JOURNAL OF INFORMATION SCIENCE AND ENGINEERING 24, 1-9 (2008)

# A Novel Approach to Comparison-Based Diagnosis for Hypercube-Like Systems*

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Interconnection network has been an active research area for parallel and distributed computer systems. The diagnosability is one of the important issues in the reliability of interconnection networks. In this paper, a novel idea on system diagnosis called local diagnosability is presented. The concept of local diagnosability is strongly related to the traditional global one. For this local sense, the status of every particular processor can be correctly identified. A sufficient condition is also proposed to determine the local diagnosability of a given processor. Following this local sense, we prove that the diagnosability of an $n$-dimensional hypercube-like network $HL_n$ is $n$ for $n \geq 5$, and show that in $HL_n$ with up to $n - 2$ faulty links, the local diagnosability of each processor equals to the connection links incident with it.

***Keywords:*** locally $t$-diagnosable, local diagnosability, comparison model, extended star, hypercube-like system

## 1. INTRODUCTION

With the rapid development of technology, multiprocessor systems are more and more popular. One of the important issues about multiprocessor systems is the reliability of the processors in it. In order to maintain the reliability of a system, if a processor is found faulty, it should be replaced by a fault-free one. The procedure of identifying faulty processors is called the diagnosis of the system. The maximum number of faulty processors that a system can guarantee to identify is called the diagnosability of the system.

There are several approaches for interconnected processors to perform self-diagnosis. One major approach is called the comparison model, first proposed by Maeng and Malek [6, 7]. This approach performs diagnosis by sending the same inputs to a pair of adjacent processors and comparing their responses. Under this model, an $n$-dimensional hypercube is proved to have diagnosability $n$ for $n \geq 5$ [11], and an $n$-dimensional enhanced hypercube has diagnosability $n + 1$ for $n \geq 6$ [12]; Fan [3, 4] showed that the diagnosability of an $n$-dimensional Crossed cube is $n$, and that of an $n$-dimensional Möbius cube is $n$ for $n \geq 4$; a $k$-ary $n$-dimensional butterfly graph has diagnosability $2k - 2$ when $k \geq 3$ and $n \geq 2$ [1]; Lai *et al.* [5] proposed that the diagnosability of the matching composition network is $n$ for $n \geq 4$; Chang *et al.* [2] investigated the diagnosabilities of regular networks.

In the previous studies on diagnosis, most investigators focused on the global diag-

Received February 2, 2007; accepted July 13, 2007.
Communicated by K. Robert Lai, Yu-Chee Tseng and Shu-Yuan Chen.
* This work was supported in part by the National Science Council of Taiwan, R.O.C. under contract No. NSC 95-2221-E-009-134-MY3.

nosis ability of a system but ignored some local systematic details. For example, if the diagnosability of a system is $t$, this system is $t$-diagnosable. That is, given any syndrome $\sigma$, all the faulty processors in a system $G$ can be precisely identified, given that there are at most $t$ faulty processors in system $G$. But in some situations, it is possible to correctly point out all faulty processors in some part of the system $G$, even if there are more than $t$ faulty processors in $G$. Thus, only considering the global status let us lose some local details of a system.

In this paper, we propose a new concept on system diagnosis, which is called local diagnosability. More local information about a system can be retrieved through this concept. By our new definition of diagnosability, every processor in a system has its own local diagnosability which states some kind of connection status around it. Moreover, we propose a sufficient condition to easily compute the local diagnosability of each processor based on the comparison model. Finally, we can get back to the original global diagnosis in the point of view of local diagnosis. As a case study, we prove that the diagnosability of an $n$-dimensional hypercube-like network $HL_n$ is $n$ for $n \geq 5$ via this local sense, and show that in an $n$-dimensional hypercube-like network with up to $n - 2$ faulty links, the local diagnosability of each processor equals to the connection links incident with it.

## 2. PRELIMINARIES

In this section, the basic graph definitions and notations are given [13]. $G = (V, E)$ is a graph if $V$ is a finite set and $E$ is a subset of $\{(u, v) \mid (u, v)$ is an unordered pair of $V\}$. The degree of node $v$ in a graph $G$ is the number of edges incident with $v$. A node cover of $G$ is a subset $Q \subset V(G)$ such that every edge of $E(G)$ has at least one end node in $Q$. A node cover set with the minimum cardinality is called a minimum node cover.

Vaidya *et al.* [10] introduced a class of hypercube-like interconnection networks, which can be defined by applying a $\oplus$ operation repeatedly as follows: $HL_0 = \{K_1\}$, where $K_1$ is a graph with just one node and no edges; for $m \geq 1$, $HL_m = \{HL^0_{m-1} \oplus HL^1_{m-1} \mid HL^0_{m-1}, HL^1_{m-1} \in HL_{m-1}\}$, which has node set $V(HL^0_{m-1}) \cup V(HL^1_{m-1})$ and edge set $E(HL^0_{m-1}) \cup E(HL^1_{m-1}) \cup M$, where $M$ is an arbitrarily perfect matching between the node sets of $HL^0_{m-1}$ and $HL^1_{m-1}$. That is, $M$ is a set of edges connecting the nodes in $HL^0_{m-1}$ and in $HL^1_{m-1}$ with a bijection.

For the purpose of self-diagnosis, several different models had been proposed. The comparison model, also called the MM model, proposed by Maeng and Malek [7, 8], is considered as a major approach for fault diagnosis in multiprocessor systems. In this approach, every processor performs diagnosis by sending two identical signals to two other linked processors, and comparing their returning responses. The agreement is denoted by 0, whereas the disagreement is denoted by 1. After the completion of diagnosis on every processor, a syndrome is generated by collecting all these testing results. For a given syndrome $\sigma$, a subset of processors $F \subset V(G)$ is said to be consistent with $\sigma$ if the syndrome $\sigma$ can be produced when all processors in $F$ are faulty and all processors in $V - F$ are fault-free. We say that a system is diagnosable if, for every syndrome $\sigma$, an unique set of processors $F \subset V$ is consistent with it. A system is defined to be $t$-diagnosable if the system is diagnosable as long as the number of faulty processors does not exceed $t$. The maximum number $t$ for a system being $t$-diagnosable is called the diagnosability of the system. Two distinct subsets of processors $F_1$, $F_2 \subset V$ are distinguishable if and only if every syndrome consistent with $F_1$ is different to those consistent with $F_2$.

A labeled multigraph $M = (V, C)$ is usually used to model this diagnosis strategy, where $V$ represents the set of all processors in $G$ and $C$ represents the set of labeled edges. Each labeled edge $(u, v)_w \in C$ implies that processors $u$ and $V$ are being compared by processor $w$.

The following is a characterization presented by Sengupta and Dahbura to determine the distinguishability of two sets of processors in a system.

**Lemma 1** [9] For every two distinct subsets of nodes $F_1$ and $F_2$, $(F_1, F_2)$ is a distinguishable pair if and only if at least one of the following conditions is satisfied:

(1) $\exists u, w \in V - F_1 - F_2$ and $\exists v \in (F_1 - F_2) \cup (F_2 - F_1)$ such that $(u, v)_w \in C$,
(2) $\exists u, v \in F_1 - F_2$ and $\exists w \in V - F_1 - F_2$ such that $(u, v)_w \in C$, or
(3) $\exists u, v \in F_2 - F_1$ and $\exists w \in V - F_1 - F_2$ such that $(u, v)_w \in C$.

## 3. LOCAL DIAGNOSABILITY

In this section, we define the concept of local diagnosability, and provide some practical theorems about it. Applying these theorems, the traditional global diagnosability of a system can be easily computed.

**Definition 1** A system $G(V, E)$ is locally $t$-diagnosable at node $x \in V(G)$ if, given a test syndrome $\sigma_F$ produced by the system under the presence of a set of faulty nodes $F$ containing node $x$ with $|F| \le t$, every set of faulty nodes $F'$ consistent with $\sigma_F$ and $|F'| \le t$, must also contain node $x$.

**Definition 2** The local diagnosability $t_l(x)$ of a node $x \in V(G)$ in a system $G(V, E)$ is defined to be the maximum number of $t$ for $G$ being locally $t$-diagnosable at $x$.

The concept of a system being locally $t$-diagnosable at a node $x$ is consistent with the traditional concept of a system being $t$-diagnosable in the global sense. We can see the relationship between these two concepts by the following theorem.

**Theorem 1** A system $G(V, E)$ is $t$-diagnosable if and only if $G$ is locally $t$-diagnosable at $x$, for every $x \in V(G)$.

In the following, we propose a sufficient condition for verifying whether a system $G$ is locally $t$-diagnosable at a given node $x$. Before discussing it, we need some definitions of terminologies. For any set of nodes $S \subset V(G)$, let $G - S$ denote the subgraph of $G$ induced by the node subset $V(G) - S$. Let $S$ be a set of nodes and $x$ be a node *not* in $S$, we use $C_{x,S}$ to denote the connected component which $x$ belongs to in $G - S$.

**Theorem 2** A system $G(V, E)$ is locally $t$-diagnosable at a given node $x \in V(G)$ if, for every set of nodes $S \subset V(G)$, $|S| = p$, $0 \le p \le t - 1$, and $x \notin S$, the cardinality of every node cover including $x$ of the component $C_{x,S}$ is at least $2(t - p) + 1$.

***Proof:*** We prove this theorem by ways of contradiction. Suppose on the contrary that $G$ is *not* locally $t$-diagnosable at node $x$. By Definition 1, we know that there exists two distinct sets of nodes $F_1 \ne F_2 \subset V$ with $|F_i| \le t$, $i = 1, 2$, and $x \in (F_1 \cup F_2) - (F_1 \cap F_2)$, such that $(F_1, F_2)$ is an indistinguishable pair. Let $S = F_1 \cap F_2$, then $|S| = p \le t - 1$. According to the condition, the cardinality of a node cover including $x$ of the component $C_{x,S}$ is at least $2(t - p) + 1$. Since $|(F_1 \cup F_2) - (F_1 \cap F_2)| \le 2(t - p)$ and $x \in (F_1 \cup F_2) - (F_1 \cap F_2)$, there is at least one node in the node cover of $C_{x,S}$ lying in $C_{x,S} - [(F_1 \cup F_2) - (F_1 \cap F_2)]$, and there is at least one edge of $C_{x,S}$ lying in $C_{x,S} - [(F_1 \cup F_2) - (F_1 \cap F_2)]$, consequently. Then, $(F_1, F_2)$ is a distinguishable pair since it satisfies condition 1 of Lemma 1. Therefore $G$ is locally $t$-diagnosable at node $x$, which is a contradiction. ❑

We now propose a substructure at a node to guarantee its local diagnosability.

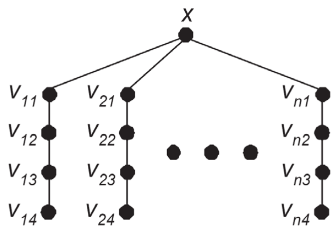



Fig. 1. Extended star sturcture $ES(x; n)$.

**Definition 3** Let $x$ be a node in a graph $G(V, E)$. For $n \le deg(x)$, an extended star $ES(x; n)$ of order $n$ at node $x$ is defined as $ES(x; n) = (V(x; n), E(x; n))$, where the set of nodes $V(x; n) = \{x\} \cup \{v_{ij} \mid 1 \le i \le n, 1 \le j \le 4\}$ and the set of edges $E(x; n) = \{(x, v_{k1}), (v_{k1}, v_{k2}), (v_{k2}, v_{k3}), (v_{k3}, v_{k4}) \mid 1 \le k \le n\}$.

We say that there is an *extended star* structure $ES(x; n) \subseteq G$ at node $x$ if $G$ contains an extended star $ES(x; n)$ of order $n$ at node $x$ as a subgraph.

As stated in the following theorem, the extended star is an useful structure to check the local diagnosability of a node in a system. Then, the global diagnosability can be also derived consequently.

**Theorem 3** Let $x$ be a node in a system $G(V, E)$ with $deg(x) = n$. The local diagnosability of $x$ is $n$ if there exists an extended star $ES(x; n) \subseteq G$ at $x$.

***Proof:*** For the first part of this proof, we claim that the local diagnosability of $x$ is at most $n$. Suppose on the contrary that the local diagnosability of $x$ is larger than $n$, *i.e.* $t_l(x) > n$. Let the nodes adjacent to $x$ be $v_k$, for all $k$, $1 \le k \le n$. Let $F_1$ be the set $\{x\} \cup \{v_k \mid k = 1 \text{ to } n\}$ and $F_2$ be the set $\{v_k \mid k = 1 \text{ to } n\}$. Then, $(F_1, F_2)$ is not a distinguishable pair according to Lemma 1. So this system is not locally $(n + 1)$-diagnosable at node $x$, which is a contradiction. Therefore, the local diagnosability of $x$ is at most $n$, that is, $t_l(x) \leq n$.

Now, we claim that the local diagnosability of $x$ is at least $n$ if there exists an extended star $ES(x; n) \subseteq G$ at $x$. We use Theorem 2 to prove this result. First, we define $l_k = (v_{k1}, v_{k2}, v_{k3}, v_{k4})$ to be a quadruple of four consecutive nodes for any $k$, $1 \leq k \leq n$, with respect to $ES(x; n)$. We note that $l_k$ is a path of length 3. Accordingly, the cardinality of a node cover of each $l_k$ is at least 2. Let $S \subset V(G)$ be a set of nodes in $G$ with $|S| = p \leq n - 1$ and $x \notin S$. After deleting $S$ from $V(G)$, there are at least $(n - p)$ complete $l_k$'s still remaining in $ES(x; n)$, where the word "complete" means that all $v_{k1}$, $v_{k2}$, $v_{k3}$, and $v_{k4}$ of an $l_k$ have not been deleted in $G - S$. Thus, the cardinality of a node cover including $x$ of the connected component $C_{x,S}$ is at least $1 + 2(n - p)$. Therefore, the system $G$ with an extended star $ES(x; n)$ is locally $n$-diagnosable at $x$ by Theorem 2. By Definition 2, the local diagnosability of $x$ is at least $n$, that is, $t_l(x) \geq n$.

Therefore, $t_l(x) = n$, and the proof is complete. ❑

## 4. DIAGNOSABILITY OF HYPERCUBE-LIKE NETWORKS

For many practical interconnection networks, an extended star structure does exist and can be found without too much effort. As an example, we apply the concept of local diagnosability to the class of hypercube-like networks.

**Theorem 4** The diagnosability of an $n$-dimensional hypercube-like network $HL_n$ is $n$ for $n \geq 5$.

***Proof:*** We will find an extended star structure at every node, and then the result follows from Theorems 1 and 3. We now prove it by induction on $n$, the dimension of a hypercube-like network $HL_n$.

**BASIS** For the basis of this theorem, we claim that $HL_5$ is 5-diagnosable. For every node $x$ in $HL_5$, we can easily find an extended star subgraph $ES(x, 5)$ at $x$ since $n$ is greater than or equal to five. By Theorem 3, the system $HL_5$ is locally 5-diagnosable at every node. So $HL_5$ is 5-diagnosable by Theorem 1.

**HYPOTHESIS** Suppose $HL_{n-1}$ contains an extended star of order $n - 1$ at every node as a subgraph, which implies that this theorem holds for $HL_{n-1}$.

**INDUCTION** Consider an $n$-dimensional hypercube-like network $HL_n$. We want to show that there is an extended star structure at each node in $HL_n$. Consider an arbitrary node $x$ in $HL_n$, we can separate $HL_n$ into two $HL_{n-1}$, denoted by $HL^0_{n-1}$ and $HL^1_{n-1}$. Without loss of generality, we may assume that $x$ is in $HL^0_{n-1}$. By hypothesis, there is an extended star subgraph $ES(x, n - 1)$ at $x$ in $HL^0_{n-1}$. Consider the corresponding node $x'$ in $HL^1_{n-1}$, that is, the node in $HL^1_{n-1}$ which is adjacent to $x$. There is also an extended star subgraph $ES(x', n - 1)$ at $x'$ in $HL^1_{n-1}$. Hence there is an extended star subgraph $ES(x, n)$ at $x$ in $HL_n$. So, the system $HL_n$ is locally $n$-diagnosable at node $x$ by Theorem 3, and $HL_n$ is $n$-diagnosable by Theorem 1. ❑

Furthermore, we consider the case of an interconnection network with some communication link faults, and study the effect on its local diagnosability.

**Theorem 5** In an $n$-dimensional hypercube-like network $HL_n$ ($n \geq 5$) with at most $n - 2$ edge faults, the local diagnosability of each node equals to its degree.

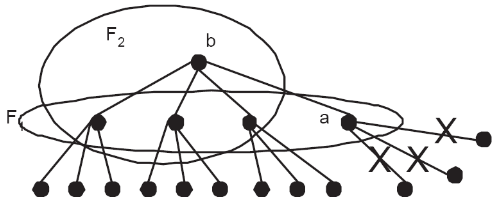
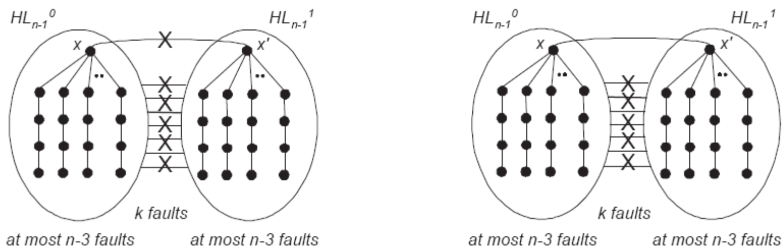



Fig. 2. An indistinguishable pair.

***Proof:*** First we explain why the local diagnosability of each node in $HL_n$ may not equal to its degree if there are $n - 1$ faulty edges. We give an example in Fig. 2. Let $b$ be an arbitrary node and $a$ be another node adjacent to it. Suppose all edges exclusive $\{(a, b)\}$ incident with node $a$ are faulty. We can see that $(F_1, F_2)$ is an indistinghushable pair by Lemma 1. In this situation, $|F_1| = |F_2| = n$, the degree of $b$ equals to $n$, but the local diagnosability of node $b$ does not equal $n$.

Now we are going to prove this theorem by induction on $n$. For the base case $n = 5$, if the number of edge faults in an $HL_n$ is less than or equal to three ($n - 2 = 3$), for each node with degree $k$ where $k \leq n$, it is straightforward though tedious to find an extended star structure of order $k$ at this node. As a consequence, the local diagnosability of each node can be easily checked by this extended star structure, and it will be the same as its degree. For induction hypothesis, we suppose for each node with degree $k$ on an $HL_{n-1}$ with $n - 3$ edge faults ($(n - 1) - 2 = n - 3$), there exists an extended star of order $k$ at this node, which implies that the local diagnosability of every node equals to its degree. Consider an $HL_n$ which has $n - 2$ faulty edges and can be constructed with two copies, that is, $HL^0_{n-1}$ and $HL^1_{n-1}$. We say an edge is crossed edge if its one end is in $HL^0_{n-1}$ and the other end is in $HL^1_{n-1}$. Let $x$ be an arbitrary node. Without loss of generality, we let $x$ be in $HL^0_{n-1}$ and $deg(x) = m$, and $x'$ be its corresponding node in $HL^1_{n-1}$ and $deg(x') = m'$. Our proof is divided into the following two major cases.




(a) (b)

Fig. 3. Case1 of the proof in Theorem 5.

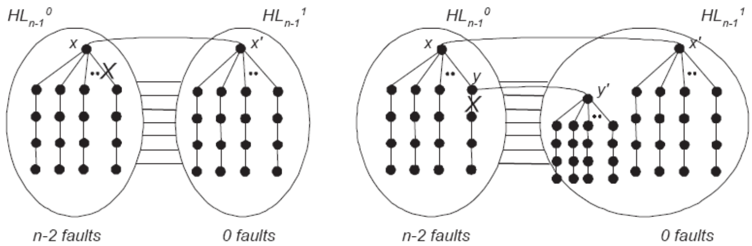
**Case 1:** There are $k$ faulty edges that are crossed edges, where $1 \le k \le n-2$.
**Case 1.1:** Edge $(x, x')$ is faulty. See Fig. 3 (a).

Since there are $k$ faulty edges in the crossed edge, where $1 \le k \le n-2$, the number of faulty edges in $HL_{n-1}^{0}$ is at most $n-3$. By induction hypothesis, there exists an extended star of order $m$ at node $x$, and then the local diagnosability of $x$ in this $HL_n$ with $n-2$ faulty edges equals to its degree $m$.

**Case 1.2:** Edge $(x, x')$ is fault-free. See Fig. 3 (b).

As in Case 1.1, the number of faulty edges in both $HL_{n-1}^{0}$ and $HL_{n-1}^{1}$ are at most $n-3$. Hence there is a subgraph $ES(x, m-1)$ at $x$ in $HL_{n-1}^{0}$ and a subgraph $ES(x', m'-1)$ in $HL_{n-1}^{1}$. So there is a subgraph $ES(x, m)$ at $x$ in $HL_n$. By Theorem 3, the local diagnosability of $x$ in $HL_n$ with $n-2$ faulty edges equals to its degree $m$.

**Case 2:** None of the faulty edges are crossed edges.
**Case 2.1:** All faulty edges are in $HL_{n-1}^{0}$ (respectively, or $HL_{n-1}^{1}$). That is, there are $n-2$ faulty edges in $HL_{n-1}^{0}$ (respectively, or $HL_{n-1}^{1}$).



(a) (b)
Fig. 4. Case 2.1 of the proof in Theorem 5.

If there is a faulty edge $s$ incident with $x$ (see Fig. 4 (a)), we may treat edge $s$ as fault-free temporarily. Hence there are $n-3$ faulty edges in $HL_{n-1}^{0}$. By hypothesis, we can find an extended star $ES(x, m-1)$ in $HL_{n-1}^{0}$. Consider the corresponding node $x'$ in $HL_{n-1}^{1}$, we can also find an $ES(x', m'-1)$ in $HL_{n-1}^{1}$. Therefore, we can easily find an extended star $ES(x, m)$ of order $m$ in $HL_n$. Since the edge $s$ is faulty, there is still an $ES(x, m-1)$ in this $HL_n$. So the local diagnosability of $x$ in $HL_n$ with $n-2$ faulty edges equals to its degree by Theorem 3.

Now we follow the notations in Definition 3. If there is a faulty edge $s$ belonging to $\{(v_{11}, v_{12}), (v_{21}, v_{22}), \ldots, (v_{n1}, v_{n2})\}$ (see Fig. 4 (b)), we may treat edge $s$ as fault-free temporarily, and then there are $n-3$ faulty edges in $HL_{n-1}^{0}$. By hypothesis, there is an $ES(x, m-1)$ at $x$ in $HL_{n-1}^{0}$. Consider the corresponding node $x'$ in $HL_{n-1}^{1}$, we can also find an $ES(x', m'-1)$ at $x'$ in $HL_{n-1}^{1}$. Consider node $y'$ in $HL_{n-1}^{0}$, we can find an $ES(y', deg(y')-1)$ in $HL_{n-1}^{0}$. Therefore, we can easily find an $ES(x, m)$ in $HL_n$. Although the edge $s$ is faulty, the local diagnosability of $x$ in $HL_n$ with $n-2$ faulty edges equals to its degree by Theorem 3.

If there is a faulty edge $s$ belonging to $\{(v_{12}, v_{13}), (v_{22}, v_{23}), \ldots, (v_{n2}, v_{n3})\}$ or $\{(v_{13}, v_{14}), (v_{23}, v_{24}), \ldots, (v_{n3}, v_{n4})\}$, it can be proved by using the same argument.

**Case 2.2:** There are $k_0$ faulty edges in $HL^0_{n-1}$ and $k_1$ faulty edges in $HL^1_{n-1}$, where $1 \le k_0, k_1 \le n - 3$.

Because the number of faulty edges in $HL^0_{n-1}$ is at most $n - 3$ by induction hypothesis, there exists an extended star $ES(x, m - 1)$ in $HL^0_{n-1}$, and there exists an extended star $ES(x', m' - 1)$ in $HL^0_{n-1}$. So we can find an $ES(x, m)$ in this $HL_n$. Therefore the local diagnosability of $x$ in $HL_n$ with $n - 2$ faulty edges equals to its degree by Theorem 3.

In Cases 1 and 2, we proved all possible distributions of faulty edges. Therefore, the proof is complete. ❑

## 5. CONCLUSIONS

The reliability of interconnection networks is an important issue. The diagnosability is also an important factor in measuring the reliability of interconnection networks. In this paper, we proposed a new concept called the local diagnosability, and presented an useful local structure called extended star structure to find the local diagnosability of multiprocessor systems under the comparison-based diagnosis model. Then we prove the diagnosability of an $n$-dimensional hypercube-like network is $n$ for $n \ge 5$, and show that the local diagnosability of each node in an $n$-dimensional hypercube-like network equals to its degree if there are $n - 2$ faulty edges in it. An interesting problem worth studying will be to identify the faulty or fault-free status of a given node using this useful local structure.

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