



# Inelastic electron dephasing scattering times in disordered metals

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## Abstract

We have studied and compared the inelastic electron dephasing scattering times in disordered metals with significantly different characteristics. Both the temperature dependence and electron elastic mean-free-path ( $l$ ) dependence of the electron–phonon scattering times,  $\tau_{ep}$ , are determined from weak-localization studies. Our experimental results suggest that  $\tau_{ep}^{-1} \sim T^2$  in numerous crystalline disordered dilute  $Ti_{1-x}Al_x$ , dilute  $Ti_{1-x}Sn_x$ ,  $Au_{50}Pd_{50}$ , and  $Ti_{73}Al_{27}$  alloys. However, our results do not support a universal dependence of  $\tau_{ep}^{-1}$  on  $l$  among these various metals. Our observation of the  $T^2$ -law is in disagreement with current theoretical concept for electron–phonon interaction in disordered metals. In addition, we have inferred the critical electron–electron scattering times,  $\tau_{EE}$ , in a number of very low-diffusivity thick Sc,  $RuO_2$ , and  $IrO_2$  films. We find that  $\tau_{EE}^{-1} \sim T$  and also that  $\tau_{EE}^{-1}$  is independent of  $l$ . This observation is understood in terms of the current theory for inelastic electron–electron scattering in bulk metals near the mobility edge. Our results altogether establish a crossover of the inelastic electron dephasing from electron–phonon scattering to electron–electron scattering as the amounts of disorder greatly increase and the systems move significantly toward the Anderson transition. © 2000 Elsevier Science B.V. All rights reserved.

**Keywords:** Dephasing times; Electron–phonon relaxation; Electron–electron relaxation; Disordered metals

## 1. Introduction

Electron dephasing scattering time,  $\tau_\phi$ , is a quantity of fundamental interest and importance in disordered metals and mesoscopic systems. Both the theoretical and experimental investigations of  $\tau_\phi$  have advanced significantly over the years, largely due to the realization of weak-localization (WL) effects in the presence of strong impurity scattering. In practice,  $\tau_\phi$  totally controls the magnitude and temperature dependence of the WL effects and is given by [1,2]

$$\tau_\phi^{-1}(T) = \tau_c^{-1} + \tau_i^{-1}(T), \quad (1)$$

where  $\tau_c$  is a constant, and  $\tau_i$  is the relevant inelastic electron scattering time(s) in question. Owing to extensive theoretical and experimental studies over the past 20 years, it is now known that the microscopic processes responsible for  $\tau_c^{-1}$  and  $\tau_i^{-1}$  can essentially be characterized and ascribed to four different origins. It is also known that a few scattering processes will usually coexist in a given system, but one or two processes would dominate over the others under certain conditions, depending on the system *dimensionality*, *amounts of disorder*, and the measuring temperatures involved. The four electron dephasing scattering mechanisms and the particular (experimental) circumstances where each individual process might dominate are categorized as follows.

(a) In three-dimensional (3D) weak-disordered metals, electron–phonon (e–ph) scattering is the sole, dominant inelastic dephasing process, causing  $\tau_i^{-1} \approx \tau_{ep}^{-1}$ . Both the temperature dependence and electron elastic mean-free-path  $l$  dependence of  $\tau_{ep}^{-1}$  have been studied in, e.g., thick granular films, metal–insulator composites,

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and amorphous metals [1,2]. However, the behavior of  $\tau_{ep}^{-1}$  in the presence of disorder is still poorly understood. In particular, experimental results vary widely and are frequently in disagreement with theoretical predictions [3–6]. (b) In 3D strong-disordered metals, electron–electron (e–e) scattering is very sensitive to the critical current dynamics, resulting in an e–e rate  $\tau_{EE}^{-1}$  dominating over the e–ph rate [7]. Thus far, studies of  $\tau_i^{-1} \approx \tau_{EE}^{-1}$  in metals near the mobility edge have not attracted much attention. (c) In mesoscopic systems at very low temperatures, a saturation of  $\tau_\phi^{-1}$  ( $\approx \tau_c^{-1}$ ) is very often observed. The underlying physics of  $\tau_c^{-1}$  is currently under much debate [8]. While some authors argue for a  $\tau_c^{-1}$  caused by magnetic spin–spin scattering or external microwave fields, others argued for a  $\tau_c^{-1}$  caused by a more intrinsic quantum origin (e.g., the zero-point fluctuations of the electrons). (d) In thin metal and semiconductor wires and films at low temperatures, small energy-transfer (‘quasielastic’) e–e scattering is the dominant inelastic process [1,2], causing  $\tau_i^{-1} \approx \tau_{ee}^{-1}$  (or,  $\tau_i^{-1} \approx \tau_{ee}^{-1} + \tau_{ep}^{-1}$  if the temperature is not too low so that e–ph scattering is not negligible). Among the four dephasing processes just discussed, this last scattering rate  $\tau_{ee}^{-1}$  is the most widely studied and best-understood electron dephasing rate.

In this work, we shall concentrate on our experimental results for  $\tau_{ep}^{-1}$  and  $\tau_{EE}^{-1}$ ; experimental results concerning the other two dephasing rates  $\tau_c^{-1}$  and  $\tau_{ee}^{-1}$  will not be discussed. Our experimental results are derived from *three-dimensional* WL studies of various impure metals with *significantly different* characteristics.

## 2. $\tau_{ep}^{-1}(T, l)$ in disordered metals

The e–ph scattering time is a very fundamental quantity in solid state physics. In the case of clean metals, the cubic temperature dependence of  $\tau_{ep}^{-1}$  is well established. In the case of disordered metals, the e–ph interaction has been calculated by a number of authors [3–6,9–11] for over two decades; it is widely accepted that a consensus has recently been reached in theoretical efforts [3–6]. Experimentally, however, few measurements have successfully provided an overall consistency check for the various aspects of the theoretical predictions. One difficulty in this area of study is the problem of making and comparing samples with significantly different characteristics. In order to advance our understanding of the e–ph interaction in disordered metals, we have, in the last several years, launched stage by stage, and fairly quantitative measurements of  $\tau_{ep}^{-1}$  using carefully selected and tailored 3D metal samples, including dilute  $Ti_{1-x}(Al,Sn)_x$ ,  $Au_{50}Pd_{50}$ , and relatively disordered  $Ti_{73}Al_{27}$  alloys. Our experimental approaches have numerous advantages over previous experimental approaches. Therefore, we are able to provide crucial information about  $\tau_{ep}^{-1}$  on  $T$  and on  $l$ . Such information is

of prime importance for a stringent justification for the current e–ph interaction theory of dirty metals.

To facilitate a close comparison of our 3D experiments with theory, we rewrite Eq. (1) in the form

$$\tau_\phi^{-1}(T) = \tau_c^{-1} + \tau_{ep}^{-1}(T) = \tau_c^{-1} + AT^p, \quad (2)$$

where  $A$  is the strength of e–ph interaction, and  $p$  is a positive integer. Usually,  $p = 2, 3$ , or 4. As mentioned, for the case of *three* dimensions, e–ph scattering is the *sole*, significant inelastic process while the quasielastic e–e scattering is not important. Therefore, we have identified  $\tau_i^{-1}$  with  $\tau_{ep}^{-1}$  in Eq. (1) – this is very much unlike that in the cases of thin wires and films in which  $\tau_i^{-1} \approx \tau_{ee}^{-1} + \tau_{ep}^{-1}$  at liquid-helium temperatures, making difficult a quantitative separation of  $\tau_{ep}^{-1}$  from the total  $\tau_i^{-1}$ . This is one of the main reasons why we have focused our measurements on three dimensions. Furthermore, the phonon dimensionality is clearly 3D in our case while it is not very well defined in the cases of thin wires and films.

### 2.1. Compositional disorder: dilute titanium alloys

In the first stage of our studies of  $\tau_{ep}^{-1}$ , we have used 3D crystalline disordered dilute titanium alloys,  $Ti_{1-x}Al_x$  and  $Ti_{1-x}Sn_x$ , prepared by a standard arc-melting method [12,13]. These alloys are metal samples essentially *microscopically homogeneous* and having a *wide* range of *high* impurity resistivity  $\rho_0 \equiv \rho(10K)$  ( $\approx 40$ – $160 \mu\Omega\text{cm}$ , corresponding to  $k_F l \approx 5$ – $20$ ). Such a wide range of  $\rho_0$  makes feasible a reliable experimental determination of the dependence of  $\tau_{ep}^{-1}$  on  $l$ , in addition to the dependence on  $T$ .

Fig. 1 shows the variations of  $\tau_{ep}^{-1}$  with temperature for several  $Ti_{1-x}Al_x$  alloys. The solid line is drawn proportional to  $T^2$  and guides the eye. This figure clearly illustrates that  $\tau_{ep}^{-1} \sim T^2$ . Furthermore, a close inspection of Fig. 1 indicates that the magnitude of  $\tau_{ep}^{-1}$  at a given temperature increases with increasing  $x$  (or,  $\rho_0$ , which increases essentially linearly with increasing  $x$ ). A similar behavior has also been observed in several  $Ti_{1-x}Sn_x$  alloys. Fig. 2 shows the variation of our experimental  $\tau_{ep}^{-1}(10K)$  with  $\rho_0$  for  $Ti_{1-x}Al_x$  (○) and  $Ti_{1-x}Sn_x$  (●) alloys. The solid lines guide the eye. Inspection of this figure clearly suggests that  $\tau_{ep}^{-1}$  is disorder dependent and, more precisely,  $\tau_{ep}^{-1}(10K) \sim \rho_0 \sim l^{-1}$ .

It is of importance to check whether our experimental results for  $\tau_{ep}^{-1}$  satisfy the disorder criterion for e–ph interaction  $ql \lesssim 1$ , where  $q \approx k_B T / \hbar v_s$  is the wave number of the thermal phonons, and  $v_s$  is the sound velocity. For our dilute titanium alloys studied,  $l \approx 2.6$ – $11 \text{ \AA}$ . We take  $v_s \approx 5000 \text{ m/s}$ . Then,  $ql \approx (0.0068 - 0.029) T$ , indicating that the e–ph processes in these alloys are *well* within the disordered limit at our measuring temperatures of 3–15 K.

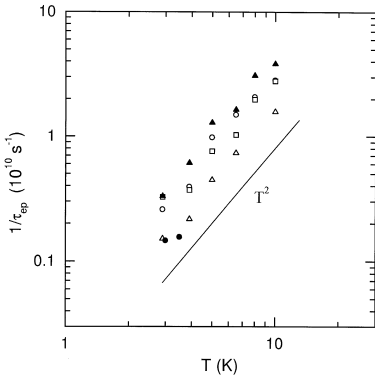


Fig. 1.  $\tau_{ep}^{-1}$  versus  $T$  for five  $Ti_{1-x}Al_x$  alloys with  $x = 0.053$  ( $\Delta$ ),  $0.072$  ( $\bullet$ ),  $0.080$  ( $\square$ ),  $0.102$  (solid triangles), and  $0.150$  ( $\circ$ ).

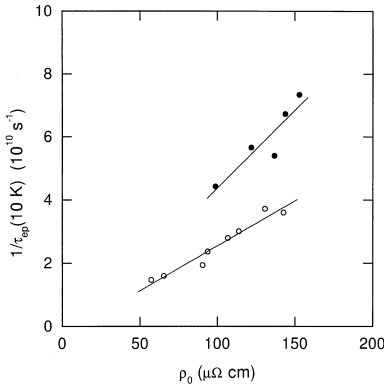


Fig. 2. Variation of  $\tau_{ep}^{-1}(10\text{K})$  with  $\rho_0$  for  $Ti_{1-x}Al_x$  ( $\circ$ ) and  $Ti_{1-x}Sn_x$  ( $\bullet$ ) alloys.

### 2.2. Structural disorder: $Au_{50}Pd_{50}$ alloys

In the second stage of our systematic investigations of  $\tau_{ep}^{-1}$ , we have used several thick  $Au_{50}Pd_{50}$  films having a wide range of high  $\rho_0$  ( $\approx 70\text{--}230 \mu\Omega \text{ cm}$ ). Our films were prepared by DC sputtering deposition; the deposition rate was varied to ‘tune’ the amount of disorder  $\rho_0$  [14]. Unlike in the case of dilute titanium alloys where the degree of disorder was controlled by the amounts of the impurity atoms (Al or Sn) doped, the composition in this case is fixed (i.e.,  $Au_{50}Pd_{50}$ ) while the degree of disorder is controlled by the structural arrangement of Au and Pd atoms. Therefore, this series of samples possess very different qualities of disorder from that in the dilute titanium alloys.

A careful analysis of the variation of  $\tau_{ep}^{-1}$  with  $T$  in our AuPd samples also reveals that  $\tau_{ep}^{-1} \sim T^2$  (not shown).

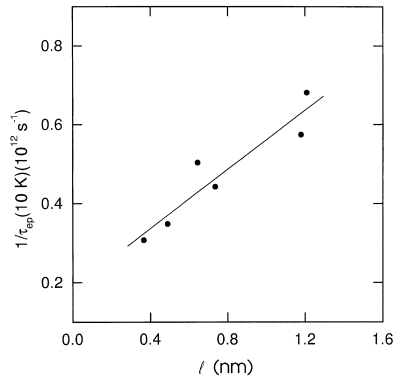


Fig. 3. Variation of  $\tau_{ep}^{-1}(10\text{K})$  with  $l$  for several thick  $Au_{50}Pd_{50}$  films.

Fig. 3 shows the e–ph rate  $\tau_{ep}^{-1}(10\text{K})$  as a function of the electron mean free path  $l$  for several AuPd films. This figure clearly illustrates that  $\tau_{ep}^{-1} \sim l$ .

For our thick AuPd films,  $v_s \approx 2.0 \times 10^3 \text{ m/s}$ , and  $l \approx 3.7\text{--}12 \text{ \AA}$ . Then,  $ql \approx (0.024\text{--}0.078)T$ , indicating that the e–ph processes in our dirty films are well within the disordered limit even at temperatures as high as  $\sim 20\text{K}$ .

### 2.3. Dirty limit: $Ti_{73}Al_{27}$ alloys

Apart from the experimental results obtained above, there are a number of measurements in various metal films which also report a  $\tau_{ep}^{-1} \sim T^2$  [15–18]. Theoretically, however, the current calculations predict a  $T^4$  law in the dirty limit, which is essentially unseen in experiments [19]. Some authors have argued that the  $T^4$  law is absent in most measurements because the materials previously studied are *not yet* strongly disordered enough for the e–ph interactions to strictly satisfy the disorder criterion of  $ql \ll 1$ . Information about  $\tau_{ep}^{-1}$  on  $T$  in samples having smaller values of  $ql$  than ever reported is therefore of crucial importance for a profound understanding of the nature of the e–ph interaction in disordered metals.

In the third stage of our investigations of  $\tau_{ep}^{-1}$ , we have therefore used a series of 3D crystalline disordered Sn-doped  $Ti_{73}Al_{27}$  alloys [20]. The resistivities of these alloys are fairly large ( $\rho_0 \approx 225 \mu\Omega \text{ cm}$ ) and barely changed upon Sn doping. Such an extremely high value of  $\rho_0$ , corresponding to *an l being on the order of the interatomic spacing*, put the e–ph interaction in this material *much closer* to the dirty limit than *ever* obtained in any previous material. Noticeably, our least-squares fits of experimental  $\tau_{ep}^{-1}$  with Eq. (1) result in  $p \approx 1.9 \pm 0.2$  for this series of  $Ti_{73-x}Al_{27}Sn_x$  alloys (not shown). That is, even in this very low- $ql$  metal, our experimental value of  $p$  implies again that  $\tau_{ep}^{-1}$  varies quadratically with  $T$ .

For  $\text{Ti}_{73}\text{Al}_{27}$  alloys, we estimate that  $v_s \approx 4 \times 10^3 \text{ m s}^{-1}$  and  $l \approx 1.6 \text{ \AA}$ , and thus a very small value of  $ql \approx 0.0056T$ . This indicates that the e–ph process in these alloys is *very well* within the dirty limit at our measuring temperatures. This experimental observation therefore provides a very strong evidence in support of the  $T^2$ -law for  $\tau_{\text{ep}}^{-1}$  in impure metals. (Recall that  $l$  already takes the smallest possible value, i.e., the interatomic spacing, in this experiment, and thus  $ql$  cannot be readily made any smaller, presuming that  $v_s$  does not change significantly with disorder.)

#### 2.4. Comparison with theory

Schmid and coworkers [3] have recently evaluated the problem of e–ph interaction in disordered metals. They consider impurity atoms that move in phase with the other lattice atoms. They find that there is a subtle compensation between the coupling of the electrons to the vibrating impurities and the interaction of the electrons with the deformed lattice vibrations. Consequently, the e–ph scattering rate in the dirty limit is ‘weakened’ and is given by  $\tau_{\text{ep}}^{-1} \sim (ql)(\tau_{\text{ep},0}^{-1}) \sim T^4/l$ , where  $\tau_{\text{ep},0}^{-1} \sim T^3$  is the e–ph rate in the pure metals. This theory has received wide acceptance from the theoretical community, and it has been independently confirmed by Reizer and Sergeev, and Belitz [4–6].

As discussed, the e–ph interaction in the various samples that we studied falls well within the dirty-limit regime. However, the theoretically expected  $T^4$ -law is not observed in any of our experiments. On the contrary, we find that  $\tau_{\text{ep}}^{-1} \sim T^2$ . The origin for the  $T^2$  law is still not understood. In fact, a quadratic temperature dependence of  $\tau_{\text{ep}}^{-1}$  has previously been observed in both thin films and bulk samples [15–18]; it has caused much confusion and controversy on the nature of the e–ph interaction in disordered metals. Moreover, apart from the WL experimental results, there exist a good number of tunneling measurements in literature that suggest an Eliashberg function  $\alpha^2F(\omega) \sim \omega$  in dirty metals [21]. An  $\alpha^2F(\omega)$  linear in  $\omega$  implies that  $\tau_{\text{ep}}^{-1} \sim T^2$ . Our systematic establishment of the  $T^2$  dependence in the dirty limit of  $ql \ll 1$  should therefore cause us to rethink what heretofore has been taken for granted concerning e–ph interaction in impure metals.

To the author’s knowledge, a linear dependence of  $\tau_{\text{ep}}^{-1}$  on  $l^{-1}$  as observed in our  $\text{Ti}_{1-x}(\text{Al},\text{Sn})_x$  alloys has never been reported in previous experiments. Also, a linear dependence of  $\tau_{\text{ep}}^{-1}$  on  $l$  as observed in our AuPd alloys has not been reported in previous 3D experiments. The former disorder behavior is consistent with an ‘old’ e–ph interaction theory which considered only the scattering of electrons by impurity vibrations [9–11]. The latter disorder behavior is in agreement with the prediction of the Rammer–Schmid theory [3]. It is quite surprising that opposite (linear and inversely linear)

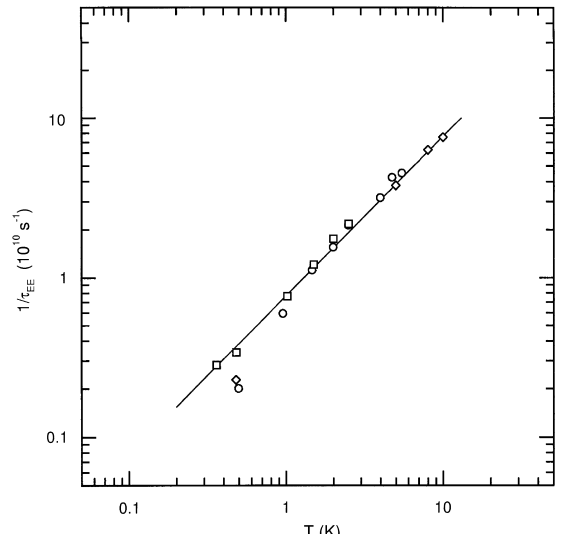


Fig. 4.  $\tau_{\text{EE}}^{-1}$  as a function of  $T$  for three thick Sc films with  $D \approx 0.25$  ( $\diamond$ ),  $0.36$  ( $\square$ ), and  $0.59 \text{ cm}^2/\text{s}$  ( $\circ$ ).

dependencies of  $\tau_{\text{ep}}^{-1}$  on  $l$  are realized in our different experiments. Measurements are under way to clarify the underlying physics.

### 3. $\tau_{\text{EE}}^{-1}$ near the mobility edge

In the course of our studies of  $\tau_i^{-1}$  in disordered metals, in addition to the  $\tau_{\text{ep}}^{-1}$  in AuPd and titanium alloys discussed above, we have also systematically measured  $\tau_i^{-1}$  in very low-diffusivity thick Sc,  $\text{RuO}_2$ , and  $\text{IrO}_2$  films [22,23]. We find that in these low-diffusivity samples,  $\tau_i^{-1} \approx \tau_{\text{EE}}^{-1} \sim T$  and there is essentially *no* dependence of  $\tau_{\text{EE}}^{-1}$  on  $l$ . Such temperature and disorder behavior of  $\tau_{\text{EE}}^{-1}$  is distinctly different from that expected for the e–ph scattering rate discussed above.

Our main results of  $\tau_{\text{EE}}^{-1}$  as a function of  $T$  for three thick Sc films are plotted in Fig. 4, where the straight solid line is drawn proportional to  $T$  and guides the eye. This figure reveals that  $\tau_{\text{EE}}^{-1} \sim T$  for well over a decade of temperature. In addition, the magnitudes of  $\tau_{\text{EE}}^{-1}$  are very similar for all the three films studied, regardless of the difference in the amount of disorder (see the caption to Fig. 4) in the various films. This result is strongly suggestive of a disorder-insensitive dephasing scattering mechanism operating in 3D dirty Sc films. In fact, we have also found similar results in a number of very low-diffusivity thick  $\text{RuO}_2$  and  $\text{IrO}_2$  films. Thus, our observations altogether indicate that the temperature and disorder behavior of  $\tau_{\text{EE}}^{-1}$  as shown in Fig. 4 is common to very low-diffusivity metals.

In the case of *three* dimensions, the behavior of  $\tau_{\text{EE}}^{-1} \sim Tl^0$  has been theoretically explored and is quali-

tatively understood. Recently, this observation has been attributed to the e–e scattering in impure metals near the mobility edge. Belitz and Wysokinski [4] have calculated the inelastic quasiparticle lifetime due to a Coulomb interaction in strongly disordered 3D metals. They found that the inelastic e–e scattering is very sensitive to the critical (as opposed to diffusive) current dynamics in systems near the Anderson transition. Particularly, they observed a linear  $T$  dependence of the inelastic scattering rate. They also predicted that the inelastic scattering rate should be disorder independent. Their predictions are in line with our experimental results for thick Sc, RuO<sub>2</sub> and IrO<sub>2</sub> films, which, on the one hand have very low values of  $D$  in common ( $D \approx 0.25\text{--}0.75 \text{ cm}^2 \text{ s}^{-1}$ ), but otherwise have significantly different characteristics. Very recently, we have also observed low values of  $p$  ( $\sim 1.3$ ) in disordered Cu<sub>x</sub>Ge<sub>100-x</sub> alloys which are close to the metal–insulator transition [24]. It is worth stressing that our observed linear  $T$  behavior of  $\tau_i^{-1} \approx \tau_{EE}^{-1}$  is *not* due to the more familiar 2D quasielastic e–e processes usually operating in weakly disordered thin metal films in which  $\tau_i^{-1} \approx \tau_{ee,2D}^{-1} \sim T$  has been very well established [1, 2].

#### 4. Conclusion

We have measured both the temperature dependence and disorder dependence of  $\tau_{ep}^{-1}$  in various impure metals. Our experimental results suggest a quadratic temperature dependence of  $\tau_{ep}^{-1}$  in the dirty limit of  $ql \ll 1$  for e–ph interaction. However, our results do *not* support a universal dependence of  $\tau_{ep}^{-1}$  on  $l$ . Our observations should cause us to rethink what heretofore has been taken for granted concerning e–ph scattering in dirty metals. In addition, we have observed a linear dependence of  $\tau_{EE}^{-1}$  on  $T$  in numerous very low-diffusivity metals. We also find that  $\tau_{EE}^{-1}$  depends very weakly on  $l$  in such materials. Our observation is understood in terms of the current theoretical concept that considers the dynamics of the inelastic e–e scattering in metals near the mobility edge. Our results altogether establish a crossover of the

inelastic electron dephasing from e–ph scattering to e–e scattering as the amounts of disorder greatly increase and the systems move significantly toward the Anderson transition.

#### Acknowledgements

The author is grateful to C.Y. Wu, W.B. Jian, T.J. Li, Y.L. Zhong, W. Xu, P.J. Sheng, and S.Y. Hsu for their help in the various stages of the experiment. This work was supported by Taiwan National Science Council through Grant No. NSC 88-2112-M-009-027.

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