Temperature Prediction Using Fuzzy Time Series

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Abstract—A drawback of traditional forecasting methods is that they can not deal with forecasting problems in which the historical data are represented by linguistic values. Using fuzzy time series to deal with forecasting problems can overcome this drawback. In this paper, we propose a new fuzzy time series model called the two-factors time-variant fuzzy time series model to deal with forecasting problems. Based on the proposed model, we develop two algorithms for temperature prediction. Both algorithms have the advantage of obtaining good forecasting results.

Index Terms—Main-factor fuzzy time series, second-factor fuzzy time series, temperature prediction, time-invariant fuzzy time series, time-variant fuzzy time series.

I. INTRODUCTION

I T is obvious that forecasting activities play an important role in our daily life. Every day the weather forecast tells us what the weather will be like tomorrow. We can prevent huge damage by forecasting the coming of storms or typhoons. We usually forecast many things concerned with our daily life, such as economy, stock market, population growth, weather, etc. To make a forecast with 100% accuracy may be impossible, but we can do our best to reduce the forecasting errors or increase the speed of the forecasting process. To solve the forecasting problems, many researchers have proposed many different methods or models. A drawback of traditional forecasting methods is that they can not deal with forecasting problems in which the historical data are linguistic values. In order to overcome the drawback of the traditional forecasting methods, in [17], Song and Chissom proposed the concepts of fuzzy time series to deal with the forecasting problem in which the historical data are linguistic values. In [18] and [19], they proposed two fuzzy time series models to deal with the forecasting problems for forecasting enrollments of the University of Alabama.

In [18], Song *et al.* use the following method to forecast enrollments of the University of Alabama:

$$F(t) = F(t-1) \circ R \tag{1}$$

where

$F(t-1)$ fuzzified enrollment of year $t-1$;
$F(t)$ forecasted enrollment of year t represented by fuzzy sets;

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R union of fuzzy relations;
"∘"   Max-Min composition operator.

It takes a lot of time to compute the union of fuzzy relations R, but it is computed only once. In [19], Song *et al.* proposed the time-variant fuzzy time series model. The model also uses (1), but the calculation of the union of fuzzy relations R is different from the method in [18]. The method presented in [19] takes less time to compute the union of fuzzy relations R, but it must recalculate R when we want to forecast the enrollments of different years. The time complexities of the methods in [18] and [19] are all $O(kn^2)$, where k is the number of fuzzy logical relationships, and n is the number of elements in the universe of discourse. Although some researchers have proposed new fuzzy time series models to improve Song's model, such as [4], [5], [7], and [21], all of these models can only deal with one-factor forecasting problems. In [4], we presented a time-invariant fuzzy time series method to forecast university enrollments. In [5], Day proposed a time-variant fuzzy time series model which used expert opinions and genetic algorithms. In [7], we presented a one-factor time-variant fuzzy time series model and proposed an algorithm for handling forecasting problems. In [21], Sullivan *et al.* proposed the Markov model which used linguistic labels with probability distributions.

In this paper, we propose a new fuzzy time series model which is called the "two-factors time-variant fuzzy time series model" to deal with forecasting problems. Based on the proposed model, we develop two algorithms for temperature prediction. The proposed algorithms have the advantage that they can obtain good forecasting results.

The rest of this paper is organized as follows. In Section II, we briefly review some basic concepts of fuzzy set theory [8]–[11], [28] and fuzzy time series [17]–[19]. In Section III, we propose the two-factors time-variant fuzzy time series model and propose two efficient algorithms for temperature prediction. The conclusions are discussed in Section IV.

II. FUZZY SET THEORY AND FUZZY TIME SERIES

In the following, we briefly review some basic concepts of fuzzy set theory from [8]–[11], and [28]. A fuzzy set is a class with fuzzy boundaries. A fuzzy set can be characterized by a membership function defined as follows.

Definition 2.1: Let U be the universe of discourse. A fuzzy subset A on the universe of discourse U can be defined as follows:

$$A = \{(u_i, \mu_A(u_i)) \,|\, u_i \in U\} \tag{2}$$

where μ_A is the membership function of A, $\mu_A : U \rightarrow [0, 1]$, and $\mu_A(u_i)$ is the degree of membership of the element u_i in the fuzzy set A.

Definition 2.2: Let U be the universe of discourse, $U = \{u_1, u_2, \ldots, u_n\}$, and U be a finite set. A fuzzy set A can be expressed as follows:

$$A = \sum_{i=1}^{n} \mu_A(u_i)/u_i$$
$$= \mu_A(u_1)/u_1 + \mu_A(u_2)/u_2 + \cdots + \mu_A(u_n)/u_n \quad (3)$$

where the symbol " $+$ " means the operation of union instead of the operation of summation, and the symbol "$/$" means the seperator rather than the commonly used algebraic symbol of division.

Definition 2.3: Let U be the universe of discourse, where U is an infinite set. A fuzzy set A of U can be expressed as follows:

$$A = \int_U \mu_A(u_i)/u_i, \quad \forall u_i \in U. \quad (4)$$

Definition 2.4: Let A be a fuzzy set of the universe of discourse U. The α-significance level [27] A_α of the fuzzy set A is defined as follows:

$$\mu_{A_\alpha}(u) = \begin{cases} \mu_A(u), & \text{if } \mu_A(u) \geq \alpha \\ 0, & \text{otherwise} \end{cases} \quad (5)$$

where $\alpha \in [0, 1]$.

The concept of fuzzy time series was proposed by Song *et al.* [17]. In a traditional time series, the values of observations of a special dynamic process are represented by crisp numerical values. However, in a fuzzy time series, the values of observations of a special dynamic process are represented by linguistic values. In [18] and [19], Song *et al.* used the fuzzy time series to forecast the enrollments of the University of Alabama, and they got good forecasting results. In the following, we briefly review the concept of fuzzy time series from [17].

Definition 2.5: Assume that $Y(t)$ $(t = \ldots, 0, 1, 2, \ldots)$ is a subset of real numbers and is the universe of discourse, and assume that fuzzy sets $\mu_i(t)$ $(i = 1, 2, \ldots)$ are defined on $Y(t)$. Let $F(t)$ be a collection of $\mu_i(t)$ $(i = 1, 2, \ldots)$. Then, $F(t)$ is called a fuzzy time series of $Y(t)$ $(t = \ldots, 0, 1, 2, \ldots)$.

We can see that $F(t)$ is a function of time t, and $\mu_i(t)$ are linguistic values of $F(t)$, where $\mu_i(t)$ $(i = 1, 2, \ldots)$ are represented by fuzzy sets. In [17], Song *et al.* divided the fuzzy time series into two categories which are the time-invariant fuzzy-time series and the time-variant fuzzy time series defined as follows.

Definition 2.6: If $F(t)$ is caused by $F(t-1)$ denoted by $F(t) \to F(t-1)$, then this relationship can be represented by

$$F(t) = F(t-1) \circ R(t, t-1) \quad (6)$$

where $R(t, t-1)$ is a fuzzy relationship between $F(t)$ and $F(t-1)$ and is called the first-order model of $F(t)$.

Definition 2.7: Assume that $F(t)$ is a fuzzy time series, and $R(t, t-1)$ is a first-order model of $F(t)$. If $R(t, t-1) = R(t-1, t-2)$ for any time t, then $F(t)$ is called the time-invariant fuzzy time series. If $R(t, t-1)$ is dependent on time t, that is $R(t, t-1)$ may be different from $R(t-1, t-2)$ for any t, then $F(t)$ is called the time-variant fuzzy time series.

In [18], Song and Chissom proposed the time-invariant fuzzy time series model. In [19], Song *et al.* proposed the time-variant fuzzy time series model. Both of these two models only used one factor to forecast the enrollments of the University of Alabama. In fact, an event may be influenced by many factors. In the following, we propose a new fuzzy time series called the two-factors fuzzy time series.

Definition 2.8: Assume that fuzzy time series $F(t)$ and $G(t)$ are the factors of the forecasting problems. If we only use $F(t)$ to solve the forecasting problems, then it is called a one-factor fuzzy time series. If we use both $F(t)$ and $G(t)$ to solve the forecasting problems, then it is called a two-factors fuzzy time series.

III. A NEW FUZZY TIME SERIES MODEL FOR TEMPERATURE PREDICTION

In [7], we have presented a one-factor time-variant fuzzy time series model and proposed an algorithm (called Algorithm-A) for handling the forecasting problems. However, in the real-world, an event can be affected by many factors. For example, the temperature can be affected by the wind, the sunshine duration, the cloud density, the atmospheric pressure, $\ldots$, etc. If we only use one factor of them to forecast the temperature, the forecasting results may lack accuracy. We can get better forecasting results if we consider more factors for temperature prediction. In [4], [18], [19], and [21], the researchers only use the one-factor fuzzy time series model to deal with the forecasting problems. In this section, we propose a new forecasting model which is a two-factors time-variant fuzzy time series model. Based on the proposed model, we develop two algorithms which use two factors (i.e., the daily average temperature and the daily cloud density) for temperature prediction.

Definition 3.1: Assume that $Y(t)$ $(t = \ldots, 0, 1, 2, \ldots)$ is a subset of R and is the universe of discourse. Let $F(t)$ and $G(t)$ $(t = 1, 2, \ldots)$ be two fuzzy time series on $Y(t)$, where $F(t) = \{\mu_{11}(t), \mu_{12}(t), \ldots, \mu_{1n}(t)\}$, $G(t) = \{\mu_{21}(t), \mu_{22}(t), \ldots, \mu_{2n}(t)\}$, $\mu_{1i}(t)$ is a fuzzy set on $Y(t)$, $\mu_{2i}(t)$ is a fuzzy set on $Y(t)$, and $1 \leq i \leq n$. Assume that we want to forecast $F(t)$ and use $G(t)$ to aid the forecasting of $F(t)$, then $F(t)$ and $G(t)$ are called the main-factor fuzzy time series and the second-factor fuzzy time series of the two-factors time-variant fuzzy time series model, respectively.

For example, in the proposed two-factors time-variant fuzzy time series model for temperature prediction, "the daily average temperature" and "the daily cloud density" are called the main-factor and the second-factor of the two-factors time-variant fuzzy time series model, respectively.

The fuzzified variation $f(t)$ of the main-factor fuzzy time series $F(t)$ between time t and time $t-1$ can be described as follows. Assume that the universe of discourse U has been

divided into m intervals (i.e., $u_1, u_2, \ldots,$ and u_m). Furthermore, assume that there are k linguistic terms (i.e., $A_1, A_2, \ldots,$ and A_k) described by fuzzy sets shown as follows:

$$A_1 = \mu_{A1}(u_1)/u_1 + \mu_{A1}(u_2)/u_2 + \cdots + \mu_{A1}(u_m)/u_m$$
$$A_2 = \mu_{A2}(u_1)/u_1 + \mu_{A2}(u_2)/u_2 + \cdots + \mu_{A2}(u_m)/u_m.$$
$$\vdots$$
$$A_k = \mu_{Ak}(u_1)/u_1 + \mu_{Ak}(u_2)/u_2 + \cdots + \mu_{Ak}(u_m)/u_m \tag{7}$$

where the maximum membership value of A_i occurs at u_i and $1 \le i \le k$. If the difference of the historical data of the main-factor fuzzy time series $F(t)$ between time t and time $t-1$ is x and $x \in u_i$, then the fuzzified variation $f(t)$ of the historical data of the main-factor fuzzy time series $F(t)$ between time t and time $t-1$ is A_i, where $1 \le i \le k$. That is,

$$f(t) = [\mu_{Ai}(u_1) \quad \mu_{Ai}(u_2) \quad \cdots \quad \mu_{Ai}(u_m)].$$

The fuzzified data $g(t)$ of the second-factor fuzzy times series $G(t)$ at time t can be described as follows. Assume that there are k linguistic terms (i.e., $B_1, B_2, \ldots,$ and B_k) represented by fuzzy sets to describe the fuzzified historical data of the second-factor "the daily cloud density", where the daily cloud density ranges from 0% to 100%

$$B_1 = \mu_{B1}(u_1)/u_1 + \mu_{B1}(u_2)/u_2 + \cdots + \mu_{B1}(u_m)/u_m$$
$$B_2 = \mu_{B2}(u_1)/u_1 + \mu_{B2}(u_2)/u_2 + \cdots + \mu_{B2}(u_m)/u_m$$
$$\vdots$$
$$B_k = \mu_{Bk}(u_1)/u_1 + \mu_{Bk}(u_2)/u_2 + \cdots + \mu_{Bk}(u_m)/u_m. \tag{8}$$

If the data y of the second-factor fuzzy time series $\in (90, 100]$, then the fuzzified data of y is B_1. If $y \in [75, 90)$, then the fuzzified data of y is B_2. If $y \in [60, 75)$, then the fuzzified data of y is B_3. If $y \in [45, 60)$, then the fuzzified data of y is B_4. If $y \in [30, 45)$, then the fuzzified data of y is B_5. If $y \in [15, 30)$, then the fuzzified data of y is B_6. If $y \in [0, 15)$, then the fuzzified data of y is B_7.

To forecast the data of time t, we must decide the window basis w. Then, we can get the criterion vector $C(t)$ and the operation matrix $O^w(t)$ at time t which are expressed as follows:

$$C(t) = f(t-1) = [C_1 \quad C_2 \quad \cdots \quad C_m] \tag{9}$$

$$O^w(t) = \begin{bmatrix} f(t-2) \\ f(t-3) \\ \vdots \\ f(t-w) \end{bmatrix}$$
$$= \begin{bmatrix} O_{11} & O_{12} & \cdots & O_{1m} \\ O_{21} & O_{22} & \cdots & O_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ O_{(w-1)1} & O_{(w-1)2} & \cdots & O_{(w-1)m} \end{bmatrix} \tag{10}$$

where $f(t-1)$ is the fuzzified variation of the main-factor fuzzy time series $F(t)$ between time $t-1$ and time $t-2$, m is the

TABLE I
HISTORICAL DATA OF THE DAILY
AVERAGE TEMPERATURE FROM JUNE 1996 TO SEPTEMBER 1996
IN TAIPEI (UNIT: °C) [2]

Day \ Month	June	July	August	September
1	26.1	29.9	27.1	27.5
2	27.6	28.4	28.9	26.8
3	29.0	29.2	28.9	26.4
4	30.5	29.4	29.3	27.5
5	30.0	29.9	28.8	26.6
6	29.5	29.6	28.7	28.2
7	29.7	30.1	29.0	29.2
8	29.4	29.3	28.2	29.0
9	28.8	28.1	27.0	30.3
10	29.4	28.9	28.3	29.9
11	29.3	28.4	28.9	29.9
12	28.5	29.6	28.1	30.5
13	28.7	27.8	29.9	30.2
14	27.5	29.1	27.6	30.3
15	29.5	27.7	26.8	29.5
16	28.8	28.1	27.6	28.3
17	29.0	28.7	27.9	28.6
18	30.3	29.9	29.0	28.1
19	30.2	30.8	29.2	28.4
20	30.9	31.6	29.8	28.3
21	30.8	31.4	29.6	26.4
22	28.7	31.3	29.3	25.7
23	27.8	31.3	28.0	25.0
24	27.4	31.3	28.3	27.0
25	27.7	28.9	28.6	25.8
26	27.1	28.0	28.7	26.4
27	28.4	28.6	29.0	25.6
28	27.8	28.0	27.7	24.2
29	29.0	29.3	26.2	23.3
30	30.2	27.9	26.0	23.5
31		26.9	27.7	

number of elements in the universe of discourse, C_j and O_{ij} are crisp values, $0 \le C_j \le 1$, $0 \le O_{ij} \le 1$, $1 \le i \le w-1$, and $1 \le j \le m$.

From (9) and (10), we can see that both the criterion vector $C(t)$ and the operation vector $O^w(t)$ are caused by the main-factor fuzzy time series $F(t)$. In the following, we define the second-factor vector $S(t)$ which is composed by the second-factor fuzzy time series $G(t)$.

Definition 3.2: Let $G(t)$ be a second-factor fuzzy time series in the universe of discourse. The second-factor vector $S(t)$ is described by

$$S(t) = g(t-1) = [S_1 \quad S_2 \quad \cdots \quad S_m] \tag{11}$$

where $S(t)$ is the second-factor vector at time t, $g(t-1)$ is the fuzzified data of the second-factor fuzzy time series $G(t)$ at time $t-1$, m is the number of elements in the universe of discourse, $S_i \in [0, 1]$, and $1 \le i \le m$.

The fuzzy relationship of the one-factor time-variant fuzzy time series model is the relationship between the operation matrix and the criterion matrix. In the two-factors time-variant fuzzy time series model, we must decide the fuzzy relationship between the criterion vector, the operation matrix, and the second-factor vector. The new fuzzy relationship matrix is defined as follows.

Definition 3.3: Assume that $F(t)$ is the main-factor fuzzy time series and $G(t)$ is the second-factor fuzzy time series, $C(t)$ is a criterion vector on $F(t)$, $O^w(t)$ is an operation matrix on $F(t)$, and $S(t)$ is a second-factor vector on $G(t)$. The fuzzy re-

TABLE II
HISTORICAL DATA OF THE DAILY CLOUD DENSITY FROM JUNE 1996 TO
SEPTEMBER 1996 IN TAIPEI (UNIT: %) [2]

Day \ Month	June	July	August	September
1	36	15	100	29
2	23	31	78	53
3	23	26	68	66
4	10	34	44	50
5	13	24	56	53
6	30	28	89	63
7	45	50	71	36
8	35	34	28	76
9	26	15	70	55
10	21	8	44	31
11	43	36	48	31
12	40	13	76	25
13	30	26	50	14
14	29	44	84	45
15	30	25	69	38
16	46	24	78	24
17	55	26	39	19
18	19	25	20	39
19	15	21	24	14
20	56	35	25	3
21	60	29	19	38
22	96	48	46	70
23	63	53	41	71
24	28	44	34	70
25	14	100	29	40
26	25	100	31	30
27	29	91	41	34
28	55	84	14	59
29	29	38	28	83
30	19	46	33	38
31		95	26	

TABLE III
FUZZIFIED HISTORICAL DATA OF THE MAIN-FACTOR (THE DAILY AVERAGE
TEMPERATURE) AND THE SECOND-FACTOR (THE DAILY CLOUD DENSITY)
IN JUNE 1996 IN TAIPEI

Day	Variation of the Main-Factor (Unit: °C)	Fuzzified Variation of the Main-Factor	Data of the Second-Factor (Unit: %)	Fuzzified Data of the Second-Factor
1			36	B_5
2	1.5	A_7	23	B_6
3	1.4	A_6	23	B_6
4	1.5	A_7	10	B_7
5	-0.5	A_3	13	B_7
6	-0.5	A_3	30	B_5
7	0.2	A_4	45	B_4
8	-0.3	A_4	35	B_5
9	-0.6	A_3	26	B_6
10	0.6	A_5	21	B_6
11	-0.1	A_4	43	B_5
12	-0.8	A_3	40	B_5
13	0.2	A_4	30	B_5
14	-1.2	A_2	29	B_6
15	2.0	A_7	30	B_5
16	-0.7	A_3	46	B_4
17	0.2	A_4	55	B_4
18	1.3	A_6	19	B_6
19	-0.1	A_4	15	B_6
20	0.7	A_5	56	B_4
21	-0.1	A_4	60	B_3
22	-2.1	A_1	96	B_1
23	-0.9	A_3	63	B_3
24	-0.4	A_3	28	B_6
25	0.3	A_5	14	B_7
26	-0.6	A_3	25	B_6
27	1.3	A_6	29	B_6
28	-0.6	A_3	55	B_4
29	1.2	A_6	29	B_6
30	1.2	A_6	19	B_6

lationship matrix $R(t)$ between $C(t)$, $O^w(t)$, and $S(t)$ is equal to $R(t) = O^w(t) \otimes S(t) \otimes C(t)$, where we have (12), shown at the bottom of the page. m is the number of elements in the universe of discourse, $R_{ij} = O_{ij} \times S_j \times C_j$, $1 \leq i \leq w - 1$, $1 \leq j \leq m$, and "$\times$" is the multiplication operator.

From the fuzzy relationship matrix $R(t)$, we can get the fuzzified forecasted variation $f(t)$ between time t and time $t - 1$ described by

$$f(t) = \big[\text{Max}\big(R_{11}, R_{21}, \ldots, R_{(w-1)1}\big)$$
$$\text{Max}\big(R_{12}, R_{22}, \ldots, R_{(w-1)2}\big)$$
$$\cdots \quad \text{Max}\big(R_{1m}, R_{2m}, \ldots, R_{(w-1)m}\big)\big] \quad (13)$$

where $f(t)$ is the fuzzified variation of the fuzzy time series $F(t)$ between time t and time $t - 1$.

In the following, we propose a new algorithm called Algorithm-B based on the proposed two-factors time-variant fuzzy time series model and we will use two factors (i.e., the daily average temperature and the daily cloud density) for temperature prediction from June 1996 to September 1996 in Taipei, Taiwan, where "the daily average temperature" and "the daily cloud density" are called the main-factor and the second-factor of the two-factors time-variant fuzzy time series model, respectively. The algorithm is now presented as follows.

Algorithm-B:

Step 1) Partition the historical data into suitable groups and perform the following forecasting steps to each group, respectively. For example, we partition the historical data into four groups, i.e., June, July, August, and September, as shown in Table I.

$$R(t) = \begin{bmatrix} O_{11} \times S_1 \times C_1 & O_{12} \times S_2 \times C_2 & \cdots & O_{1m} \times S_m \times C_m \\ O_{21} \times S_1 \times C_1 & O_{22} \times S_2 \times C_2 & \cdots & O_{2m} \times S_m \times C_m \\ \vdots & \vdots & \vdots & \vdots \\ O_{(w-1)1} \times S_1 \times C_1 & O_{(w-1)2} \times S_2 \times C_2 & \cdots & O_{(w-1)m} \times S_m \times C_m \end{bmatrix}$$
$$= \begin{bmatrix} R_{11} & R_{12} & \cdots & R_{1m} \\ R_{21} & R_{22} & \cdots & R_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ R_{(w-1)1} & R_{(w-1)2} & \cdots & R_{(w-1)m} \end{bmatrix} \quad (12)$$
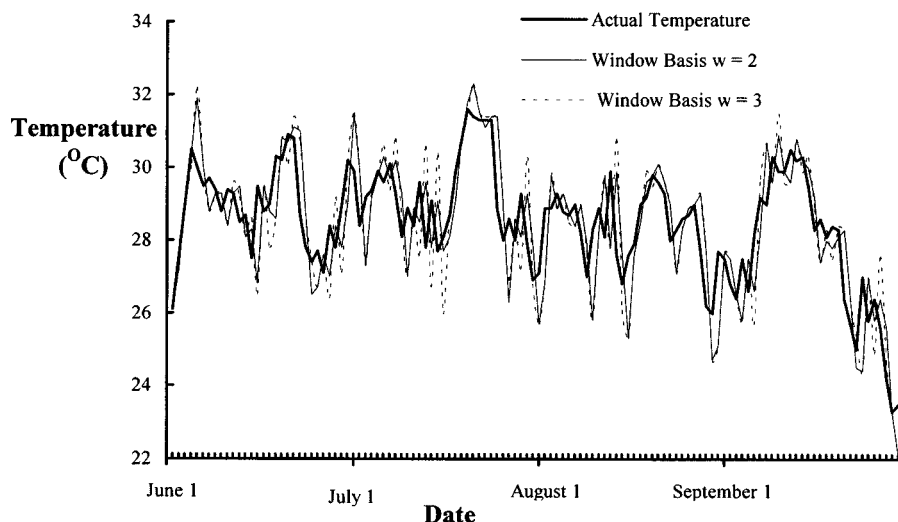
Fig. 1.   Curve of the actual temperature and forecasted temperature of Algorithm-A with the window bases $w = 2$ and $w = 3$.

Step 2) Compute the variations of the main-factor fuzzy time series between any two continuous data. Find the maximum decrease D_L and the maximum increase D_R between any two continuous data, and define the universe of discourse $U, U = [D_L - D_1, D_R + D_2]$, where D_1 and D_2 are suitable positive numbers. For example, assume that we want to forecast the temperature of June 1996 in Taipei. From Table I, we can see that $D_L = -2.1$, and $D_R = 2.0$. We set $D_1 = 0.1, D_2 = 0$. Then, the universe of discourse can be defined as $U = [-2.2, 2.0]$.

Step 3) Partition the universe of discourse U into several even length intervals $u_1, u_2, \ldots, u_m$. For example, in this case, we partition the universe of discourse U into seven intervals $u_1, u_2, \ldots, u_7$, where $u_1 = [-2.2, -1.6]$, $u_2 = [-1.6, -1.0]$, $u_3 = [-1.0, -0.4]$, $u_4 = [-0.4, 0.2]$, $u_5 = [0.2, 0.8]$, $u_6 = [0.8, 1.4]$, and $u_7 = [1.4, 2.0]$.

Step 4) Define fuzzy sets on the universe of discourse U for the fuzzified variation of the main-factor fuzzy time series $F(t)$. In this case, we consider seven fuzzy sets which are $A_1 =$ (very big decrease), $A_2 =$ (big decrease), $A_3 =$ (decrease), $A_4 =$ (no change), $A_5 =$ (increase), $A_6 =$ (big increase), $A_7 =$ (very big increase), where the fuzzy sets $A_1, A_2, \ldots,$ and A_7 in the universe of discourse U are defined as follows:

$A_1 = 1/u_1 + 0.5/u_2 + 0/u_3 + 0/u_4 + 0/u_5 + 0/u_6 + 0/u_7,$
$A_2 = 0.5/u_1 + 1/u_2 + 0.5/u_3 + 0/u_4 + 0/u_5 + 0/u_6 + 0/u_7,$
$A_3 = 0/u_1 + 0.5/u_2 + 1/u_3 + 0.5/u_4 + 0/u_5 + 0/u_6 + 0/u_7,$
$A_4 = 0/u_1 + 0/u_2 + 0.5/u_3 + 1/u_4 + 0.5/u_5 + 0/u_6 + 0/u_7,$
$A_5 = 0/u_1 + 0/u_2 + 0/u_3 + 0.5/u_4 + 1/u_5 + 0.5/u_6 + 0/u_7,$
$A_6 = 0/u_1 + 0/u_2 + 0/u_3 + 0/u_4 + 0.5/u_5 + 1/u_6 + 0.5/u_7,$
$A_7 = 0/u_1 + 0/u_2 + 0/u_3 + 0/u_4 + 0/u_5 + 0.5/u_6 + 1/u_7.$

$$(14)$$

Step 5) Define fuzzy sets on the universe of discourse U for the second-factor fuzzy time series $G(t)$. In this case, the second-factor fuzzy time series is the daily cloud density which ranges from 0% to 100% as shown in Table II. We consider seven fuzzy sets which are $B_1 =$ (very, very cloudy), $B_2 =$ (very cloudy), $B_3 =$ (more or less cloudy), $B_4 =$ (cloudy), $B_5 =$ (little cloudy), $B_6 =$ (very little cloudy), $B_7 =$ (very very little cloudy) for the second-factor fuzzy time series. We describe some human knowledge as follows.

1) When the cloud density increases, it seems that the weather becomes bad and the temperature may go down. When the cloud density decreases, it seems that the weather becomes good and the temperature may go up.

2) When yesterday's cloud density was very low, that means that yesterday was a sunny day. It is more likely that today's cloud density will increase and it is less likely that the cloud density will decrease. Thus, a decrease of the temperature has a higher possibility than an increase of the temperature.

3) When yesterday's cloud density was very high, that means that yesterday was a cloudy day. It is more likely that today's cloud density will decrease and it is less likely that the cloud density will increase. Thus, an increase of the temperature has a higher possibility than a decrease of the temperature.

According to the human knowledge described above, we can define fuzzy sets $B_1, B_2, \ldots, B_7$ on the universe of discourse $U, U = \{u_1, u_2, \ldots, u_7\}$, as follows:

$B_1 = 0/u_1 + 0/u_2 + 0.5/u_3 + 1/u_4 + 1/u_5 + 1/u_6 + 1/u_7,$
$B_2 = 0/u_1 + 0.5/u_2 + 1/u_3 + 1/u_4 + 1/u_5 + 1/u_6 + 1/u_7,$
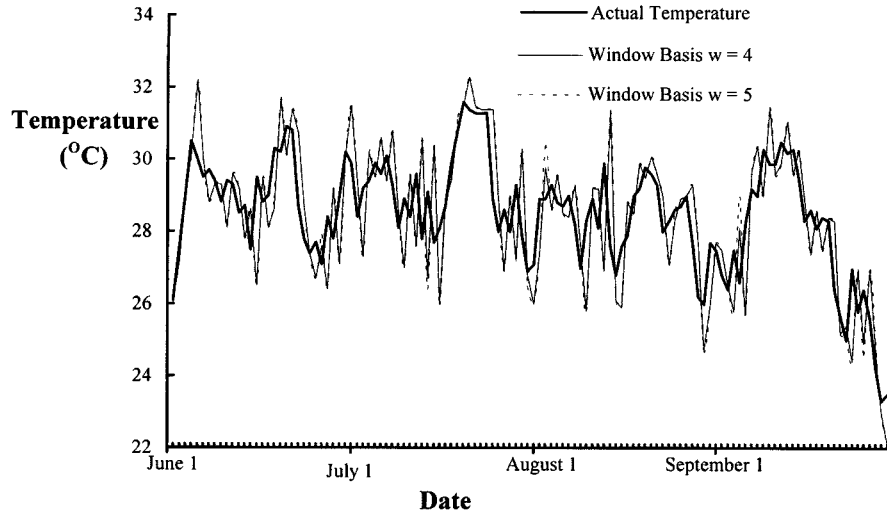
Fig. 2.   Curve of the actual temperature and forecasted temperature of Algorithm-A with the window bases $w = 4$ and $w = 5$.

$$B_3 = 0.5/u_1 + 1/u_2 + 1/u_3 + 1/u_4 + 1/u_5 + 1/u_6 + 1/u_7,$$
$$B_4 = 1/u_1 + 1/u_2 + 1/u_3 + 1/u_4 + 1/u_5 + 1/u_6 + 1/u_7,$$
$$B_5 = 1/u_1 + 1/u_2 + 1/u_3 + 1/u_4 + 1/u_5 + 1/u_6 + 0.5/u_7,$$
$$B_6 = 1/u_1 + 1/u_2 + 1/u_3 + 1/u_4 + 1/u_5 + 0.5/u_6 + 0/u_7,$$
$$B_7 = 1/u_1 + 1/u_2 + 1/u_3 + 1/u_4 + 0.5/u_5 + 0/u_6 + 0/u_7. \tag{15}$$

Step 6) Fuzzify the variation of the main-factor (i.e., the daily average temperature) fuzzy time series and fuzzify the data of the second-factor (i.e., the cloud density) fuzzy time series. The fuzzification method is described as follows:

1) Assume that the variation p of the main-factor fuzzy time series belongs to the interval u_i (i.e., $p \in u_i$), and assume that, from (14), we know that the maximum membership value of fuzzy set A_j occured at u_i, then the fuzzified variation of p is A_j.

2) If the data y of the second-factor fuzzy time series $\in [90, 100]$, then the fuzzified data of y is B_1. If $y \in [75, 90)$, then the fuzzified data of y is B_2. If $y \in [60, 75)$, then the fuzzified data of y is B_3. If $y \in [45, 60)$, then the fuzzified data of y is B_4. If $y \in [30, 45)$, then the fuzzified data of y is B_5. If $y \in [15, 30)$, then the fuzzified data of y is B_6. If $y \in [0, 15)$, then the fuzzified data of y is B_7. For example, the fuzzified variation of the main-factor (i.e., the daily average temperature) fuzzy time series and the fuzzified data of the second-factor (i.e., the cloud density) fuzzy time series in June 1996 in Taipei are shown in Table III.

Step 7) Choose a suitable window basis w and define the criterion vector $C(t)$, the operation matrix $O^w(t)$, and the second-factor vector $S(t)$. Then, calculate the fuzzy relationship matrix $R(t)$ between the criterion vector $C(t)$, the operation matrix $O^w(t)$, and the second-factor vector $S(t)$ based on (12), where t

is the date for which we want to forecast the temperature. From the fuzzy relationship matrix $R(t)$, we can get the fuzzified forecasted variation $f(t)$ based on (13). Then, we can evaluate the fuzzified forecasted output. For example, if we set window basis $w = 4$, and we want to forecast the average temperature for June 15, 1996, in Taipei. Then, we can set a 3×7 operation matrix $O^4(t)$, a 1×7 second-factor vector $S(t)$, and a 1×7 criterion vector $C(t)$ shown as follows[1]:

O^4(June 15, 1996)

$$= \begin{bmatrix} \text{fuzzy variation of the main-factor of June 13, 1996} \\ \text{fuzzy variation of the main-factor of June 12, 1996} \\ \text{fuzzy variation of the main-factor of June 11, 1996} \end{bmatrix}$$

$$= \begin{bmatrix} A_4 \\ A_3 \\ A_4 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0.5 & 1 & 0.5 & 0 & 0 \\ 0 & 0.5 & 1 & 0.5 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 1 & 0.5 & 0 & 0 \end{bmatrix}$$

S(June 15, 1996) = fuzzy data of the second-factor of June 14, 1996 = $[B_6]$

(VBD)	(BD)	(D)	(NC)	(I)	(BI)	(VBI)
$= [\,1$	1	1	1	1	0.5	$0\,]$

C (June 15, 1996) = fuzzy variation of the main-factor of June 14, 1996 = $[A_2]$

(VBD)	(BD)	(D)	(NC)	(I)	(BI)	(VBI)
$= [0.5$	1	0.5	0	0	0	$0\,].$

Calculate the relation matrix $R(t)$ by $R(t)[i, j] = O^4(t)[i, j] \otimes S(t)[j] \otimes C(t)[j]$, where $1 \leq i \leq 3$, and $1 \leq j \leq 7$. Then, based on (12), we can get

$$R(\text{June 15, 1996}) = \begin{bmatrix} 0 & 0 & 0.25 & 0 & 0 & 0 & 0 \\ 0 & 0.5 & 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.25 & 0 & 0 & 0 & 0 \end{bmatrix}$$

[1]In Step 7, the following abbreviations are used: VBD = very big decrease; BD = big decrease; D= decrease; NC = no change; I = increase; BI = big increase; and VBI = very big increase.
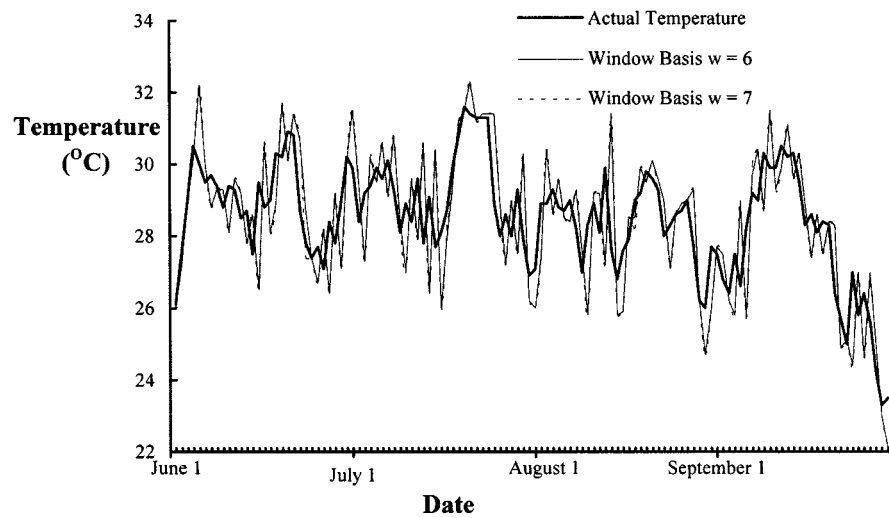
Fig. 3. Curve of the actual temperature and forecasted temperature of Algorithm-A with the window bases $w = 6$ and $w = 7$.
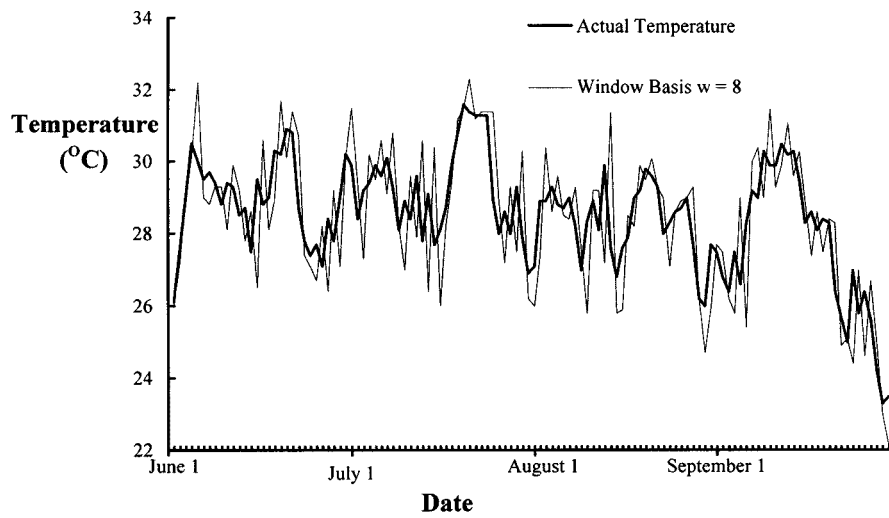

Fig. 4. Curve of the actual temperature and forecasted temperature of Algorithm-A with the window bases $w = 8$.
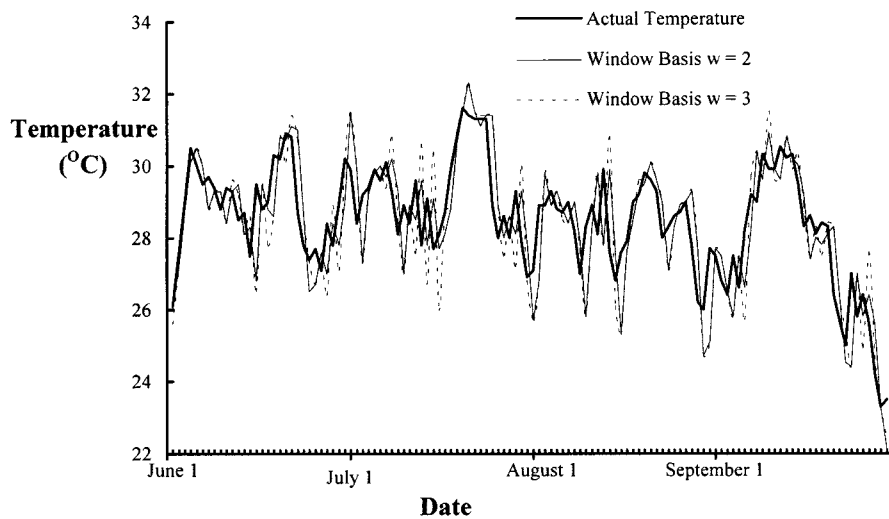

Fig. 5. Curve of the actual temperature and forecasted temperature of Algorithm-B with the window bases $w = 2$ and $w = 3$.
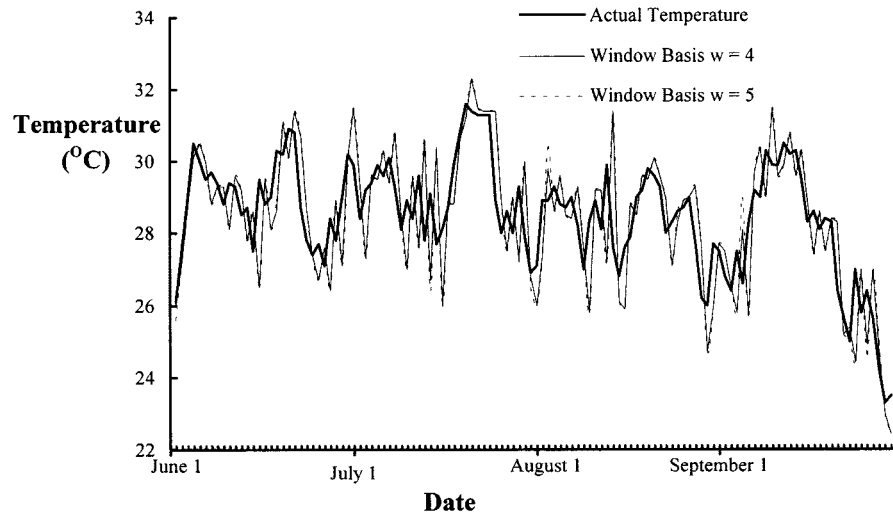
Fig. 6. Curve of the actual temperature and forecasted temperature of Algorithm-B with the window bases $w = 4$ and $w = 5$.
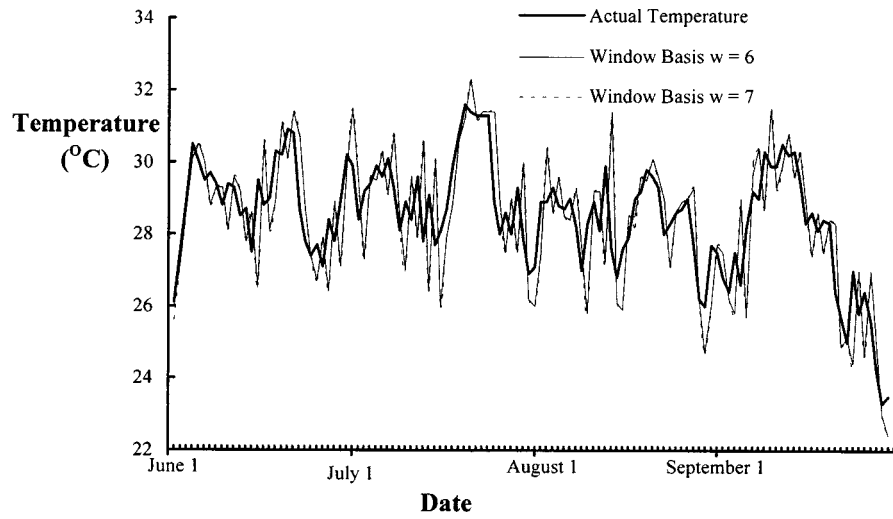

Fig. 7. Curve of the actual temperature and forecasted temperature of Algorithm-B with the window bases $w = 6$ and $w = 7$.
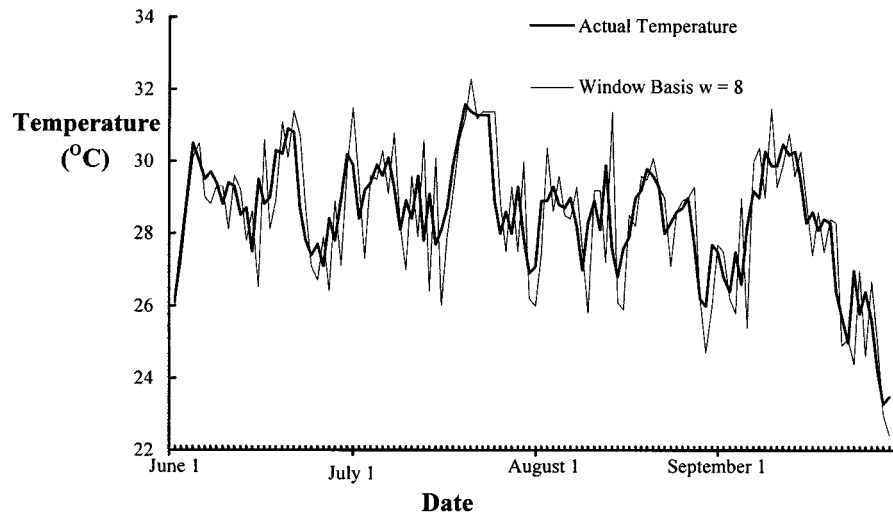

Fig. 8. Curve of the actual temperature and forecasted temperature of Algorithm-B with the window basis $w = 8$.

Based on (13), we can get the fuzzified forecasted variation f(June 15, 1996) shown as follows:

$$f(\text{June 15, 1996}) = [0 \quad 0.5 \quad 0.5 \quad 0 \quad 0 \quad 0 \quad 0].$$

Step 8) Defuzzify the fuzzified forecasted variations of the main-factor fuzzy time series obtained in Step 7. We use the following combined method to defuzzify the fuzzified forecasted variations:

1) If the grades of membership of the fuzzified forecasted variation are all 0, then we set the forecasted variation to 0.

2) If the maximum membership of the fuzzified forecasted variation $f(t)$ occurred at u_i and the midpoint of u_i is m_i, then the forecasted variation is m_i. If the maximum membership of the fuzzified forecasted variation $f(t)$ occurred at $u_1, u_2, \ldots$, and u_k, and the midpoints of $u_1, u_2, \ldots$, and u_k are $m_1, m_2, \ldots$, and m_k, respectively, then the forecasted variation is $(m_1 + m_2 + \cdots + m_k)/k$ [4].

For example, in the above, we can see that the maximum membership value of f(June 15, 1996) is 0.5 which occurs at u_2 and u_3, where the midpoint of u_2 is -1.3 and the midpoint of u_3 is -0.7. Thus, the forecasted variation of June 15, 1996 is $= -1$.

Step 9) Calculate the forecasted data of the main-factor fuzzy time series. The forecasted data is equal to the forecasted variation plus the actual data of the last day of the main-factor fuzzy time series. For example, if the forecasted variation of June 15, 1996, is -1, and the actual daily average temperature of June 14, 1996 is 27.5, then the forecasted average temperature of June 15, 1996 is equal to $27.5 + (-1) = 26.5$.

Figs. 1–4 show the forecasting results using Algorithm-A from June 1996 to September 1996, where Algorithm-A only uses one factor (i.e., the daily average temperature) for temperature prediction. Figs. 5–8 show the forecasting results using Algorithm-B from June 1996 to September 1996 with different window basis. Table IV shows the forecasting errors of the daily average temperature using Algorithm-A [7] and Algorithm-B with different window bases from June 1996 to September 1996 in Taipei. In general, the forecasting accuracy of the two-factors time-variant fuzzy time series algorithm Algorithm-B is better than the one-factor time-variant fuzzy time series algorithm Algorithm-A [7]. In order to further enhance the forecasting accuracy of the proposed algorithm Algorithm-B, we will modify Algorithm-B into Algorithm-B* to further reduce the forecasting errors.

In the following, we will propose the algorithm Algorithm-B* based on the concept of α-significance level [27], where $\alpha \in [0, 1]$, and the repeated transition method, where Algorithm-B* is essentially a modification of Algorithm-B.

Definition 3.4: Assume that $F(t)$ and $H(t)$ are fuzzy time series of the daily average temperature, where $H(t)$ is last year's $F(t)$, $f(t)$ is the data of $F(t)$ at date t, and $h(t)$ is the data of $H(t)$ at date t. The boundary of $f(t)$ ranges from $h(t-k)$ to $h(t+k)$, where k is the boundary length. Then, we can find the

TABLE IV
AVERAGE FORECASTING ERRORS OF ALGORITHM-A AND ALGORITHM-B WITH DIFFERENT WINDOW BASES FOR TEMPERATURE PREDICTION

	Window Bases						
	w = 2	w = 3	w = 4	w = 5	w = 6	w = 7	w = 8
Algorithm-A [7] to Forecast the Temperature of June	3.04%	3.48%	3.49%	3.57%	3.81%	3.88%	4.09%
Algorithm-B to Forecast the Temperature of June	2.90%	3.23%	3.37%	3.44%	3.67%	3.82%	3.97%
Algorithm-A [7] to Forecast the Temperature of July	3.09%	3.97%	4.22%	4.34%	4.47%	4.35%	4.58%
Algorithm-B to Forecast the Temperature of July	2.88%	3.86%	4.18%	4.26%	4.43%	4.35%	4.54%
Algorithm-A [7] to Forecast the Temperature of August	3.49%	3.72%	3.79%	3.83%	3.67%	3.66%	3.72%
Algorithm-B to Forecast the Temperature of August	3.49%	3.68%	3.71%	3.79%	3.58%	3.56%	3.63%
Algorithm-A [7] to Forecast the Temperature of September	3.37%	3.67%	3.72%	3.56%	3.78%	3.68%	3.60%
Algorithm-B to Forecast the Temperature of September	3.33%	3.58%	3.63%	3.47%	3.68%	3.57%	3.49%

TABLE V
DAILY AVERAGE TEMPERATURE FROM MAY 1995 TO OCTOBER 1995 IN TAIPEI (UNIT: °C) [2]

Day / Month	May	June	July	August	September	October
1	27.0	28.9	28.4	28.4	29.8	28.4
2	26.5	28.8	29.1	26.7	29.8	28.8
3	25.0	28.4	28.7	26.1	29.6	30.1
4	20.7	28.2	28.6	28.8	30.3	29.8
5	18.5	25.6	29.0	27.5	29.7	28.3
6	22.3	26.0	29.5	28.4	30.0	25.2
7	21.7	27.5	30.1	28.7	29.8	24.8
8	23.5	27.7	30.1	29.8	30.5	24.6
9	24.6	24.6	28.6	30.6	30.1	24.0
10	24.6	25.0	27.2	30.8	29.7	24.1
11	23.2	24.4	26.5	29.8	29.5	24.7
12	25.6	26.1	26.4	28.6	27.3	24.8
13	26.3	25.8	27.8	27.5	27.7	24.5
14	25.8	26.5	28.5	27.9	27.2	26.4
15	21.0	26.0	27.2	29.1	27.7	25.6
16	18.3	27.4	29.4	29.4	26.2	24.2
17	21.3	29.0	30.0	30.1	25.4	24.1
18	22.9	27.8	31.1	30.0	25.4	24.5
19	25.9	29.9	31.3	30.3	25.8	23.9
20	27.6	30.0	31.1	30.7	23.2	24.8
21	22.7	30.4	32.1	30.8	21.6	23.9
22	19.2	31.5	32.2	29.7	22.9	23.6
23	20.5	31.2	28.8	28.9	23.4	24.2
24	24.2	30.7	26.8	26.6	25.1	23.8
25	27.0	30.8	27.0	25.9	25.4	21.4
26	27.8	31.2	29.0	28.1	25.9	22.2
27	27.7	31.5	29.0	29.4	26.6	23.0
28	27.9	25.7	29.9	30.6	27.1	23.5
29	28.7	28.0	29.9	30.9	26.3	23.3
30	29.3	28.0	28.6	29.4	27.6	24.0
31	29.3		27.7	30.5		22.8

maximum $T_{\max}$ and the minimum $T_{\min}$ of these $2k+1$ data. The forecasted temperature of date t can not be bigger than $T_{\max}$ and not be smaller than $T_{\min}$. The algorithm Algorithm-B* is now presented as follows:

Algorithm-B*:

Step 1) Partition the historical data into suitable groups.

Step 2) Compute the variations of the main-factor fuzzy time series between any two continuous data. Find the maximum decrease D_L and the maximum increase D_R, and define the universe of discourse $U = [D_L - D_1, D_R + D_2]$, where D_1 and D_2 are suitable positive numbers.
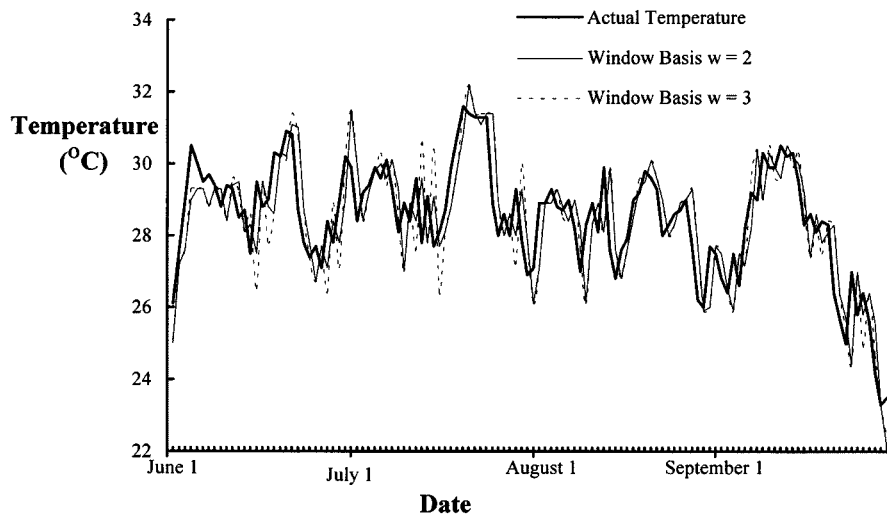
Fig. 9.   Curve of the actual temperature and forecasted temperature of Algorithm-B* with the window bases $w = 2$ and $w = 3$.
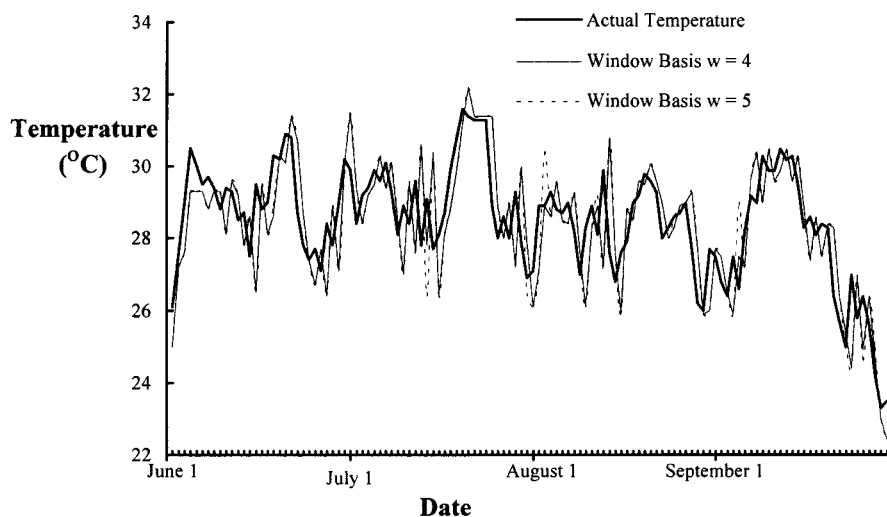

Fig. 10.   Curve of the actual temperature and forecasted temperature of Algorithm-B* with the window bases $w = 4$ and $w = 5$.

Step 3) Partition the universe of discourse U into several even length intervals.

Step 4) Define fuzzy sets on the universe of discourse U for the main-factor fuzzy time series $F(t)$.

Step 5) Define fuzzy sets on the universe of discourse U for the second-factor fuzzy time series $G(t)$.

Step 6) Fuzzify the variation of the historical data of the main-factor fuzzy time series and fuzzify the historical data of the second-factor fuzzy time series.

Step 7) Choose a suitable window basis w. Define the criterion vector $C(t)$, the operation matrix $O^w(t)$, and the second-factor vector $S(t)$. Then, calculate the fuzzy relationship matrix $R(t)$ between the criterion vector $C(t)$, the operation matrix $O^w(t)$, and the second-factor vector. The fuzzified forecasted variation of the main-factor fuzzy time series can be obtained from the fuzzy relationship matrix.

Step 8) Defuzzify the fuzzified forecasted variations of the main-factor obtained in Step 7.

1) Compute the α-significance level [27] $(f(t))_\alpha$ of the fuzzified forecasted variation $f(t)$ as follows:

$$f(t) = [\, f_1 \quad f_2 \quad \cdots \quad f_m \,],$$
$$(f(t))_\alpha = [\, f_{1\alpha} \quad f_{2\alpha} \quad \cdots \quad f_{m\alpha} \,] \qquad (16)$$

where $f(t)$ is the fuzzified forecasted variation of date t represented by a fuzzy set in the fuzzy time series $F(t)$, m is the number of elements in the universe of discourse, and $\alpha \in [0, 1]$. If $f_i \geq \alpha$, then let $f_{i\alpha} = f_i$; if $f_i < \alpha$, then let $f_{i\alpha} = 0$, where $1 \leq i \leq m$.
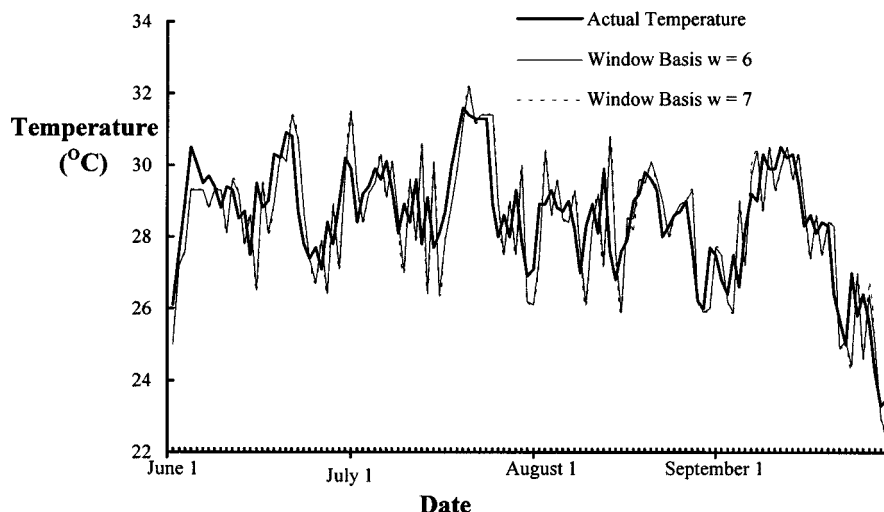
Fig. 11. Curve of the actual temperature and forecasted temperature of Algorithm-B* with the window bases $w = 6$ and $w = 7$.
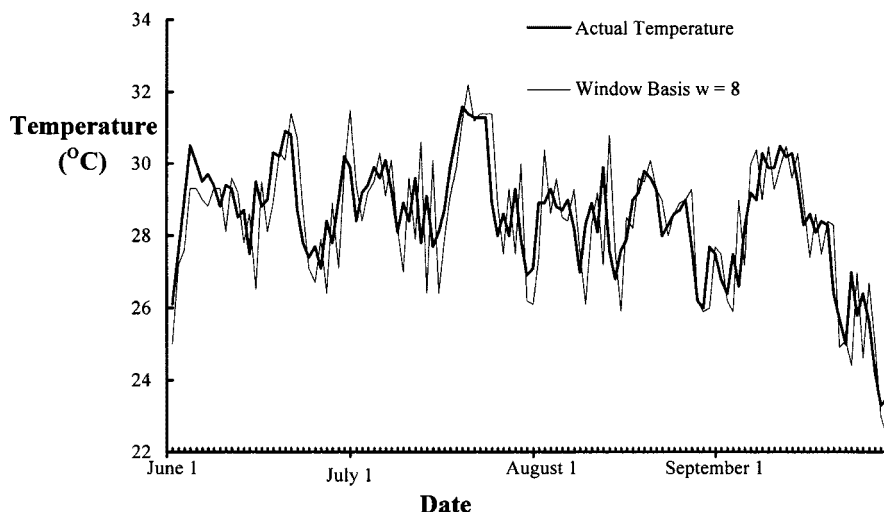

Fig. 12. Curve of the actual temperature and forecasted temperature of Algorithm-B* with the window basis $w = 8$.

In this paper, we set the value of the significance level $\alpha = 0.5$.

2) If the grades of membership of the α-significance level forecasted variation $(f(t))_\alpha$ are all 0, then we set the forecasted variation to 0.

3) If the grades of membership of the α-significance level forecasted variation $(f(t))_\alpha$ have only one maximum u_i, and the midpoint of u_i is m_i, then the forecasted variation of date t is m_i. If the grades of membership of the α-significance level forecasted variation $(f(t))_\alpha$ have more than one maximum $u_1, u_2, \ldots,$ and u_k, and the midpoints of $u_1, u_2, \ldots,$ and u_k are $m_1, m_2, \ldots,$ and m_k, respectively, then the forecasted variation of date t is $(m_1 + m_2 + \cdots + m_k)/k$.

Step 9) Calculate the forecasted data of the main-factor fuzzy time series. The forecasted data is equal to the forecasted variation plus the actual data of the last day of the main-factor fuzzy time series.

Step 10) Set the boundary length to k. Then, we can find the maximum $T_{\max}$ and the minimum $T_{\min}$ among the data from the date $t - k$ of last year to the date $t + k$ of last year. The forecasted temperature of date t obtained in Step 9 can not be bigger than $T_{\max}$ and not be smaller than $T_{\min}$. If the forecasted temperature is larger than $T_{\max}$, then we set the forecasted temperature to $T_{\max}$; if the forecasted temperature is smaller than $T_{\min}$, then we set the forecasted temperature to $T_{\min}$. For example, assume that we set the window basis $w = 4$ and the boundary length $k = 10$, and we want to forecast the temperature of June 6, 1996, in Taipei. Furthermore, assume that the forecasted temperature of June 6, 1996, by using the above steps is 30.0 °C. Table V shows the daily average temperature from May 1995 to October 1995 in Taipei. The boundary data is from May 27, 1995 to June 16, 1995. According to Table V, we can find that $T_{\max} = 29.3$ °C and $T_{\min} = 24.4$ °C. Because 30

TABLE VI
COMPARISON OF THE AVERAGE FORECASTING ERRORS OF ALGORITHM-A, ALGORITHM-B AND ALGORITHM-B* TO FORECAST THE TEMPERATURE FROM JUNE 1996 TO SEPTEMBER 1996 IN TAIPEI WITH DIFFERENT WINDOW BASES

	Window Bases						
	w = 2	w = 3	w = 4	w = 5	w = 6	w = 7	w = 8
Algorithm-A [7] to Forecast the Temperature of June	3.04%	3.48%	3.49%	3.57%	3.81%	3.88%	4.09%
Algorithm-B to Forecast the Temperature of June	2.90%	3.23%	3.37%	3.44%	3.67%	3.82%	3.97%
Algorithm-B* to Forecast the Temperature of June	2.88%	3.16%	3.24%	3.33%	3.39%	3.53%	3.67%
Algorithm-A [7] to Forecast the Temperature of July	3.09%	3.97%	4.22%	4.34%	4.47%	4.35%	4.58%
Algorithm-B to Forecast the Temperature of July	2.88%	3.86%	4.18%	4.26%	4.43%	4.35%	4.54%
Algorithm-B* to Forecast the Temperature of July	3.04%	3.76%	4.08%	4.17%	4.35%	4.38%	4.56%
Algorithm-A [7] to Forecast the Temperature of August	3.49%	3.72%	3.79%	3.83%	3.67%	3.66%	3.72%
Algorithm-B to Forecast the Temperature of August	3.49%	3.68%	3.71%	3.79%	3.58%	3.56%	3.63%
Algorithm-B* to Forecast the Temperature of August	2.75%	2.77%	3.30%	3.40%	3.18%	3.15%	3.19%
Algorithm-A [7] to Forecast the Temperature of September	3.37%	3.67%	3.72%	3.56%	3.78%	3.68%	3.60%
Algorithm-B to Forecast the Temperature of September	3.33%	3.58%	3.63%	3.47%	3.68%	3.57%	3.49%
Algorithm-B* to Forecast the Temperature of September	3.29%	3.10%	3.19%	3.22%	3.39%	3.38%	3.29%

$^{\circ}$C $> T_{\max}$, we set the forecasted temperature of June 6, 1996, to 29.3 $^{\circ}$C.

Figs. 9–12 show the forecasting results from June 1996 to September 1996 by using Algorithm-B* with different window bases. Table VI shows a comparison of the average forecasting errors of Algorithm-A, Algorithm-B, and Algorithm-B* with different window bases. We can see that the forecasting results of Algorithm-B* are better than the forecasting results of Algorithm-A and Algorithm-B. From Table VI, we can see that when window basis $w = 2$, we almost can get the best forecasting results, and the forecasting results of a smaller window basis are better than the ones of a bigger window basis. This is because the forecasted temperature usually is related to the temperature of the recent days. Because the time complexities of the algorithms (i.e., Algorithm-B and Algorithm-B*) are dominated by Step 7, i.e., calculating the fuzzy relationship matrix $R(t)$, where $R(t) = O^w(t) \otimes S(t) \otimes C(t)$ as shown in (12) and the time complexity for calculating $R(t)$ is $O(wm)$, where w is the window basis and m is the number of elements in the universe of discourse U derived in Step 3 of the proposed algorithms and because there are c groups of the historical data derived in Step 1 of the proposed algorithms, the time complexities of the proposed algorithms are $O(cwm)$, respectively.

IV. CONCLUSIONS

In this paper, we have proposed a new fuzzy time series model called the two-factors time-variant fuzzy time series model for handling forecasting problems. Based on the proposed model, we develop two algorithms (i.e., Algorithm-B and Algorithm-B*) for temperature prediction. From Table VI, we can see that the forecasting results of Algorithm-B* are better than the forecasting results of Algorithm-A and Algorithm-B. Both of these algorithms have the advantages that they can get good forecasting results. The time complexities of the proposed algorithms are $O(cwm)$, respectively, where c is the number of partitioned groups in the historical data, w is the window basis, and m is the number of elements in the universe of discourse.

REFERENCES

[1] H. Bentley, "Time series analysis with REVEAL," *Fuzzy Sets Syst.*, vol. 23, no. 1, pp. 97–118, 1987.
[2] Central Weather Bureau, *The Historical Data of the Daily Average Temperature and the Daily Cloud Density*　Taipei, Taiwan, R.O.C, 1996. (from January 1995 to September 1996).
[3] S. M. Chen, "A new method for forecasting enrollments using fuzzy time series," in *Proc. 1995 National Computer Symp.* Chungli, Taoyuan, Taiwan, R.O.C., 1995, pp. 603–609.
[4] ——, "Forecasting enrollments based on fuzzy time series," *Fuzzy Sets Syst.*, vol. 81, no. 3, pp. 311–319, 1996.
[5] M. Y. Day, "Research of Applying Genetic Algorithms to Fuzzy Forecasting—Focus on Sales Forecasting," M.S. thesis, Tamkang Univ., Taipei, Taiwan, R.O.C., 1995.

[6] J. R. Hwang, S. M. Chen, and C. H. Lee, "A new method for handling forecasting problems based on fuzzy time series," in *Proc. 7th Int. Conf. Information Management*. Chungli, Taoyuan, Taiwan, R.O.C., 1996, pp. 312–321.

[7] ——, "Handling forecasting problems using fuzzy time series," *Fuzzy Sets Syst.*, vol. 100, no. 2, pp. 217–228, 1998.

[8] A. Kandel, *Fuzzy Mathematical Techniques with Applications*. Reading, MA: Addison-Wesley, 1996.

[9] A. Kaufmann and M. M. Gupta, *Fuzzy Mathematical Models in Engineering and Management Science*. Amsterdam, The Netherlands: North-Holland, 1988.

[10] G. J. Klir and T. A. Folger, *Fuzzy Sets, Uncertainty, and Information*. Englewood Cliffs, NJ: Prentice-Hall, 1988.

[11] G. J. Klir and B. Yuan, *Fuzzy Sets and Fuzzy Logic: Theory and Applications*. Englewood Cliffs, NJ: Prentice-Hall, 1995.

[12] K. S. Leung and W. Lam, "Fuzzy concepts in expert systems," *IEEE Comput.*, vol. 21, no. 9, pp. 43–56, 1988.

[13] ——, "A fuzzy expert system shell using both exact and inexact reasoning," *J. Autom. Reasoning*, vol. 5, no. 2, pp. 207–233, 1989.

[14] S. Mabuchi, "A proposal for a defuzzification strategy by the concept of sensitivity analysis," *Fuzzy Sets Syst.*, vol. 55, no. 1, pp. 1–14, 1993.

[15] T. A. Runkler and M. Glesner, "A set of axioms for defuzzification strategies towards a theory of rational defuzzification operator," in *Proc. 2nd IEEE Int. Conf. Fuzzy Systems*, San Francisco, CA, 1993, pp. 1161–1166.

[16] Q. Song and G. Bortolan, "Some properties of defuzzification neural networks," *Fuzzy Sets Syst.*, vol. 61, no. 1, pp. 83–89, 1994.

[17] Q. Song and B. S. Chissom, "Fuzzy time series and its models," *Fuzzy Sets Syst.*, vol. 54, no. 3, pp. 269–277, 1993.

[18] ——, "Forecasting enrollments with fuzzy time series—Part I," *Fuzzy Sets Syst.*, vol. 54, no. 1, pp. 1–9, 1993.

[19] ——, "Forecasting enrollments with fuzzy time series—Part II," *Fuzzy Sets Syst.*, vol. 62, no. 1, pp. 1–8, 1994.

[20] Q. Song and R. P. Leland, "Adaptive learning defuzzification techniques and applications," *Fuzzy Sets Syst.*, vol. 81, no. 3, pp. 321–329, 1996.

[21] J. Sullivan and W. H. Woodall, "A comparison of fuzzy forecasting and Markov modeling," *Fuzzy Sets Syst.*, vol. 64, no. 3, pp. 279–293, 1994.

[22] L. X. Wang, *A Course in Fuzzy Systems and Control*. Englewood Cliffs, NJ: Prentice-Hall, 1997.

[23] W. Wu, "Fuzzy reasoning and fuzzy relational equations," *Fuzzy Sets Syst.*, vol. 20, no. 1, pp. 67–78, 1986.

[24] R. R. Yager, "Implementing fuzzy logic controllers using a neural network," *Fuzzy Sets Syst.*, vol. 48, no. 1, pp. 53–64, 1992.

[25] R. R. Yager and D. Filev, "On the issue of defuzzification and selection based on a fuzzy set," *Fuzzy Sets Syst.*, vol. 55, no. 3, pp. 255–271, 1993.

[26] ——, "SLIDE: A simple adaptive defuzzification method," *IEEE Trans. Fuzzy Systems*, vol. 1, no. 1, pp. 69–78, 1993.

[27] Y. Yuan and M. J. Shaw, "Induction of fuzzy decision trees," *Fuzzy Sets Syst.*, vol. 69, no. 2, pp. 125–139, 1995.

[28] L. A. Zadeh, "Fuzzy sets," *Inform. Contr.*, vol. 8, pp. 338–353, 1965.

[29] ——, "Outline of a new approach to the analysis of complex system and decision processes," *IEEE Trans. Syst., Man, Cybern.*, vol. SMC–3, pp. 28–44, 1973.

[30] ——, "The concept of a linguistic variable and its application to approximate reasoning, part I–III," *Inform. Sci.*, vol. 8, 9, pp. 199–249, 301–357, 43–80, 1975.

[31] Z. Li, Z. Chen, and J. Li, "A model of weather forecast by fuzzy grade statistics," *Fuzzy Sets Syst.*, vol. 26, no. 3, pp. 275–281, 1988.

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