ELSEVIER

Physica C 332 (2000) 199–202

PHYSICA C

www.elsevier.nl/locate/physc

Excitation of ''rotation'' collective modes in vortex lattice of clean type II superconductors

A. Kasatkin [a,b], B. Rosenstein [c,*]

[a] Electrophysics Department, National Chiao Tung University, Hsinchu 30043, Taiwan
[b] Institute of Metal Physics, Ukrainian Academy of Sciences, Kiev, Ukraine
[c] National Center for Theoretical Sciences and Electrophysics Department, National Chiao Tung University, Hsinchu 30043, Taiwan

Abstract

In superclean limit, the Magnus force on Abrikosov vortices is stronger than friction. Due to this nondissipative force, vortex segments rotate around pinning centers. Waves of such rotations under certain conditions are only weakly damped (not overdamped as is usually the case) and lead to resonances in ac response. Excitation of such waves by applied ac field near the surface is considered. Surface impedance, ac resistivity and magnetic permeability are calculated using elasticity theory of the vortex lattice.

Keywords: Excitation; Rotation; Vortex lattice

1. Introduction

Abrikosov vortex dynamics in type II superconductors under magnetic field is usually thought to be overdamped. Due to large vortex viscosity, the displacement waves in vortex lattice do not propagate. In high-T_c superconductors, the situation under certain conditions might be different. The dissipation during the vortex motion is, at least, to large extent due to excitation of quasiparticles inside the vortex core. At small temperatures, this process is frozen and instead of the usual Bardeen–Stephen friction force, $\eta \boldsymbol{v}$, one only has a non-dissipative Magnus force $\eta \hat{\boldsymbol{z}} \times \boldsymbol{v}$ perpendicular to the vortex velocity, where z is the direction of external magnetic field. As evidence to the increasing role of the Magnus force is the famous Hall anomaly [1,2]. In a series of direct experiments [3], it was shown that in YBCO single crystals at low temperatures, the Hall angle $\tan(\theta_H) \equiv \eta'/\eta$ diverges as T^{-1} and clearly exceeds 1 below 4 K reaching 2.5 at 3 K. This regime was termed by authors of Ref. [3] ''superclean limit''. Theoretically, such a behavior was predicted in Refs. [4–8]. In such a superclean regime, vortex dynamics might be non-overdamped and, for example, displacement waves in the vortex lattice can propagate. This type of phenomenon was used recently [9,10] to explain the magneto-absorption in BSCCO [11–14], although alternative explanations based on the Josephson plasma oscillations exist [15,16].

In this paper, we consider dynamics of vortices in ''superclean'' superconductors under applied ac field. We argue existence of weakly damped ''rotation'' waves in the pinned vortex lattice, calculate its dispersion law, and consider linear vortex response on applied ac field. We show that excitation of the

* Corresponding author.

0921-4534/00/$ - see front matter
PII: S0921-4534(99)00666-8

non-overdamped waves by applied ac field modify in an essential way the theory of the linear response developed by Brandt [17,18] and Coffey and Clem [19,20] and point out possible resonance effects in the surface impedance and ac resistivity.

2. Dispersion relation for waves in vortex lattice

For small magnetic induction values, $B/\Phi_0 < \lambda^{-2}$, one can neglect exponentially small interactions between vortices and consider single vortex dynamics. Assuming that the vortex is pinned (Fig. 1), we describe it by equation of motion for displacement **u**:

$$m\ddot{\boldsymbol{u}} + \eta\dot{\boldsymbol{u}} + \eta'\boldsymbol{z} \times \dot{\boldsymbol{u}} + \alpha\boldsymbol{u} = 0. \tag{1}$$

Here, α is the Labousch parameter describing restoring pinning force in the x–y plane. Neglecting

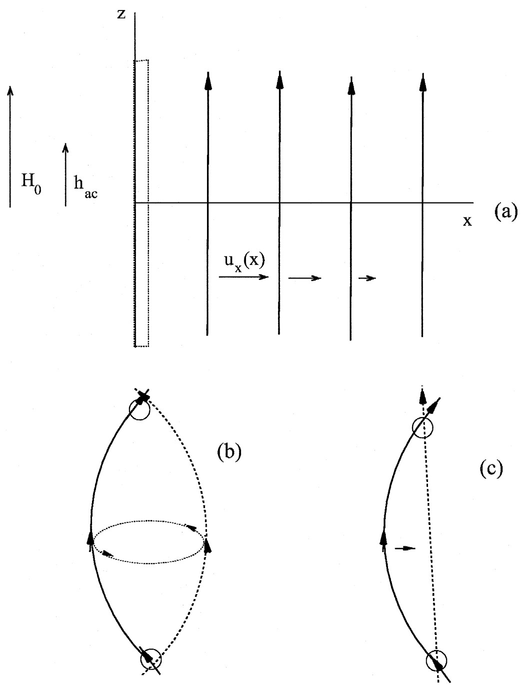

Fig. 1. (a) Positions and displacements of vortices caused by external Lorentz force. (b) Displacement of a vortex segment under influence of ac field in the superclean limit. (c) Displacement of a vortex segment under influence of ac field in the conventional overdamped case.

the vortex mass in Eq. (1) for single vortex dynamics, one obtains the following periodic solution: $u_i(t) = e^{i\Omega_\pm t}u_i$ where:

$$\Omega_\pm = \alpha\frac{i\eta \pm \eta'}{\eta^2 + \eta'^2}. \tag{2}$$

When the friction coefficient, η, is small, one obtains (clockwise and counterclockwise) circular vortex motion around the pinning center (see Fig. 1b) with the frequency $\omega_M = \alpha/\eta'$.

Contribution of interactions between vortices to the vortex dynamics can be taken into account within harmonic approximation:

$$\eta\dot{u}_i(\boldsymbol{R}^a)\eta'\epsilon_{ij}\dot{u}_j(\boldsymbol{R}^a) + \alpha u_i(\boldsymbol{R}^a) + \sum_b \Phi_{ij}(\boldsymbol{R}^a - \boldsymbol{R}^b)u_j(\boldsymbol{R}^{b)} = 0. \tag{3}$$

Here $\boldsymbol{\Phi}_{ij}$ is the dynamical matrix and $\boldsymbol{R}^a$ are locations of vortices usually arranged in the lattice and ϵ_{ij} is the totally antisymmetric tensor. Since we are using elasticity theory, the detailed nature of the vortex matter is not very important as long as correct elastic moduli are used and most of the considerations are valid in vortex liquid or glass. We will consider only external forces homogeneous in y and z directions; therefore, the only nonzero component of momentum is $k_x \equiv k$. When external force is absent displacement vector for frequency, Ω, satisfies:

$$\begin{pmatrix} (i\Omega\eta + \alpha) + c_{11}(k)k^2 & i\Omega\eta' \\ -i\Omega\eta' & (i\Omega\eta + \alpha) + c_{66}(k)k^2 \end{pmatrix}\begin{pmatrix} u_x(k) \\ u_y(k) \end{pmatrix} \equiv A_{ij}u_j = 0, \tag{4}$$

where c_{11} and c_{66} are (possibly dispersive) elastic moduli of the vortex matter. In London limit [21]:

$$c_{11}(k) = \frac{B^2}{4\pi}\frac{1}{1+\lambda^2k^2} \equiv \frac{c_{11}}{1+\lambda^2k^2};$$
$$c_{66} = \frac{B\Phi_0}{4(4\pi\lambda)^2}. \tag{5}$$

The eigenfrequencies in Eq. (2) now become branches. In the superclean limit $\eta = 0$, one has non-damped waves with dispersion law:

$$\Omega_{\pm}(k) = \pm \frac{\sqrt{\alpha^2 + \alpha k^2 \bar{c} + k^4 c_{11}(k) c_{66}}}{\eta'}, \qquad (6)$$

where $\bar{c} \equiv c_{11}(k) + c_{66}$. In the general case, when both η and η' are nonzero, the eigenfrequencies $\Omega_{\mathrm{pm}}(k)$ are complex values and dispersion law becomes rather complicated [22].

Polarization of the waves (which follows from Eq. (4)) is as follows:

$$\frac{u_y(k)}{u_x(k)} = -\tan^{-1}\theta_{\mathrm{H}} + i\frac{\alpha + c_{11}k^2}{\Omega_{\pm}(k)\eta'}. \qquad (7)$$

The fact that the ratio is imaginary means that vortices move on elliptic trajectories.

3. Linear response under applied ac field

In this section, we consider the pinned vortex system response to surface ac current caused by alternating field $h_{\mathrm{ac}}e^{i\omega t}$ in direction parallel to dc field, H, and to the surface of the superconducting half space, see Fig. 1a. Linear response for such geometry for the case $\eta' = 0$ was considered by Brandt [17,18] and Coffey and Clem [19,20], also taking into account pinning, viscosity and creep. Since we are interested mostly in the low temperature regime, flux creep can be neglected while the Magnus force term is important (creep can be taken into account in a similar manner as in Refs. [17–20]). When one performs similar calculation for $\eta' > 0$, new resonant phenomena are readily seen. We impose proper boundary conditions using the ''bulk concept'' methods of Refs. [17,18], which allow referring the problem to an equivalent problem in whole space.

The external force is:

$$F_{\mathrm{ext}}(x,t) = \frac{Bh_{\mathrm{ac}}}{4\pi\lambda} e^{-|x|\setminus\lambda} e^{i\omega t}, \qquad (8)$$

$$F_{\mathrm{ext}}(k,\omega) = \frac{Bh_{\mathrm{ac}}}{2\pi(1+\lambda^2k^2)}. \qquad (9)$$

The displacement in momentum space is obtained from Eq. (4) with external force (Eq. (9)). Very often both c_{66} and k are ''small''. If $c_{66}k^2$ is small compared to $(\omega\eta + \alpha)$, one can readily obtain displacements in the form:

$$u_x(k,\omega) = \frac{Bh_{\mathrm{ac}}}{2\pi\alpha(\omega)\left[1 + k^2\left(c_{11}/\alpha(\omega) + \lambda^2\right)\right]}$$

$$= \frac{2h_{\mathrm{ac}}\lambda_C^2(\omega)}{B\left[1 + k^2\lambda_{\mathrm{ac}}^2(\omega)\right]}, \qquad (10)$$

$$u_y(k,\omega) = \frac{i\omega\eta'}{i\omega\eta + \alpha} u_x(k,\omega), \qquad (11)$$

where $\alpha(\omega) \equiv i\omega\eta + \alpha - [(\omega^2\eta'^2)/(i\omega\eta + \alpha)]$ and the modified Campbell penetration depth $\lambda_{\mathrm{C}}^2(\omega)$ is:

$$\lambda_{\mathrm{C}}^2(\omega) \equiv \frac{c_{11}}{\alpha(\omega)} = \frac{B^2(i\omega\eta + \alpha)}{4\pi\left[(i\omega\eta + \alpha)^2 - \omega^2\eta'^2\right]}$$

$$= -\frac{B^2(i\omega + \tan\theta_{\mathrm{H}}\,\omega_M)\cos^2\theta_{\mathrm{H}}}{4\pi\eta[\omega - \Omega_+(0)][\omega - \Omega_+(0)]}. \qquad (12)$$

The frequency-dependent complex ac penetration depth was introduced in Eq. (8):

$$\lambda_{\mathrm{ac}}^2(\omega) \equiv \lambda^2 + \lambda_C^2(\omega). \qquad (13)$$

As is in the usual case [17–20], $\eta' = 0$, this quantity determines both the surface impedance:

$$Z_{\mathrm{s}}(\omega) = (4\pi i/c^2)\omega\lambda_{\mathrm{ac}}(\omega) \qquad (14)$$

and the ac resistivity $\rho_{\mathrm{ac}}(\omega) \equiv E(x)/J(x) = (4\pi i/c^2)\omega\lambda_{\mathrm{ac}}^2(\omega)$. These two quantities exhibit resonance in the clean limit. On Fig. 2, real and imaginary parts of surface impedance for various values of $\cos\theta_{\mathrm{H}}$ and $b \equiv B/H_{c1} = 10$ are shown.

The general case, when k^2c_{66} is not negligible, was also studied in our work [22]. Qualitative behavior of the $Z(\omega)$ dependence, shown in Fig. 2, does

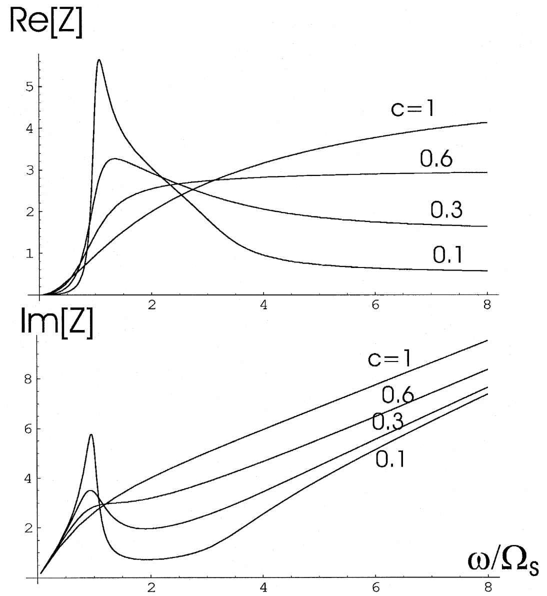

Fig. 2. Frequency dependence of the surface impedance (real and imaginary parts) for $\cos\theta_{\mathrm{H}} = 0.1$, 0.3, 0.6, 1. Magnetic induction $B = 10\ H_{c1}$.

not change, while resonance peak becomes even more pronounced in this case.

4. Discussion

In this work, we determined conditions under which a non-overdamped ''rotation'' (around pinning centers) waves exist in clean type II superconductors. There are clear indications that these conditions can be met in non-twinned YBCO single crystals [2], and some resonance effects due to vortex motion were really observed in ac experiments on HTS materials [23,24]. Excitation of such waves by applied ac field near the surface is considered. The simplest realistic geometry is the superconducting half space with the dc magnetic field creating vortices parallel to the surface. We considered the direction of the surface ac field parallel to the dc magnetic field. In this case, linear response characteristics, such as surface impedance and ac resistivity, were calculated using the elasticity theory of the vortex lattice. The most pronounced effect of the rotation waves is resonance at characteristic frequency of order $\Omega_{\mathrm{s}} = \alpha/\eta'$. It is comparable or larger than the depinning frequency, $\Omega_{\mathrm{depin}} = \alpha/\eta$, which is of order 10–100 GHZ [25].

Acknowledgements

The authors thank Professor Y.S. Gou and Dr. A. Knigavko for helpful discussions.

This work was supported by National Science Council, Republic of China through contract #NSC88-2112-M009-026. A. Kasatkin, one of the authors, is grateful to the Ministry of Education and National Science Council for their support on this work during his stay in Taiwan.

References

[1] S.J. Hagen, A.W. Smith, M. Rajeswari, J.L. Peng, Z.Y. Li, R.L. Greene, S.N. Mao, X.X. Xi, S. Bhattacharya, Q. Li, C.J. Lobb, Phys. Rev. B 47 (1993) 1064.
[2] J.M. Graybeal, J. Luo, W.R. White, Phys. Rev. B 49 (1994) 12923.
[3] J.M. Harris, Y.F. Yan, O.K.C. Tsui, Y. Matsuda, N.P. Ong, Phys. Rev. Lett. 73 (1994) 1711.
[4] N.B. Kopnin, V.E. Kravtsov, Pis'ma Zh. Eksp. Teor. Fiz. 23 (1976) 631.
[5] N.B. Kopnin, V.E. Kravtsov, JETP Lett. 23 (1996) 578.
[6] N.B. Kopnin, A.V. Lopatin, Phys. Rev. B 51 (1995) 15291.
[7] P. Ao, D.J. Thouless, Phys. Rev. Lett. 70 (1993) 2158.
[8] E.B. Sonin, Phys. Rev. B 55 (1997) 485.
[9] N.B. Kopnin, A.V. Lopatin, E.B. Sonin, K.B. Traito, Phys. Rev. Lett. 74 (1995) 4527.
[10] E.B. Sonin, Phys. Rev. Lett. 79 (1997) 3732.
[11] O.K.C. Tsui, N.P. Ong, Y. Matsuda, Y.F. Yan, B. Peterson, Phys. Rev. Lett. 73 (1994) 724.
[12] O.K.C. Tsui, N.P. Ong, Phys. Rev. Lett. 76 (1996) 819.
[13] Y. Matsuda, M.B. Gaifullin, K. Kumagai, K. Kadowaki, T. Mochiku, Phys. Rev. Lett. 75 (1995) 4512.
[14] Y. Matsuda, M.B. Gaifullin, K. Kumagai, M. Kosugi, K. Hirada, Phys. Rev. Lett. 78 (1997) 1972.
[15] L.N. Bulaevskii, M. Maley, H. Safar, D. Dominguez, Phys. Rev. B 53 (1996) 6634.
[16] L.N. Bulaevskii, V.L. Pokrovski, M. Maley, Phys. Rev. Lett. 76 (1996) 1719.
[17] E.H. Brandt, Phys. Rev. Lett. 67 (1991) 2219.
[18] E.H. Brandt, Physica C 195 (1992) 1.
[19] M.W. Coffey, J.R. Clem, Phys. Rev. Lett. 67 (1991) 386.
[20] M.W. Coffey, J.R. Clem, Phys. Rev. B 46 (1992) 11757.
[21] E.H. Brandt, Rep. Prog. Phys. 58 (1995) 1465.
[22] A. Kasatkin, B. Rosenstein, Phys. Rev. B 60 (1999) 14907.
[23] E.J. Choi, S. Kaplan, S. Wu, H.D. Drew, Q. Li, D.B. Ferrer, J.M. Phillips, S.Y. Hoi, Physica C 254 (1995) 258.
[24] T. Ichiguchi, Phys. Rev. 57 (1998) 638.
[25] M. Golosovsky, M. Tsindiekht, D. Davidov, Supercond. Sci. Technol. 9 (1996) 1.