. The results of aggregation method and watershed-aggregation method are shown in Table 3. Table 4 presents the results of compatibility method and watershed-compatibility method.

The proposed methods are also applied to Fig. 6 to recognize the spatial relation "A is left of R" for region A and region R. Results are shown in Tables 5 and 6. Here, the argument region A is the small square and another is the reference region R in each of Fig. 6(a)–(b). In each of

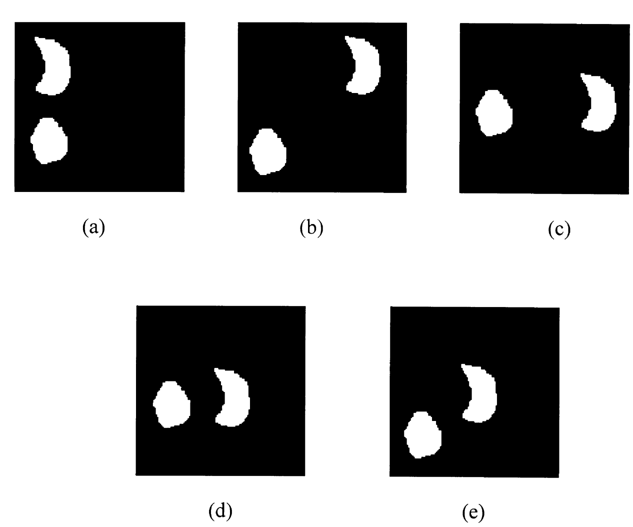


Fig. 7. Experimental data applied to relative spatial relationship (III).

Table 7
Result of degree of location between reference and argument objects for Fig. 7 using aggregation method and watershed-aggregation method

	Aggregation method					Watershed plus aggregation method				
	(a)	(b)	(c)	(d)	(e)	(a)	(b)	(c)	(d)	(e)
Right	0.030189	0.000000	0.000000	0.000000	0.000000	0.039305	0.000000	0.000000	0.000000	0.000000
Below	0.922629	0.476452	0.091483	0.124960	0.394929	0.908541	0.200685	0.098140	0.281955	0.241917
Above	0.000000	0.000000	0.024891	0.056215	0.000373	0.000000	0.000000	0.011234	0.010741	0.000000
Left	0.065923	0.543956	0.903160	0.838672	0.625039	0.058957	0.806870	0.897554	0.727235	0.772805
Time used (s)	4.55	4.71	4.37	4.43	4.64	0.18	0.54	0.28	0.21	0.32

Table 8
Result of degree of location between reference and argument objects for Fig. 7 using compatibility method and watershed-compatibility method

	Compatibility method					Watershed plus compatibility method				
	(a)	(b)	(c)	(d)	(e)	(a)	(b)	(c)	(d)	(e)
Right	0.022344	0.000000	0.000000	0.000000	0.000000	0.032028	0.000000	0.000000	0.000000	0.000000
Below	0.962466	0.449994	0.055836	0.123776	0.351887	0.955653	0.187414	0.073589	0.194040	0.176077
Above	0.000000	0.000000	0.020934	0.068795	0.004470	0.000000	0.000000	0.010848	0.123664	0.000000
Left	0.037534	0.550006	0.944164	0.876224	0.648113	0.044347	0.812586	0.921548	0.805536	0.823923
Time used (s)	4.48	4.66	4.34	4.41	4.59	0.19	0.54	0.28	0.22	0.32

Fig. 6(c)–(d), the argument region A is the smaller rectangular and the reference region R is the larger rectangular. For the concave objects, such as those in Fig. 6(a) and (b), evaluation of the spatial relationships may be affected by the size and location of the chosen window. In our experiments, the window size is determined by the up most, down most, right most, and left most object points. If we double the window width in right direction for Fig. 6(a), the results of watershed-aggregation method are 0.626457, 0.248012, 0.136127, and 0.001177 for "right of", "below", "above", and "left of" relationships, respectively. Similarly, the results of watershed-compatibility method are 0.740888, 0.240736, 0.325860, and 0.006627 for "right of", "below", "above", and "left of" relationships, respectively. However, the method used in our experiment always produce moderate results. Fig. 7 is another example; its results are shown in Tables 7 and 8. In this case, the evaluation results of the proposed methods are similar to the compatibility method and aggregation method.

We also applied the proposed method to building recognition in maps and aerial images, an application appeared in [4,5]. In this application, the relative relations with respect to several objects in the scene are used to assist to recognize or label a set of objects. An example aerial image and its segmented image are shown in Fig. 9. Five houses are segmented from the image. For the sake of clarity, we

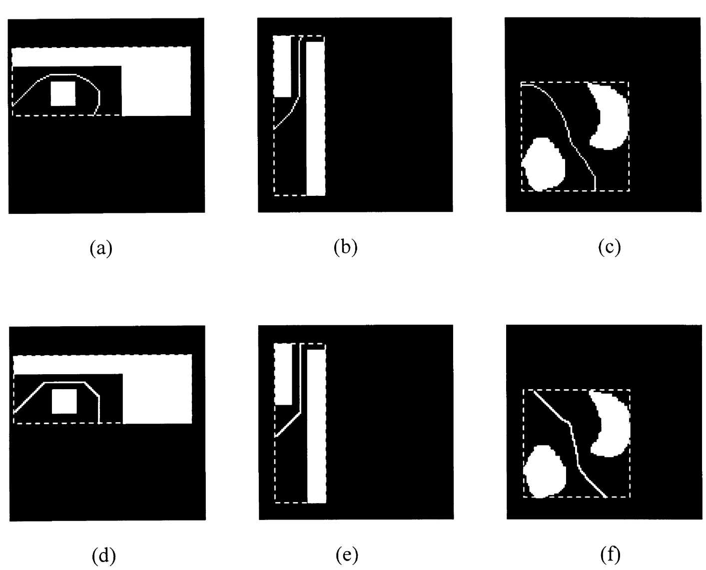


Fig. 8. (a)–(c) 8-connected SKIZ of test images; (d)–(f) 4-connected SKIZ of test images.

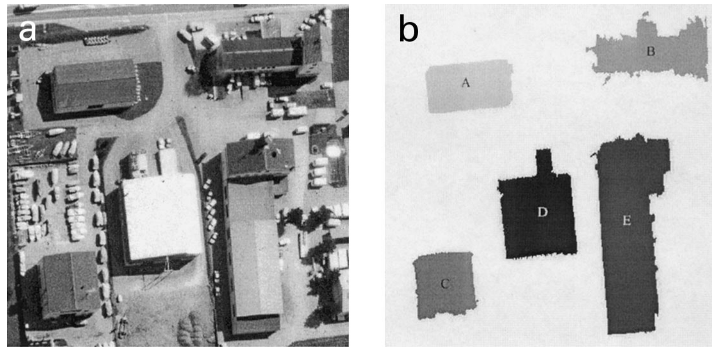


Fig. 9. (a) The aerial image. (b) The segmented image.

Table 9
Result of degree of location between house *A* and *E* for Fig. 9 using the proposed method, aggregation and compatibility method

	Right	Below	Above	Left	Time used (s)
Watershed-compatibility	0.000000	0.000000	0.346483	0.653517	4.75
Watershed-aggregation	0.000000	0.000000	0.374707	0.640281	4.74
Aggregation	0.000000	0.000000	0.460327	0.560081	1019.07
Compatibility	0.000000	0.000000	0.432821	0.567179	1027.24

name these houses house *A*, *B*, *C*, *D*, and *E* as shown in Fig. 9(b). Spatial relations obtained by the proposed methods, compatibility method and aggregation method are summarized in Table 9. In this example, we assume house *A* is the argument object and *E* is the reference object. The results produced by these four methods are very similar. However, the proposed methods are hundred times faster than the aggregation and compatibility methods.

In all cases, the computation time has been reduced by the proposed methods. The reason is that both compatibility method and aggregation method need to compute all point pairs between the objects. If the object regions are getting larger, the computation overhead of compatibility method or aggregation method is rapidly increased, since it is proportional to the number of points within the objects, i.e. the areas of the regions. However, the computation time of the proposed methods is proportional to the number of SKIZ points.

5. Conclusions

Many proposed methods for the spatial relation description rely on the angle measurements between two objects. In this paper, a new approach to angle measurements has been proposed. The proposed methods apply a modified watershed algorithm to a window contained the objects under consideration. Once the SKIZ of the objects are found, we collect all the angles formed by the directions of normal vectors of tangent lines to the SKIZ. Then we describe the spatial relations between objects using the aggregation operator or compatibility notion. The proposed methods just compute pairs of adjacency SKIZ points to detect the spatial relations between objects. Hence, the computation time can be reduced when object regions are large. The experiment results show that they are effective and fast methods.

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