

Theory of self-induced transparency in a Kerr host medium beyond the slowly-varying-envelope approximation

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The self-induced transparency in the Kerr host medium has been studied beyond the slowly-varying-envelope approximation. An analytic solution is obtained. There exists a hyperbolic-secant soliton whose power, pulse width, group velocity, and propagation constant are uniquely determined for given parameters of the medium. The reductions of the group velocities of the solitons by the self-induced-transparency effect in the resonant medium with homogeneous and inhomogeneous broadening are also studied. It is found that there is negative dispersion induced by the self-induced transparency, which is not predicted by the theory with the slowly-varying-envelope approximation. The amount of the induced dispersion is determined by the total dispersion of the system required to be compensated by the Kerr effect. Numerical examples of the self-induced transparency in an erbium-doped fiber are shown. In a typical erbium-doped fiber, the soliton solution has not been found because the inhomogeneous-broadening linewidth of the medium is too large.

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I. INTRODUCTION

The self-induced transparency (SIT) discovered by McCall and Hahn is a phenomenon of a coherent optical pulse propagating in a resonant medium without loss and distortion when the pulse energy exceeds a critical value [1,2]. It has been found that certain solutions of the coherent pulse are solitons. Such a pulse is called a SIT soliton. For the optical fiber, Hasegawa and Tappert have proposed that the dispersion of the pulse can be compensated by utilizing the Kerr effect if the dispersion of the pulse is in the negative regime [3]. Because the distortionless pulse satisfies the nonlinear Schrödinger (NLS) equation and behaves as a soliton, it is called the NLS soliton. Since the recent development of the erbium-doped fiber amplifiers [4,5], it is interesting to consider the SIT in the silica-based erbium-doped fiber (EDF). Therefore, the coexistence of a SIT soliton and a NLS soliton has been a subject of intense interest [6-8]. Reference [8] claims that an impractically large dispersion is required for the SIT-NLS soliton to exist in an EDF. Therefore, it concludes that it is not possible for the SIT soliton and the NLS soliton to coexist in EDF. In Ref. [8], the slowly-varying-envelope approximation (SVEA) is used. In this paper, we extend our previous work [7] on the theory of the SIT-NLS soliton without making the SVEA and show that the large dispersion required for the coexistence of a SIT soliton and a NLS soliton is automatically induced by the SIT. However, because of the large inhomogeneous broadening linewidth of the medium, the soliton solution has not been found in a typical EDF.

In Sec. II, the Maxwell-Bloch equations governing the

SIT-NLS soliton are derived without using SVEA. The resonant medium is assumed to be inhomogeneous broadening. In Sec. III, the equations are solved to obtain the SIT-NLS soliton solution. In Sec. IV, we study the characteristics of the SIT-NLS solitons for three special cases. The results are compared to the previous works. Some numerical examples with the typical parameters of EDF are shown. At last, the conclusions are given in Sec. V.

II. MAXWELL-BLOCH EQUATIONS WITHOUT USING SVEA

SIT is the coherent effect of the interaction between the optical pulse and the resonant atoms. This gives the upper limit on the pulse width. The pulse width T_w must be much shorter than the relaxation times of the population difference T_1 and polarization T_2 . The resonant atoms are modeled as an ensemble of two-level atoms which has the dipole moment μ and resonant frequency ω_r . The atoms are embedded in a nonresonant host medium. In this paper, the host medium is the optical fiber with Kerr effect and dispersion.

The electric field of the optical pulse propagating along the z direction in such a medium is assumed to be linearly polarized and can be expressed as

$$\mathcal{E}(z,t) = E(z,t) \exp[i(k_0 z - \omega_0 t)], \quad (1)$$

where $E(z,t)$ is the complex envelope of the pulse, ω_0 is the carrier frequency, and $k_0 \equiv k(\omega_0)$ is the wave number at ω_0 ; $k(\omega) = n(\omega)\omega/c$ and $n(\omega)$ is the linear refractive index of the fiber. The macroscopic polarization due to the resonant atoms is written as

$$\mathcal{P}_r(z,t) = P_r(z,t) \exp[i(k_0 z - \omega_0 t)]. \quad (2)$$

The complex envelopes E and P_r can be written as

$$E(z,t) = q(z,t) \exp[i\phi(z,t)], \quad (3)$$

$$P_r(z,t) = [U(z,t) + iV(z,t)] \exp[i\phi(z,t)], \quad (4)$$

where $q(z,t)$ and $\phi(z,t)$ are the amplitude and phase of the pulse, respectively, and $U(z,t)$ and $V(z,t)$ correspond to the dispersion (in phase) and absorption (in quadrature) due to the resonant atoms, respectively, and these four quantities are real.

The wave equation for this problem is

$$\frac{\partial^2 \mathcal{E}}{\partial z^2} = \mu_0 \left[\frac{\partial^2 \mathcal{D}_1}{\partial t^2} + \frac{\partial^2 \mathcal{D}_{nl}}{\partial t^2} + \frac{\partial^2 \mathcal{P}_r}{\partial t^2} \right], \quad (5)$$

where \mathcal{D}_1 is the linear displacement and $\mathcal{D}_1(z,t) = \int_{-\infty}^{\infty} \epsilon_1(t-t') \mathcal{E}(z,t') dt'$, where $\epsilon_1(t)$ is the linear dielectric constant of the fiber and its Fourier components $\xi_1(\omega) = \epsilon_0 n^2(\omega)$; ϵ_0 and μ_0 denote the permittivity and permeability in vacuum, respectively; \mathcal{D}_{nl} is the nonlinear displacement for the nonresonant host medium and $\mathcal{D}_{nl}(z,t) = 2n_2 n_0 \epsilon_0 |\mathcal{E}|^2 \mathcal{E}$ with $n_0 = n(\omega_0)$; n_2 is the Kerr coefficient. By defining $\Delta\omega = \omega_r - \omega_0$, the macroscopic polarization components contributed from the atoms of the whole range of $\Delta\omega$ are

$$U(z,t) = \int_{-\infty}^{\infty} u(\Delta\omega, z, t) g(\Delta\omega) d(\Delta\omega), \quad (6a)$$

$$V(z,t) = \int_{-\infty}^{\infty} v(\Delta\omega, z, t) g(\Delta\omega) d(\Delta\omega), \quad (6b)$$

where $g(\Delta\omega)$ is the inhomogeneous-broadening normalized line-shape function with $\int_{-\infty}^{\infty} g(\Delta\omega) d(\Delta\omega) = 1$; u and v are the polarization components of the atoms with frequency $\Delta\omega$ detuned from ω_0 . By substituting Eqs. (1) and (2) into Eq. (5) and following the same procedure in Ref. [9] to derive the linear term $\partial^2 \mathcal{D}_1 / \partial t^2$ and averaging the transverse mode function of the electric field over the cross section of the fiber, the wave equation which governs the optical pulse in the moving frame $\eta = t - k'_0 z$, $z = z$ becomes

$$\frac{\partial^2 E}{\partial z^2} - 2k'_0 \frac{\partial^2 E}{\partial \eta \partial z} + 2ik_0 \frac{\partial E}{\partial z} - k_0 k''_0 \frac{\partial^2 E}{\partial \eta^2} + \frac{n_2}{n_0} k_0^2 |E|^2 E + \mu_0 \omega_0^2 \left[P_r + \frac{2i}{\omega_0} \frac{\partial P_r}{\partial \eta} \right] = 0, \quad (7)$$

where $k'_0 = \partial k / \partial \omega|_{\omega=\omega_0}$ and $k''_0 = \partial^2 k / \partial \omega^2|_{\omega=\omega_0}$ are the

reciprocal group velocity and the dispersion of the pulse in the host medium without Kerr effect and resonant atoms. In Eq. (7), the high-order term of the polarization $\partial^2 P_r / \partial \eta^2$ is neglected.

It is known that the velocity of the pulse can be greatly reduced by the SIT. To solve Eq. (7) in the frame with the velocity of the pulse, we take the transformation $\tau = \eta - \Delta k'_0 z$, $z = z$, where $\Delta k'_0$ is the change of the reciprocal group velocity due to the resonant atoms, and Eq. (7) becomes

$$\frac{\partial^2 E}{\partial z^2} - 2(k'_0 + \Delta k'_0) \frac{\partial^2 E}{\partial \tau \partial z} + 2ik_0 \frac{\partial E}{\partial z} - 2ik_0 \Delta k'_0 \frac{\partial E}{\partial \tau} - [k_0 k''_0 - (\Delta k'^2_0 + 2k'_0 \Delta k'_0)] \frac{\partial^2 E}{\partial \tau^2} + \frac{n_2}{n_0} k_0^2 |E|^2 E + \mu_0 \omega_0^2 \left[P_r + \frac{2i}{\omega_0} \frac{\partial P_r}{\partial \tau} \right] = 0. \quad (8)$$

Hence the group velocity of the pulse is $v_g = 1 / (k'_0 + \Delta k'_0)$. It is noticed that $\Delta k'_0$ introduces the dispersion $-(\Delta k'^2_0 + 2k'_0 \Delta k'_0) / k_0$ in the fifth term in Eq. (8). This additional dispersion can be interpreted as the dispersion caused by the SIT. In the literature, the SVEA is often applied to Eq. (7) and the first two terms in Eq. (7) are neglected; then the additional dispersion shown in Eq. (8) will not appear.

If the pulse width is much less than the relaxation times T_1 and T_2 , P_r satisfies the Bloch equations [10]

$$\frac{\partial P_r}{\partial \tau} = -i \Delta\omega P_r + i \frac{\mu}{\hbar} W E, \quad (9a)$$

$$\frac{\partial W}{\partial \tau} = -i \frac{\mu}{2\hbar} (E P_r^* - E^* P_r), \quad (9b)$$

where $W = (N_1 - N_2) \mu$ is the macroscopic population difference multiplied by μ between the ground state (N_1) and upper state (N_2) of the resonant atoms. We take N_0 as the density of the resonant atoms and $N_0 = N_1 + N_2$.

III. SIT-NLS SOLITON SOLUTION

In this section, we solve the Maxwell-bloch equations to find the SIT-NLS soliton solution. We assume the amplitude q is independent of z , i.e., $\partial q / \partial z = 0$, but the phase ϕ may depend on both τ and z . Under these assumptions and by substituting Eqs. (3), (4), and (6) into Eqs. (8) and (9), the wave equation becomes

$$\gamma \frac{\partial^2 q}{\partial \tau^2} - \left\{ \left[\frac{\partial \phi}{\partial z} \right]^2 + 2k_0 \frac{\partial \phi}{\partial z} + 2 \left[k_0 \left[k'_0 - \frac{1}{v_g} \right] - \frac{1}{v_g} \frac{\partial \phi}{\partial z} \right] \frac{\partial \phi}{\partial \tau} + \gamma \left[\frac{\partial \phi}{\partial \tau} \right]^2 \right\} q + \kappa q^3 + \mu_0 \omega_0^2 \int_{-\infty}^{\infty} \left[u - \frac{2}{\omega_0} \left[\frac{\partial \phi}{\partial \tau} u + \frac{\partial v}{\partial \tau} \right] \right] g(\Delta\omega) d(\Delta\omega) = 0, \quad (10a)$$

$$2 \left[k_0 \left[k'_0 - \frac{1}{v_g} \right] - \frac{1}{v_g} \frac{\partial \phi}{\partial z} + \gamma \frac{\partial \phi}{\partial \tau} \right] \frac{\partial q}{\partial \tau} + \gamma q \frac{\partial^2 \phi}{\partial \tau^2} + \mu_0 \omega_0^2 \int_{-\infty}^{\infty} \left[v - \frac{2}{\omega_0} \left[\frac{\partial \phi}{\partial \tau} v - \frac{\partial u}{\partial \tau} \right] \right] g(\Delta\omega) d(\Delta\omega) = 0, \quad (10b)$$

where

$$\gamma = - \left[k_0 k_0'' + k_0'^2 - \frac{1}{v_g^2} \right], \quad (11a)$$

$$\kappa = \frac{n_2}{n_0} k_0^2. \quad (11b)$$

and the Bloch equations become

$$\frac{\partial u}{\partial \tau} = \left[\frac{\partial \phi}{\partial \tau} + \Delta \omega \right] v, \quad (12a)$$

$$\frac{\partial v}{\partial \tau} = - \left[\frac{\partial \phi}{\partial \tau} + \Delta \omega \right] u + \frac{\mu}{\hbar} q w, \quad (12b)$$

$$\frac{\partial w}{\partial \tau} = - \frac{\mu}{\hbar} q v, \quad (12c)$$

where $w = w(\Delta\omega, z, \tau)$ is the component of the macroscopic population difference contributed from the atoms with frequency $\Delta\omega$ detuned from ω_0 and

$$W(z, \tau) = \int_{-\infty}^{\infty} w(\Delta\omega, z, \tau) g(\Delta\omega) d(\Delta\omega). \quad (13)$$

In order to obtain the analytic solution we assume $v(\Delta\omega, z, \tau)$ is in a factorized form

$$v(\Delta\omega, z, \tau) = v_1(z, \tau) f(\Delta\omega), \quad (14)$$

where $f(\Delta\omega)$ is known as the dipole spectra-response function [11] and is normalized as $f(0) = 1$. Integrating Eq. (12a), we have

$$u(\Delta\omega, z, \tau) = [u_1(z, \tau) + u_2(z, \tau) \Delta\omega] f(\Delta\omega), \quad (15)$$

where u_1 and u_2 are defined as

$$\frac{\partial u_1}{\partial \tau} = v_1 \frac{\partial \phi}{\partial \tau}, \quad (16a)$$

$$\frac{\partial u_2}{\partial \tau} = v_1. \quad (16b)$$

Similarly, by integrating Eq. (12c), we obtain

$$w(\Delta\omega, z, \tau) = w_0 - w_1(z, \tau) f(\Delta\omega), \quad (17)$$

where w_0 is the initial population difference and is assumed in ground state, i.e., $w_0 = N_0 \mu$; $w_1(z, \tau)$ is defined by

$$\frac{\partial w_1}{\partial \tau} = \frac{\mu}{\hbar} q v_1. \quad (18)$$

Substituting Eqs. (14)–(18) into Eq. (12b), we have

$$\begin{aligned} \frac{\partial v_1}{\partial \tau} = & \left[-u_2 \Delta\omega^2 - \left(u_1 + u_2 \frac{\partial \phi}{\partial \tau} \right) \Delta\omega + \frac{\mu w_0}{\hbar} \frac{q}{f(\Delta\omega)} \right] \\ & - u_1 \frac{\partial \phi}{\partial \tau} - \frac{\mu}{\hbar} q w_1. \end{aligned} \quad (19)$$

The terms in square brackets in Eq. (19) should be independent of $\Delta\omega$ because the other terms in Eq. (19) are independent of $\Delta\omega$. Therefore, we have

$$f(\Delta\omega) = (1 + c_1 \Delta\omega + c_2 \Delta\omega^2)^{-1}, \quad (20)$$

and

$$u_1 = c_1 \frac{\mu w_0}{\hbar} q - u_2 \frac{\partial \phi}{\partial \tau}, \quad (21a)$$

$$u_2 = c_2 \frac{\mu w_0}{\hbar} q, \quad (21b)$$

where c_1 and c_2 are the constants to be determined. Then Eq. (19) reduces to

$$\frac{\partial v_1}{\partial \tau} = \frac{\mu}{\hbar} (w_0 - w_1) q - u_1 \frac{\partial \phi}{\partial \tau}. \quad (22)$$

The population difference can be found by substituting Eqs. (16b) and (21b) into Eq. (18) and

$$w = w_0 - \frac{c_2 \mu^2}{2 \hbar^2} q^2 f(\Delta\omega) w_0. \quad (23)$$

It is noticed that, in Eq. (23), the population difference is depleted by the square of the pulse amplitude.

Because the steady-state solution is considered, we can write $\partial q / \partial \tau = dq / d\tau$ and $\partial^2 q / \partial \tau^2 = d^2 q / d\tau^2$. By substituting $w_1(z, \tau)$, $v_1(z, \tau)$, and $u_1(z, \tau)$ as functions of $q(\tau)$ derived above into Eq. (22), we have

$$c_2 \frac{d^2 q}{d\tau^2} = \left[1 - c_1 \frac{\partial \phi}{\partial \tau} + c_2 \left[\frac{\partial \phi}{\partial \tau} \right]^2 \right] q - \frac{c_2 \mu^2}{2(\hbar)^2} q^3. \quad (24)$$

From Eq. (24), we know that $\partial \phi / \partial \tau$ is a function of τ only; from Eq. (10a) for distortionless propagation, it is seen that $\partial \phi / \partial z$ is also a function of τ only. Since $\partial^2 \phi / \partial \tau \partial z = \partial(\partial \phi / \partial z) / \partial \tau = \partial(\partial \phi / \partial \tau) / \partial z = 0$, which implies $\partial \phi / \partial z$ must be a constant, we may write

$$\frac{\partial \phi}{\partial z} = \Delta k_0, \quad (25)$$

where Δk_0 is a constant and represents the change of the propagation constant due to the resonant atoms and Kerr effect. For the distortionless amplitude solution, Eq. (10) reduces to

$$\begin{aligned} \gamma \frac{d^2 q}{d\tau^2} - \left[\alpha + \beta \frac{\partial \phi}{\partial \tau} + \gamma \left[\frac{\partial \phi}{\partial \tau} \right]^2 \right] q + \kappa q^3 + s \left[\left(c_1 I_1 + c_2 I_2 - c_2 I_1 \frac{\partial \phi}{\partial \tau} \right) q \right. \\ \left. - \frac{2}{\omega_0} \left\{ (c_1 I_1 + c_2 I_2) \frac{\partial \phi}{\partial \tau} q - c_2 I_1 \left[q \left[\frac{\partial \phi}{\partial \tau} \right]^2 - \frac{d^2 q}{d\tau^2} \right] \right\} \right] = 0, \end{aligned} \quad (26a)$$

$$\left[\beta + 2\gamma \frac{\partial \phi}{\partial \tau} \right] \frac{dq}{d\tau} + \gamma q \frac{\partial^2 \phi}{\partial \tau^2} + s \left[c_2 I_1 \frac{dq}{d\tau} + \frac{2}{\omega_0} \left[(c_1 I_1 + c_2 I_2) \frac{dq}{d\tau} - c_2 I_1 \left[2 \frac{\partial \phi}{\partial \tau} \frac{dq}{d\tau} + q \frac{\partial^2 \phi}{\partial \tau^2} \right] \right] \right] = 0, \quad (26b)$$

where

$$\alpha = \Delta k_0^2 + 2k_0 \Delta k_0, \quad (27a)$$

$$\beta = 2k_0 \left[k_0' - \frac{1}{v_g} \right] - 2\Delta k_0 \frac{1}{v_g}, \quad (27b)$$

$$s = \mu_0 \omega_0^2 \left[\frac{\mu w_0}{\hbar} \right], \quad (27c)$$

$$I_1 = \int_{-\infty}^{\infty} f(\Delta\omega) g(\Delta\omega) d(\Delta\omega), \quad (27d)$$

$$I_2 = \int_{-\infty}^{\infty} \Delta\omega f(\Delta\omega) g(\Delta\omega) d(\Delta\omega). \quad (27e)$$

Equation (26b) can be rewritten as

$$\left[s_1 + 2s_2 \frac{\partial \phi}{\partial \tau} \right] \frac{dq}{d\tau} + s_2 q \frac{\partial^2 \phi}{\partial \tau^2} = 0, \quad (28)$$

where

$$s_1 = \beta + s \left[c_2 I_1 + \frac{2}{\omega_0} (c_1 I_1 + c_2 I_2) \right], \quad (29a)$$

$$s_2 = \gamma - \frac{2}{\omega_0} s c_2 I_1. \quad (29b)$$

Integrating Eq. (28), we obtain $\partial \phi / \partial \tau = -\Omega + d_1 q^{-2}$, where Ω and d_1 are constants. With this result and substituting Eq. (24) into Eq. (26a), Eq. (26a) then becomes a polynomial equation including only the terms q^{-3}, q^{-1}, q^1, q^3 . This polynomial equation is valid only when the coefficient of each term is zero. These conditions give the following results: (i) $d_1 = 0$, i.e.,

$$\frac{\partial \phi}{\partial \tau} = -\Omega. \quad (30a)$$

This means that the carrier frequency of the pulse is shifted to $\omega = \omega_0 + \Omega$ and the pulse is unchirped. (ii)

$$\left[\gamma - \frac{2}{\omega_0} s I_1 c_2 \right] (1 - \Omega^2 c_2) = (\alpha - \beta \Omega + \gamma \Omega^2) c_2 + s (\Omega I_1 - I_2) \left[1 + \frac{2\Omega}{\omega_0} \right] c_2^2. \quad (30b)$$

Here we have used the relation

$$c_1 = -2\Omega c_2, \quad (30c)$$

which can be obtained by the relations of u_1 and u_2 given by Eqs. (16) and (21). (iii)

$$\kappa = \frac{\mu^2}{2\hbar^2} \left[\gamma - \frac{2}{\omega_0} s I_1 c_2 \right]. \quad (30d)$$

By using Eqs. (30a) and (30c), Eq. (26b) has a nontrivial solution only when

$$\beta - 2\gamma \Omega + s c_2 \left[I_1 + \frac{2}{\omega_0} I_2 \right] = 0. \quad (31)$$

With Eq. (30a), the solution of the electric field can be obtained from Eq. (24) and

$$E(z, \tau) = \frac{2\hbar}{\mu T_c} \operatorname{sech} \left[\frac{\tau}{T_c} \right] \exp[i(\Delta k_0 z - \Omega \tau)], \quad (32)$$

where T_c is the characteristic time scale and

$$T_c = (1/c_2 - \Omega^2)^{-1/2}. \quad (33)$$

The pulse full width at half maximum (FWHM) of the soliton is $T_w = 1.763 T_c$. It is seen that the constant c_2 relates to the pulse width and the detuned frequency. The group velocity v_g , characteristic time scale T_c , and the change of the propagation constant Δk_0 in Eq. (32) can be uniquely determined from the coupled nonlinear algebraic equations (30b), (30d), and (31). That is, for a given medium, only one unique solution exists. It is noticed that this result is different from McCall and Hahn's finding that the group velocity varies with the pulse width. In a recent paper, Brains, Martin, and Birmam [12,13] have found that the group velocity of SIT without using SVEA in a linear host medium has a discrete set of values. In our system, only a single velocity is allowed. In Sec. IV we will consider three special cases used to solve the coupled nonlinear algebraic equations and study the characteristics of the SIT-NLS solitons.

As for the expressions for u , v , and w , we substitute Eq. (32) into Eqs. (21), (16b), and (23) and obtain

$$u(\Delta\omega, z, \tau) = f(\Delta\omega)(\Delta\omega - \Omega) \frac{2N_0\mu}{T_c(T_c^{-2} + \Omega^2)} \operatorname{sech} \left[\frac{\tau}{T_c} \right], \quad (34a)$$

$$v(\Delta\omega, z, \tau) = -f(\Delta\omega) \frac{2N_0\mu}{T_c^2(T_c^{-2} + \Omega^2)} \times \operatorname{sech} \left[\frac{\tau}{T_c} \right] \tanh \left[\frac{\tau}{T_c} \right], \quad (34b)$$

$$w(\Delta\omega, z, \tau) = N_0\mu - f(\Delta\omega) \frac{2N_0\mu}{T_c^2(T_c^{-2} + \Omega^2)} \operatorname{sech}^2 \left[\frac{\tau}{T_c} \right], \quad (34c)$$

where

$$f(\Delta\omega) = \frac{1}{1 - 2\Omega(T_c^{-2} + \Omega^2)^{-1} \Delta\omega + (T_c^{-2} + \Omega^2)^{-1} \Delta\omega^2}. \quad (34d)$$

Corresponding to the steady-state solution given by Eq. (32), with Eqs. (34), the macroscopic polarization and population difference can be written as

$$P_r(z, \tau) = \frac{N_0\mu^2}{(T_c^{-2} + \Omega^2)\hbar} \left[(I_2 - 2\Omega I_1) E(z, \tau) + i I_1 \frac{\partial E(z, \tau)}{\partial \tau} \right], \quad (35a)$$

$$W(z, \tau) = N_0\mu - \frac{2N_0\mu I_1}{T_c^2(T_c^{-2} + \Omega^2)} \operatorname{sech} \left[\frac{\tau}{T_c} \right], \quad (35b)$$

where I_1 and I_2 are the integration constants given by

Eqs. (27d) and (27e), respectively. The constants will be calculated in the following section.

IV. APPROXIMATE SOLUTIONS FOR CHARACTERISTIC CONSTANTS

In this section we will solve the characteristic time scale T_c , group velocity v_g , and propagation constant $k_0 + \Delta k_0$ from the three coupled nonlinear algebraic equations, Eqs. (30b), (30d), and (31) for three special cases. For the parameters of the EDF, we take the following values [8]: $\lambda_0 = 1.53 \mu\text{m}$, $k_0'' = -25 \text{ psec}^2/\text{km}$, $n_2 = 1.2 \times 10^{-22} \text{ m}^2/\text{V}^2$, $\mu = 1.4 \times 10^{-32} \text{ C m}$, effective area $A_{\text{eff}} = 80 \mu\text{m}^2$, and $T_1 = 10 \text{ msec}$, $T_2 = 10 \text{ nsec}$ at 4.2 K. In the fiber, because the value of k_0' is very close to $n(\omega_0)/c$, we taken $k_0' = n(\omega_0)/c$ and $n(\omega_0) = 1.46$.

In Eq. (32), the amplitude $2\hbar/\mu T_c$ corresponds to a 2π SIT soliton, where 2π is the normalized area of the SIT soliton [14]. For the fundamental NLS soliton without the resonant atoms, the amplitude is

$$I_1 = \frac{1}{c_2(c_2^{-1} - \Omega^2)^{1/2}} \frac{\Omega^2 \Delta\omega_a + \left[(c_2^{-1} - \Omega^2)^{1/2} - \frac{\Delta\omega_a}{2} \right] \left[c_2^{-1} - \left[\frac{\Delta\omega_a}{2} \right]^2 \right]}{[c_2^{-1} - (\Delta\omega_a/2)^2]^2 + \Omega^2 \Delta\omega_a^2}, \quad (38a)$$

$$I_2 = \frac{-\Omega \frac{\Delta\omega_a}{2}}{c_2(c_2^{-1} - \Omega^2)^{1/2}} \frac{\Delta\omega_a (c_2^{-1} - \Omega^2)^{1/2} - c_2^{-1} - \left[\frac{\Delta\omega_a}{2} \right]^2}{[c_2^{-1} - (\Delta\omega_a/2)^2]^2 + \Omega^2 \Delta\omega_a^2}. \quad (38b)$$

From Eq. (32), it is seen that the pulse energy is inversely proportional to the pulse width. When the pulse width is large enough, the pulse energy will be too small to induce large phase change. Therefore, we can assume the change of the propagation constant $\Delta k_0 \ll k_0$ to simplify Eq. (27a) as $\alpha = 2k_0 \Delta k_0$. This assumption will be justified in the following.

Case (i): $\Delta\omega_a \rightarrow 0$ and $\Omega = 0$

In this case, the resonant medium is homogeneous broadening, i.e., all the doped atoms have the same resonant frequency ω_r , and the carrier frequency ω_0 of the soliton coincides with the resonant frequency, i.e. $\omega_r = \omega_0$. From Eqs. (38), $I_1 = 1$ and $I_2 = 0$. Solving Eqs. (30b), (30d), and (31), we have

$$T_c = T_0 \left[1 + \left[1 + \frac{\omega_0}{k_0'} (k_0'' + k_t'') \right]^{1/2} \right]^{1/2}, \quad (39a)$$

$$\Delta k_0' = k_0' \left[\frac{T_c}{T_0} \right]^2, \quad (39b)$$

$$\Delta k_0 = \frac{1}{2} k_t'' T_c^{-2}, \quad (39c)$$

where

$$T_0 = \left[\frac{2\hbar k_0^2}{\mu_0 \omega_0^3 \mu^2 N_0} \right]^{1/2}, \quad (39d)$$

$(-2n_0 k_0''/n_2 k_0)^{1/2} T_c$ [15]. The required dispersion $k_{\text{SIT-NLS}}''$ for the coexistence of the two solitons can be obtained by equalizing these two amplitudes [8], i.e.,

$$k_{\text{SIT-NLS}}'' = -\frac{2\hbar^2 n_2 k_0}{\mu^2 n_0}. \quad (36)$$

With the numerical parameters given above, $k_{\text{SIT-NLS}}'' = -5.6 \times 10^7 \text{ psec}^2/\text{km}$, which is six orders of magnitude larger than the typical value. In the following, we will show that the SIT can induce such a high negative dispersion.

For the following discussions, the inhomogeneous broadening line shape is assumed to be Lorentzian

$$g(\Delta\omega) = \frac{\Delta\omega_a}{2\pi} \frac{1}{(\Delta\omega_a/2)^2 + \Delta\omega^2}, \quad (37)$$

where $\Delta\omega_a$ is the spectral width (FWHM) of $g(\Delta\omega)$. Given Eq. (37), the constants I_1 and I_2 defined in Eqs. (27d) and (27e) become

$$k_t'' = \frac{2\hbar^2 n_2 k_0}{\mu^2 n_0}. \quad (39e)$$

It is noticed that $k_t'' = |k_{\text{SIT-NLS}}''|$. Because $\Delta k_0'$ is positive, the group velocity of the soliton v_g is reduced from $1/k_0'$ to $1/(k_0' + \Delta k_0')$. To simplify the expressions, we have assumed $k_0' \cong k_0/\omega_0$ in the expressions of T_c , Δk_0 , and $\Delta k_0'$ given above. From Eq. (32), the peak power of the SIT-NLS soliton is

$$P_0 = \frac{1}{2} n_0 \sqrt{\epsilon_0/\mu_0} \left[\frac{2\hbar}{\mu T_c} \right]^2 A_{\text{eff}}. \quad (40)$$

With the numerical parameters given above, we have

$$T_0 = 4.06 N_0^{-1/2} \text{ psec}, \quad (41a)$$

$$k_t'' = 5.6 \times 10^7 \text{ psec}^2/\text{km}, \quad (41b)$$

where N_0 is in units of 10^{25} m^{-3} , and

$$T_w = 0.785 N_0^{-1/2} \text{ nsec}, \quad (42a)$$

$$\Delta k_0' = 120 k_0', \quad (42b)$$

$$\Delta k_0 = 0.141 N_0 \text{ m}^{-1}, \quad (42c)$$

$$P_0 = 178 N_0 \text{ W}. \quad (42d)$$

Because $k_0 = 6.0 \times 10^6 \text{ m}^{-1}$, the value of the given Δk_0 is of the order of a few m^{-1} and the assumption $\Delta k_0 \ll k_0$

is justified in this case. From Eq. (42a), the pulse width is inversely proportional to the square root of the doping density because lower doping density corresponds to less interaction between the field and the resonant atoms and, therefore, longer pulse width is required to increase the interaction. It is noticed that, in Eq. (42b), $\Delta k'_0$ is independent of N_0 and the group velocity $v_g = 1/121k'_0$. Because it requires $T_w \ll T_1, T_2$, we take $T_w = 0.5$ nsec ($N_0 = 2.46 \times 10^{25} \text{ m}^{-3}$) and the required peak power is 437 W. As mentioned above, in Ref. [8], a soliton of such a high peak power requires the dispersion k''_0 of six orders of magnitude larger than that of a typical silica fiber. Here, we will show such a high dispersion can be induced by the SIT in the resonant atoms. In this case, from Eqs. (35a) and (39), the macroscopic polarization P_r corresponds to the soliton solution given by Eq. (32) can be written in terms of the electric field envelope E as

$$P_r = \frac{2ik_0\Delta k'_0}{\mu_0\omega_0^2} \frac{\partial E}{\partial \tau}. \quad (43)$$

Substituting Eq. (43) into Eq. (8), we see that the term $-2ik_0\Delta k'_0\partial E/\partial \tau$ in Eq. (8) is canceled out by the term with P_r because the pulse is in the moving frame with its group velocity and the term with $\partial P_r/\partial \tau$ in Eq. (8) introduces an additional dispersion $4k'_0\Delta k'_0/k_0$. Including the dispersion due to $\partial P_r/\partial \tau$, the total dispersion caused by the resonant atoms are

$$k''_r = -(\Delta k'^2_0 - 2k'_0\Delta k'_0)k_0^{-1}. \quad (44)$$

Because $\Delta k'_0$ is positive, k''_r is negative for $\Delta k'_0 > 2k'_0$. Substituting the solution of $\Delta k'_0$ given by Eq. (39b) into Eq. (44), we have the total dispersion $k''_0 + k''_r = -k''_t$, which is exactly the required dispersion $k''_{\text{SIT-NLS}}$ given by Eq. (36) and is independent of k''_0 . It is clear that, for the coexistence of the SIT soliton and the NLS soliton, the value of the change of the reciprocal group velocity $\Delta k'_0$

due to the SIT is self-adjusted such that the value of the total dispersion is given by Eq. (36), where both the SIT and the NLS solitons are fundamental solitons. It is known that the bright NLS soliton cannot exist in the positive dispersion regime where $k''_0 > 0$ [3]. For the SIT-NLS soliton, it can exist in the positive dispersion regime because the total dispersion is independent of k''_0 .

From Eq. (35b) with $I_1 = 1$ and $\Omega = 0$, the macroscopic population difference W corresponds to the soliton solution given by Eq. (32) can be written as

$$W(z, \tau) = N_0\mu - 2N_0\mu \text{sech}^2 \left[\frac{\tau}{T_c} \right]. \quad (45)$$

At the peak intensity of the pulse at $\tau = 0$, $W = -N_0\mu$, i.e., the resonant atoms are all pumped to the upper state.

It is noticed that, from Eq. (44), if there is no Kerr effect and dispersion in the host medium, i.e., $n_2 = 0$ and $k''_0 = 0$, the soliton solution exists when $k''_r = 0$ or $\Delta k'_0 = 2k'_0$. In such a case, $v_g = 1/3k'_0$, which is called the "preferred velocity in SIT" [16]. Therefore, the change of the velocity is the result of the combined effect of the host dispersion, Kerr effect and the resonant medium. In Refs. [11] and [17], it is found that a SIT pulse is possible in a nonlinear Kerr host, but the pulse is necessarily chirped. From Eqs. (32) and (39), we know the solution is unchirped. The difference arises because the dispersion terms are neglected in Refs. [11] and [17].

Case (ii): $\Delta\omega_a \rightarrow 0$ and $|\Omega| \ll \omega_r$

In this case, the resonant medium is again homogeneous broadening, but the carrier frequency of the soliton is $\omega_0 + \Omega$ and differs from the resonant frequency ω_r , where $\omega_0 = \omega_r$. To simplify the problem, we assume the absolute value of the detuned frequency $|\Omega| \ll \omega_r$. Also, from Eqs. (38a) and (38b), $I_1 = 1$ and $I_2 = 0$. The approximate solution is

$$T_c = \left\{ \frac{(1+r)^{-1}T_0^{-2}}{1 + \{1 + (1+r)^{-2}[(\omega_0/k'_0)(k''_0 + k''_t) - r^2 + 2r]\}^{1/2}} - \Omega^2 \right\}^{-1/2}, \quad (46a)$$

$$\Delta k'_0 = k'_0 \left[\frac{T_c}{T_0} \right]^2 \frac{1}{1 + (\Omega T_c)^2} - k''_t \Omega, \quad (46b)$$

$$\Delta k_0 = \frac{k''_t}{2T_c^2} [1 + (\Omega T_c)^2] - 2k'_0 \left[\frac{T_c}{T_0} \right]^2 \frac{\Omega}{1 + (\Omega T_c)^2}, \quad (46c)$$

where $r = k''_t \Omega / k'_0$.

With the numerical parameters of the EDF given above, Fig. 1 shows the pulse width $T_w (= 1.763T_c)$ versus the detuned frequency for various doping density N_0 . For a given detuned frequency, as the doping density N_0 increases, the pulse width T_w decreases as in case (i). For the same pulse width, the doping density increases with the detuned frequency because the frequency detuning reduces the interaction between the field and the resonant atoms and higher doping density is required to in-

crease the interaction. From the figure, it is seen that there exist cutoff detuned frequencies because the value of the square root in Eq. (46a) must be positive to make the pulse width real. Because the order of the detuned frequency shown is less than 1 GHz and $k''_t/k'_0 = 1.15 \times 10^{-11}$ sec, the ratio $|r| \ll 1$ and Eq. (46a) reduces to

$$T_c \cong (T_{ci}^{-2} - \Omega^2)^{-1/2}, \quad (47a)$$

where T_{ci} is the T_c defined by Eq. (39a), which is

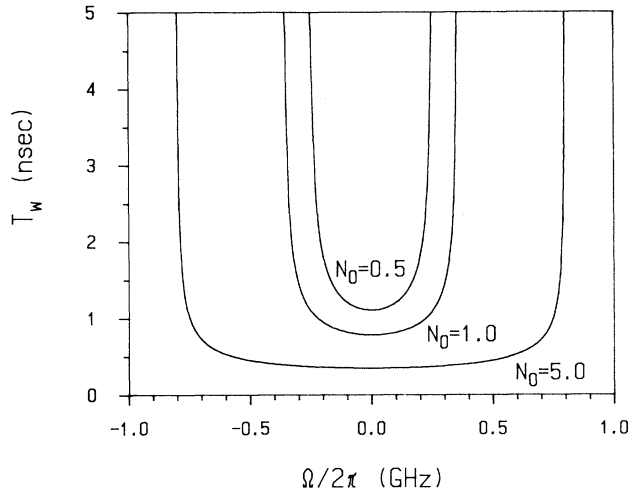


FIG. 1. The pulse width $T_w = 1.763T_c$ vs the detuned frequency $\Omega/2\pi$ for various doping density N_0 (in unit of 10^{25} m^{-3}), where T_c is the characteristic time scale given by Eq. (44a).

the characteristic time scale for the nondetuned case ($\Omega=0$). From Eq. (47a), the cutoff frequency $f_c \cong \pm(T_{ci}^{-2} - T_c^{-2})^{1/2}/2\pi$. The cutoff frequencies are 0.253, 0.358, and 0.8 GHz for the doping density $N_0 = 0.5 \times 10^{25}$, 1×10^{25} , and 5×10^{25} m^{-3} , respectively. From Eq. (46a) or (47a), at the cutoff frequency, the pulse width becomes infinite. However, the pulse width is valid only under the assumption $T_w \ll T_2$, where $T_2 = 10$ nsec at 4.2 K. For $T_w < 1$ nsec, the corresponding frequencies $|\Omega|/2\pi$ should be less than 0.23 and 0.75 GHz for the doping densities $N_0 = 1 \times 10^{25}$ and 5×10^{25} m^{-3} , respectively. For $N_0 = 0.5 \times 10^{25}$ m^{-3} , the minimum pulse width shown in Fig. 1 is 1.11 nsec. Figures 2 and 3 show the corresponding $\Delta k'_0$ and Δk_0 in Fig. 1, respectively. In Figs. 2 and 3, the three lines corresponding to $N_0 = 0.5 \times 10^{25}$, 1×10^{25} , and 5×10^{25} m^{-3} overlap. Therefore, $\Delta k'_0$ and Δk_0 are almost independent of N_0 in these cases. From Fig. 3, $|\Delta k_0|$ increases with $|\Omega|$ and the assumption $|\Delta k_0| \ll k_0$ is valid in these cases. From Fig. 2, $\Delta k'_0$ slightly increases with the detuned frequency Ω . It is noticed that the values of $\Delta k'_0$ are almost the same as the nondetuned case. For $|r| \ll 1$, Eqs. (46b) and (46c) can be reduced to

$$\Delta k'_0 = \Delta k'_{0i}, \quad (47b)$$

$$\Delta k_0 = \Delta k_{0i} - 2\Omega \Delta k'_{0i}, \quad (47c)$$

where $\Delta k'_{0i}$ and Δk_{0i} are the $\Delta k'_0$ and Δk_0 defined by Eqs. (39b) and (39c), respectively, which are the $\Delta k'_0$ and Δk_0 for the nondetuned case ($\Omega=0$). The approximate equations, Eqs. (47a)–(47c), agree very well with the data shown in Figs. 1–3. It is noticed that Eq. (47b) shows $\Delta k'_0$ is independent of the detuned frequency.

Corresponding to the steady-state solution given by Eq. (32), from Eqs. (35) and (46), the macroscopic polariza-

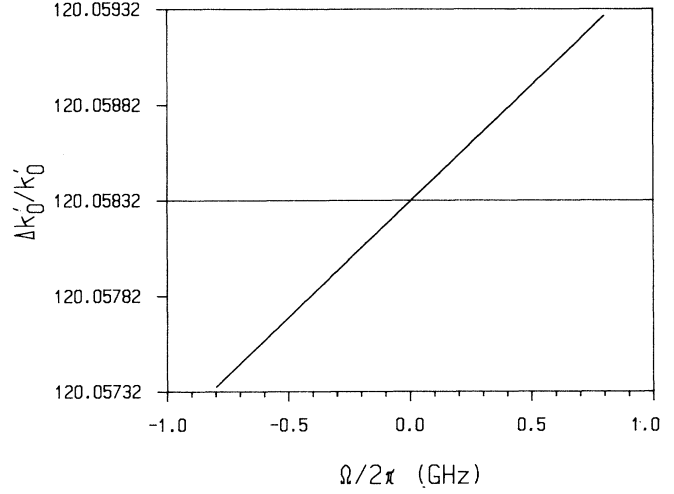


FIG. 2. The corresponding change of the reciprocal group velocity $\Delta k'_0$ for the cases shown in Fig. 1, where $\Delta k'_0$ is given by Eq. (44b) and is shown by the ratio $\Delta k'_0/k'_0$. $k'_0 = 4.868 \times 10^{-9}$ sec/m is the reciprocal group velocity without resonant atoms and Kerr effect. The three lines corresponding to $N_0 = 0.5 \times 10^{25}$, 1×10^{25} , and 5×10^{25} m^{-3} overlap.

tion and population difference can be written as

$$P_r = \frac{2k_0}{\mu_0 \omega_0^2} (\Delta k'_0 + k'_i \Omega) \left[-2\Omega E + i \frac{\partial E}{\partial \tau} \right], \quad (48a)$$

$$W = N_0 \mu - \frac{2N_0 \mu}{1 + (\Omega T_c)^2} \text{sech}^2 \left[\frac{\tau}{T_c} \right]. \quad (48b)$$

Substituting Eq. (48a) into Eq. (8), we have that, for $|\Omega| \ll \omega_0$ and $|r| \ll 1$, (a) the term $-2ik_0 \Delta k'_0 \partial E / \partial \tau$ in Eq. (8) is canceled out by $\mu_0 \omega_0^2 P_4$; (b) the total induced

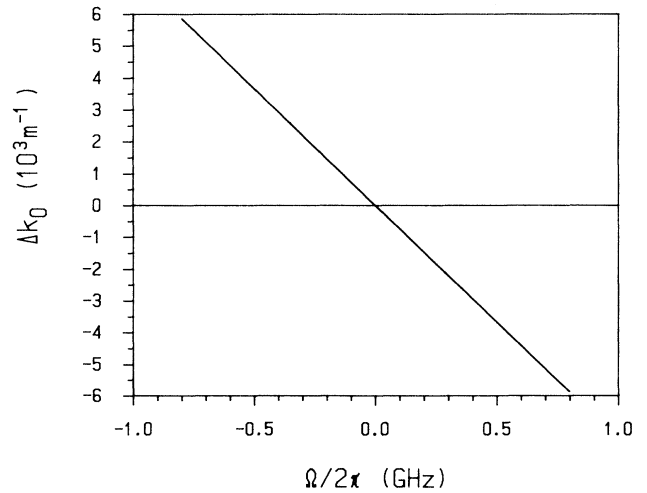


FIG. 3. The change of the propagation constant Δk_0 for the cases shown in Fig. 1, where Δk_0 is given by Eq. (43c). The three lines corresponding to $N_0 = 0.5 \times 10^{25}$, 1×10^{25} , and 5×10^{25} m^{-3} overlap.

dispersion is the same as Eq. (44) and the required dispersion for the coexistence of the SIT and NLS soliton is also contributed from the host dispersion and the total induced dispersion; (c) the macroscopic polarization additionally introduces a change of the propagation constant $-2\Omega(\Delta k'_0 + k''_0)$, which is equal to the second term in Eq. (46c) as can be seen from Eq. (46b). It is noticed that, in case (i), from Eqs. (8) and (43), no approximations are required to arrive at the results (a) the cancellation of the term $-2ik_0\Delta k'_0\partial E/\partial\tau$ in Eq. (8), (b) the total dispersion to be $-k''_0$, while in this case, the approximations $|\Omega| \ll \omega_0$ and $|r| \ll 1$ are required. This is because, in this case, Eqs. (46) and (48a) are all approximate results. From Eq. (48b), as the frequency is detuned, at the peak intensity of the pulse at $\tau=0$, the resonant atoms are not all pumped to the upper state.

Case (iii): $\Delta\omega_a \gg T_c^{-1}$ and $\Omega=0$

In this case, we consider the SIT-NLS soliton solution in an inhomogeneous broadening resonant medium. Because the expressions of the I_1 and I_2 defined in Eqs. (38a) and (38b) is not so simple as the homogeneous broadening case, we assume the spectral width of the pulse is much less than the spectral width of the inhomogeneous broadening line shape and the central resonant frequency coincides the carrier frequency. The large inhomogeneous broadening linewidth assumption is true for the EDF. With these assumptions, we have the simple expressions $I_1=2/(c_2^{1/2}\Delta\omega_a)$ and $I_2=0$. The approximate solution is

$$T_c = \frac{\Delta\omega_a}{2} T_{ci}^2, \quad (49a)$$

$$\Delta k'_0 = \Delta k'_{0i}, \quad (49b)$$

$$\Delta k_0 = \Delta k_{0i}, \quad (49c)$$

where T_{ci} , $\Delta k'_{0i}$, and Δk_{0i} are the T_c , $\Delta k'_0$, and Δk_0 defined by Eqs. (39a), (39b), and (39c), respectively, which are the corresponding values for the nondetuned and homogeneous broadening case. It is noticed that $\Delta k'_0$ and Δk_0 are the same as the homogeneous broadening case. To compare with case (i), the pulse width given by Eq. (49a) increases by a factor of $\Delta\omega_a T_{ci}/2$ because the inhomogeneous broadening reduces the interaction between the field and the resonant atoms, and a longer pulse width is required to increase the interaction.

Corresponding to the steady-state solution given by Eq. (32), from Eqs. (35) and (49), the macroscopic polarization and population difference can be written in terms of the electric field E as

$$P_r = \frac{2ik_0\Delta k'_0}{\mu_0\omega_0^2} \frac{\partial E}{\partial\tau}, \quad (50a)$$

$$W = N_0\mu - 2N_0\mu \left[\frac{2}{T_c\Delta\omega_a} \right] \text{sech}^2 \left[\frac{\tau}{T_c} \right]. \quad (50b)$$

Because Eq. (50a) and $\Delta k'_0$ is the same as case (i), the required dispersion for the SIT soliton and NLS soliton to

coexist is also contributed from the host dispersion and the induced dispersion by the SIT. From Eq. (50b), as the inhomogeneous-broadening linewidth $\Delta\omega_a$ increases, the population of the pumped resonant atoms decreases. In fact, because we have assumed $\Delta\omega_a \gg T_c^{-1}$, most of the atoms are in the ground state even at the peak intensity of the pulse.

With the numerical parameters of the EDF given above, we have the pulse width

$$T_w = 1.1\Delta f_a N_0^{-1} \text{ nsec}, \quad (51)$$

where the spectral width of the inhomogeneous-broadening line shape is expressed as $\Delta f_a = \Delta\omega_a/2\pi$ and Δf_a is in units of GHz. T_w in Eq. (51) is proportional to Δf_a and inversely proportional to N_0 . For the condition $T_w \ll T_2 = 10$ nsec, from Eq. (51), we have $\Delta f_a N_0^{-1} \ll 9.1$. For the erbium-doped fiber, the inhomogeneous broadening linewidth at $\lambda = 1.53 \mu\text{m}$ is $\Delta f_a = 1472$ GHz (11.5 nm) [18] and we have $N_0 \gg 1.62 \times 10^{27} \text{ m}^{-3}$. Because 1 ppm (part per million) corresponds to $8 \times 10^{21} \text{ m}^{-3}$ [8], $N_0 = 1.62 \times 10^{27} \text{ m}^{-3}$ corresponds to about 2×10^5 ppm, which is impractical. For the typical value 1000 ppm, $N_0 = 8 \times 10^{24} \text{ m}^{-3}$ and, from Eq. (51), $T_w = 2.02 \mu\text{sec}$. Therefore, in the practical erbium-doped fiber, the solution of the SIT-NLS soliton is not found because its pulse width is too long to compare with the relaxation time of the polarization T_2 of the resonant atoms and the pulse energy will be incoherently absorbed by the medium.

V. CONCLUSIONS

The SIT-NLS soliton in the composite medium of Kerr host doped with resonant atoms has been studied beyond the SVEA. The analytic solution is obtained. There exists a suitable hyperbolic-secant soliton whose power, pulse width, group velocity, and propagation constant are uniquely determined for given parameters of the medium. The required power of the soliton only depends on the dipole moment of the resonant atom and the pulse width. The other characteristic constants pulse width, group velocity, and propagation constant are determined by three coupled nonlinear algebraic equations. Three special cases of these equations are considered: (i) homogeneous broadening without frequency detuning, (ii) homogeneous broadening with frequency detuning, and (iii) inhomogeneous broadening without frequency detuning. Without frequency detuning, the reduction of the group velocity is the same for both the homogeneous- and inhomogeneous-broadening cases. As the carrier frequency is detuned from the resonant frequency, the reduction of the group velocity is only slightly changed for the homogeneous-broadening case. Because the power of the SIT-NLS soliton is very high, extremely large negative dispersion is required to be compensated by the Kerr effect. It is found that there is a dispersion induced by the SIT, which is not predicted by the theory with SVEA. The amount of the induced dispersion is self-adjusted such that the total dispersion, which includes the dispersion of the nonresonant host medium and the induced dispersion, is equal to the required

dispersion for the coexistence of the SIT and the NLS soliton. For the typical parameters of the EDF, the SIT-NLS soliton has not been found because its inhomogeneous-broadening linewidth is too large.

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