Dissipation in a partially coherent flux-driven ring

M. T. Liu and C. S. Chu
Department of Electrophysics, National Chiao Tung University, Hsinchu 30050, Taiwan, Republic of China
(Received 9 July 1999)

We have studied a mesoscopic ring threaded by a magnetic flux that increases linearly with time. The ring is partially coherent such that conduction electrons in the ring will encounter incoherent scatterings. We have treated both the incoherent scatterings and the coherent inelastic processes on the same footing. A *S*-matrix model, as proposed by Büttiker[1,2] for incoherent scatterings, has been adopted for our situation. This allows us to solve exactly, and analytically, the coherent inelastic processes caused by the time-varying magnetic flux. Our results demonstrate unequivocally that, for the electrons emanating out of incoherent scatterings, the lower the energies of these electrons the greater will be their net contribution to the dc component of the induced current. A physical explanation is presented.

I. INTRODUCTION

A mesoscopic conducting ring threaded by a magnetic flux has been of great interest to physicists because it provides a paradigm allowing issues of fundamental importance to be tested experimentally.[3] For a fixed magnetic flux, the single-electron states in a one-dimensional (1D) ring can be identified with the Bloch states in a 1D crystal that has a periodic potential $V(x)=V(x+L)$, where $V(x)$ is the potential along the ring and L is the circumference of the ring.[4] The threading magnetic flux Φ plays the role of a wave vector k such that the dependence to Φ of the electronic eigen-energies in the ring can be obtained through the dispersion relation $E_n(k)$ of the corresponding 1D crystal, where $k=-(2\pi/L)\Phi/\Phi^*$. Here $\Phi^*=hc/e$ is a flux quantum, and the band index n in $E_n(k)$ denotes the spectrum of eigen-energies for the electron states in the ring. Futhermore, the persistent current in the ring, given by $j=-(2/L)\Sigma_n f_n j_n=-c\Sigma_n f_n \partial E_n/\partial\Phi$, where f_n represents the occupation number for the nth eigenstate, is a periodic function of Φ. It is due to the periodic dependence of E_n on k.

This concept of analogy between the states in a mesoscopic ring and that in a one dimension crystal was extended by Büttiker, Imry, and Landauer[4] to the case when the flux is changing linearly in time. This analogy has taken the adiabatic point of view such that the time evolution of the states are interpreted in terms of the instantaneous eigenstates of the ring.[5] The intuitive picture that arises is essentially that the eigenstates in the ring, when driven by the induced field F, evolve along their respective dispersion curves according to the relation $\hbar\dot{k}=-eF$. The modification of this picture due to the possible transitions between different energy bands have also been discussed.

This intuitive picture has since become the basics for the discussions of a number of physical properties proposed for mesoscopic rings. In a disordered ring, the eigen-energies over the entire range of Φ can be grouped into bands separated by energy gaps. For the case when the induced electric field is small, such that the Zener tunneling between bands can be neglected, the states are caused to traverse the same band periodically, with a frequency $\omega=eFL/\hbar$. It was hence predicted that the induced current has no dc component but has a Josephson-like ac component.[4] On the other hand, when the induced electric field is large enough to bring about noticeable Zener tunneling, Lenstra *et al.*[5–8] showed that the phase randomization in the Zener tunneling alone can generate a nonzero dc component in the current, and hence a resistive behavior, in the ring. This result was controverted by Landauer[9,10] that, in the absence of inelastic scattering, all energies stored in the system are retrievable in later times so that the dc component in the induced current must be zero.

Meanwhile, Gefen and Thouless[11] studied the same problem by taking the weak localization point of view and showed that the states are localized in energy whenever the driven ring has elastic scatterings. They concluded that no energy can be supplied to the system in the steady state and that the system would exhibit a resistive behavior only in the presence of inelastic scattering. Blatter and Browne[12] found out that the process of phase randomization in the Zener tunneling amplitudes leads to the localization of the electrons in energy space but not to the resistive behavior. More recently, Gorelik *et al.*[13] proposed the possibility of fractional pumping of energy into the ring. All these studies demonstrate beyond doubt that a driven ring is a complicated problem and that the physics depends intricately upon both the elastic and inelastic scatterings.

These many different predictions to the physical properties of a driven ring are based on the interpretation of the wave functions expanded in terms of the instantaneous eigenstates of the ring, as we have mentioned earlier. This approach, though formally correct, is numerically very involved and usually approximations were introduced to simplify the problem. However, in the case of a mesoscopic ring where phase coherence is important, we feel that the time-evolution of the states should be treated more carefully. Hence we propose to solve the problem fully quantum mechanically by invoking a different expansion scheme for the wave functions of the states in the ring. In addition, to address the resistive behavior, we have adopted the incoherent scatterer model[2] for the incoherent processes in the ring. As an illustration to the insights obtained from this approach, and to the effectiveness of this approach, we consider a par-

tially coherent flux-driven smooth ring. We note that even though we have not included other coherent processes, such as those that arise from disorder, in the ring, we have included in this case the coherent, but inelastic, processes caused by the time-varying flux. We have solved these coherent inelastic processes exactly. The extension of our approach to the case when the ring has impurities is straightforward and is currently under investigation.

In this work, the basic wave functions in a driven ring is shown similar to that in a biased one dimension crystal except for a phase factor that has coupled the *spatial* to the time coordinates. We note, however, that this phase factor and the periodic boundary condition in the ring together constitute the essential causes for the difference in the physics between a ring and a one dimension crystal. In fact, with directions accounted for, the phase factor provides a way for the system to count the number of turns that the electron has traversed. The wave function of the electron then becomes a sum over many terms, each has its own time dependence and each associates with a different number of net clockwise turns traversed by the electron. Hence, our results are different from what one would expect if one adheres to the aforementioned intuitive picture—that a driven ring behaves analogously to a biased one dimension crystal. Since the intuitive picture for the one dimension crystal has assumed the adiabatic viewpoint, the discrepancy we find is an example showing that an adiabatic approximation that works in one dimension does not necessarily work in a mesoscopic ring.

In addition to the fully quantum-mechanical treatment, our approach differs from previous works on driven rings by the method of implementing incoherent scatterings. Incoherent scatterings have previously been introduced by way of a relaxation time[14,13] or a cut-off time.[11] But in transport phenomena, the electrons that have suffered incoherent scattering should not be discarded. Rather, these electrons should be allowed to continue their contribution to the transport current, albeit incoherently. We have adopted an incoherent scatterer model for our time-dependent situation. This model has the incoherently scattered electrons coupled to a reservoir through a unitary coherent scatterer.[1] The model also has the nice feature that the incoherently scattered electrons can be reintroduced back into the ring systematically. Finally, in the calculation of the current in the ring, we have included contributions from all electrons below the Fermi energy, we find that states below Fermi energy contribute significantly.

The paper is organized as follows. In Sec. II, we present our formulation that incorporates the incoherent scatterings into the coherent states of the driven ring. In Sec. III, we present our numerical results. Finally, in Sec. IV, we present a conclusion.

II. THEORY

The Schrödinger equation for an electron in a ring of radius ρ, and threaded by a magnetic flux Φ_B, is given by

$$\frac{1}{2m_e^*}\left(\frac{\hbar}{i}\frac{1}{\rho}\frac{\partial}{\partial\phi}+\frac{e}{c}\frac{\Phi_B}{2\pi\rho}\right)^2\tilde{\Psi}(\phi,t)=i\hbar\frac{\partial}{\partial t}\tilde{\Psi}(\phi,t), \quad (1)$$

where m_e^* is the effective mass of the electron and $e>0$.

In the following we select the length unit $R^*=\rho$, the energy unit $E^*=\hbar^2/(2m_e^*R^{*2})$, the time unit $t^*=\hbar/E^*$, the angular frequency unit $\omega^*=E^*/\hbar$, and the flux unit $\Phi^*=hc/e$. Furthermore, for a linearly increasing magnetic flux, we have $\Phi_B=\omega t\Phi^*$. With this, the electron would have, respectively, gained, or lost, an energy $2\pi\omega$ if it moves around the ring once counter-clockwisely, or clockwisely. According to the above choice of units, the Schrödinger equation is made dimensionless, given by

$$\left(-i\frac{\partial}{\partial\phi}+\omega t\right)^2\tilde{\Psi}(\phi,t)=i\frac{\partial}{\partial t}\tilde{\Psi}(\phi,t), \quad (2)$$

which, when invoking a transformation

$$\tilde{\Psi}(\phi,t)=e^{-i\omega t\phi}\Psi(\phi,t), \quad (3)$$

can also be cast into the form

$$\left(-\frac{\partial^2}{\partial\phi^2}-\omega\phi\right)\Psi(\phi,t)=i\frac{\partial}{\partial t}\Psi(\phi,t). \quad (4)$$

From the above equations, we see that Ψ describes a one-dimensional particle acted upon by a constant electric field whereas $\tilde{\Psi}$, together with the periodic boundary condition at all times, describes a particle in a driven ring. The two wave functions differ by a phase factor $e^{-i\omega t\phi}$, which has coupled the time with the *spatial* coordinate. This phase factor contributes nontrivially to the deviation of the physics in a driven ring from that in a driven one dimension system. This effect of the phase factor enters when we impose the single-valueness in $\tilde{\Psi}$ by matching the wave function at $\phi=0$ to that at $\phi=2\pi$. To facilitate the matching, we define in the following a basic set of wave functions for the driven ring. The actual wave function for the driven ring that satisfy the single valueness can be constructed out of this basic set of wave functions. The basic wave functions that represent particle moving counter clockwisely is given by

$$\tilde{\Psi}^{(+)}(\phi,t;\varepsilon)=\frac{\sqrt{\zeta(\phi,\varepsilon)}\,\mathrm{H}_{\frac{1}{3}}^{(1)}\left[\frac{2}{3}\zeta^{3/2}(\phi,\varepsilon)\right]}{(6\sqrt[3]{\omega}/\pi)^{1/2}}e^{-i(\varepsilon+\omega\phi)t}, \quad (5)$$

and the basic wave functions that represent particle moving clockwisely is given by

$$\tilde{\Psi}^{(-)}(\phi,t;\varepsilon)=\frac{\sqrt{\zeta(\phi,\varepsilon)}\,\mathrm{H}_{\frac{1}{3}}^{(2)}\left[\frac{2}{3}\zeta^{3/2}(\phi,\varepsilon)\right]}{(6\sqrt[3]{\omega}/\pi)^{1/2}}e^{-i(\varepsilon+\omega\phi)t}. \quad (6)$$

Here $\zeta(\phi,\varepsilon)=\omega^{\frac{1}{3}}(\phi+\varepsilon/\omega)$, and $\mathrm{H}_{\frac{1}{3}}^{(1)}(z),\mathrm{H}_{\frac{1}{3}}^{(2)}(z)$ are Hankel functions. The energy parameter ε is a continuous variable.

The normalization of these basic wave functions is chosen such that their particle current is unity, according to our units. The current j, given by the expression $j=\tilde{\Psi}^*(-i\partial/\partial\phi+\omega t)\tilde{\Psi}+\text{c.c.}$, is in the unit of $-eE^*/h$. This choice of the normalization for the basic wave functions is

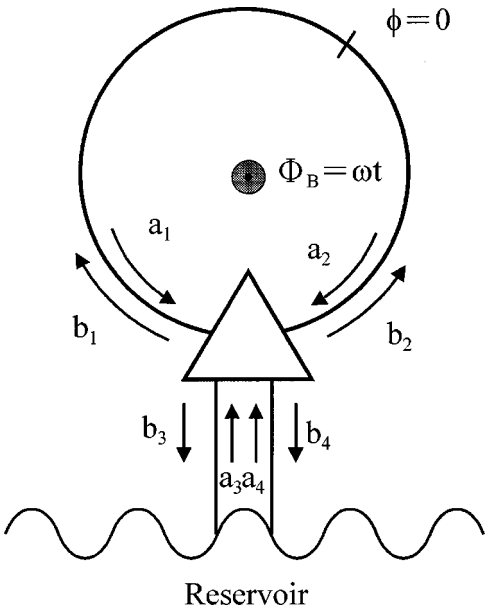

FIG. 1. A partially coherent flux-driven ring. The flux, represented by the center shaded circle and directed out of the page, is linear in time with $\Phi_B=\omega t$. The ring is coupled via a coupler, depicted by the triangle, to a reservoir, depicted by the wavy line. The coordinate ϕ measures the counter-clockwise displacement of an electron along the ring.

deemed necessary, as pointed out by Stone and Szafer,[15] if we want to invoke the incoherent scatterer model of Büttiker[1] for the incoherent processes in our system.

The incoherent scatterer model consists of a coupler that couples the electrons in the system to a reservoir. Current that flows into the reservoir will be reinjected back into the system according to the distribution in the reservoir. There is no phase correlation between the currents that flow in and out of the reservoir. It is through this process that phase coherence in the particles is lost. The unitary property of the coupler warrants the conservation of current. Furthermore, by describing the coupler in terms of an energy-independent $\underline{\mathbf{S}}$, the incoherent processes can be cast into a scattering problem and is readily treated on the same footing with other coherent processes in the system. The $\underline{\mathbf{S}}$ matrix,[1] given by

$$\underline{\mathbf{S}}=\begin{bmatrix} 0 & \sqrt{1-\alpha} & \sqrt{\alpha} & 0 \\ \sqrt{1-\alpha} & 0 & 0 & \sqrt{\alpha} \\ \sqrt{\alpha} & 0 & 0 & -\sqrt{1-\alpha} \\ 0 & \sqrt{\alpha} & -\sqrt{1-\alpha} & 0 \end{bmatrix}, \tag{7}$$

couples incoming waves, with amplitudes $\mathbf{a}=(a_1,a_2,a_3,a_4)$, to the outgoing waves, with amplitudes $\mathbf{b}=(b_1,b_2,b_3,b_4)$, through the relation $\mathbf{b}^T=\underline{\mathbf{S}}\mathbf{a}^T$. The notation convention for these amplitudes is shown in Fig. 1.

The form of $\underline{\mathbf{S}}$ is chosen such that it is unitary as long as the basic wave functions, both incoming and outgoing, are normalized to give a unit current. The coupling parameter α, with $0\leqslant\alpha\leqslant 1$, gives the extent the ring couples with the reservoir. For $\alpha=1$, the particle will lose track of its phase entirely once it encounters an incoherent process. For $\alpha=0$, the ring and the reservoir are decoupled. The $\underline{\mathbf{S}}$ matrix also has the nice feature that the outgoing amplitudes do not depend on the location of the coupler. For definitness, in our calculation, we choose the coupler to locate at $\phi=\phi_0$.

We also define, in leads $i=3,4$, basic wave functions representing particles that incident upon the coupler as

$$Y(x_i,t;\varepsilon)=\frac{e^{-i(\varepsilon t-\sqrt{\varepsilon}x_i)}}{(2\sqrt{\varepsilon})^{1/2}}, \tag{8}$$

and that emanating from the coupler as

$$Z(x_i,t;\varepsilon)=\frac{e^{-i(\varepsilon t+\sqrt{\varepsilon}x_i)}}{(2\sqrt{\varepsilon})^{1/2}}, \tag{9}$$

where the coupler is at $x_i=0$.

When electrons are incident from the reservoir within an energy interval $d\varepsilon$ via either lead 3 or 4, the incident current is $N(\varepsilon)=2f(\varepsilon)d\varepsilon$, where $f(\varepsilon)$ is the Fermi-Dirac distribution for the reservoir with a chemical potential μ, and a spin degeneracy has been included. Hence, we choose the incident amplitude to be $\sqrt{N(\varepsilon)}$.

Now, for the case when the electrons incident from the lead $i=4$, the wave functions in the leads 4 and 3 are, respectively,

$$\tilde{\Psi}_4=\sqrt{N(\varepsilon)}Y(x_4,t;\varepsilon), \tag{10}$$

and

$$\tilde{\Psi}_3=\sum_{n=0}^{\infty} t_{34}(n)Z(x_3,t;\varepsilon_n). \tag{11}$$

We stress here that the coefficients $\sqrt{N(\varepsilon)}$ and $t_{34}(n)$ are more appropriately interpreted as the current amplitudes, with the former as the incident current amplitude and the latter as the reflected current amplitudes. The use of the term *wave function* for the states in the leads 3 and 4 is to maintain the uniformity in our formulation. The index n denotes the possible reflected electron energies that are resulted from the action of the time-varying flux in the ring.

Meanwhile, the wave functions in the ring, and in the region $\phi_0\leqslant\phi\leqslant 2\pi$, is of the form

$$\tilde{\Psi}_I=\sum_n [A_n(\varepsilon_n)\tilde{\Psi}^{(+)}(\phi,t;\varepsilon_n-\omega\phi_0) + B_n(\varepsilon_n)\tilde{\Psi}^{(-)}(\phi,t;\varepsilon_n-\omega\phi_0)], \tag{12}$$

and, in the region $0\leqslant\phi\leqslant\phi_0$, the wave function is given by

$$\tilde{\Psi}_{II}=\sum_n [C_n(\varepsilon_n)\tilde{\Psi}^{(+)}(\phi,t;\varepsilon_n-\omega\phi_0) + D_n(\varepsilon_n)\tilde{\Psi}^{(-)}(\phi,t;\varepsilon_n-\omega\phi_0)]. \tag{13}$$

Here, $\varepsilon_n=\varepsilon+2\pi\omega n$. These wave functions are written in the form that facilitates the matching of the wave functions at all times.

From matching the wave functions at $\phi=0$ and $\phi=2\pi$ at all times, we obtain the relations $C_n(\varepsilon_n)=A_{n-1}(\varepsilon_{n-1})$, and $B_{n-1}(\varepsilon_{n-1})=D_n(\varepsilon_n)$. At the coupler, we obtain two more relations,

$$A_n(\varepsilon_n)=\sqrt{\alpha N(\varepsilon)}\,\delta_{n,0}+\sqrt{1-\alpha}\,C_n(\varepsilon_n), \tag{14}$$

and

$$D_n(\varepsilon_n)=\sqrt{1-\alpha}\,B_n(\varepsilon_n). \tag{15}$$

We note here that $C_n(\varepsilon_n)=0$ for $n\leqslant 0$. Solving these relations, we find $B_n=D_n=0$, and A_n is nonzero for $n\geqslant 0$, given by $A_n(\varepsilon_n)=\sqrt{\alpha N(\varepsilon)}(1-\alpha)^{n/2}$.

This solution can be understood according to the following physical picture. First of all, the increasing flux induces an electric field that points clockwisely along the ring. Second, the electrons incident from lead $i=4$ enter the ring as counter-clockwise moving states. These electrons will maintain their counter-clockwiseness under the action of the induced field while their *energy* increases because of the phase factor $e^{-i\omega t\phi}$. This increasing in *energy* is reflected by the increasing in the index n in the relation $C_n(\varepsilon_n)=A_{n-1}(\varepsilon_{n-1})$. At the coupler, there is a probability amplitude $\sqrt{1-\alpha}$ for the electrons to maintain their phase coherence and this condition is contained in Eq. (14).

The time-averaged current $\langle dj_4\rangle_t$ in the ring is then evaluated to give

$$\langle dj_4\rangle_t=\sum_{n=0}^{\infty}\alpha N(\varepsilon)(1-\alpha)^n=N(\varepsilon). \tag{16}$$

For the case when the electrons incident from the lead $i=3$, the wave functions in the lead 3 and 4 are, respectively,

$$\tilde{\Psi}_3=\sqrt{N(\varepsilon)}Y(x_3,t;\varepsilon)+\sum_n r_{33}(n)Z(x_3,t;\varepsilon_n), \tag{17}$$

and

$$\tilde{\Psi}_4=\sum_n t_{43}(n)Z(x_4,t;\varepsilon_n). \tag{18}$$

The coefficients $r_{33}(n)$ and $t_{43}(n)$ denotes the reflection current amplitudes in the leads 3 and 4, respectively. The wavefunctions in the ring are of the same form as in Eqs. (12) and (13), except that the coefficients A_n, B_n, C_n, and D_n are replaced by $\tilde{A}_n,\tilde{B}_n,\tilde{C}_n$, and $\tilde{D}_n$. Following the same matching procedure, we obtain the relations $\tilde{C}_n(\varepsilon_n)=\tilde{A}_{n-1}(\varepsilon_{n-1}),\tilde{B}_{n-1}(\varepsilon_{n-1})=\tilde{D}_n(\varepsilon_n)$, $\tilde{A}_n(\varepsilon_n)=\sqrt{1-\alpha}\tilde{A}_{n-1}(\varepsilon_{n-1})$, and also the relation

$$\tilde{D}_n(\varepsilon_n)=\sqrt{\alpha N(\varepsilon)}\,\delta_{n,0}+\sqrt{1-\alpha}\tilde{D}_{n+1}(\varepsilon_{n+1}), \tag{19}$$

where $\tilde{D}_n=0$ for $n>0$. We note that the clockwise states that are injected into the ring would become evanescent after they have traversed N turns, with $N=[\varepsilon/(2\pi\omega)]$, where $[x]$ denotes the largest integer smaller than or equal to x. At the Nth turn, we have $\tilde{C}_{-N}=\tilde{D}_{-N}e^{i\delta(\varepsilon)}$. Finally, we obtain the expressions of $\tilde{D}_{-n}$ for $0\leqslant n\leqslant N$,

$$\tilde{D}_{-n}(\varepsilon_{-n})=(1-\alpha)^{n/2}\sqrt{\alpha N(\varepsilon)}, \tag{20}$$

and of $\tilde{C}_n$ for $n\geqslant -N$,

$$\tilde{C}_n(\varepsilon_n)=(1-\alpha)^{N+n/2}e^{i\delta(\varepsilon)}\sqrt{\alpha N(\varepsilon)}. \tag{21}$$

Again, the solution can be understood according to the following physical picture. The electrons incident from lead $i=3$ enter the ring as clockwisely moving states. Under the action of the induced electric field, the *energy* of these electrons decreases as they maintain their clockwiseness. This feature is reflected in the relation $\tilde{B}_{n-1}(\varepsilon_{n-1})=\tilde{D}_n(\varepsilon_n)$. In the Nth clockwise turn, the wave functions become evanescent and the electrons are reflected into counter-clockwisely moving states. This feature is given by the relation $\tilde{C}_{-N}=\tilde{D}_{-N}e^{i\delta(\varepsilon)}$. The electrons will then maintain their counter-clockwiseness while their *energy* increases by the action of the induced field. This feature is given by the relation $\tilde{C}_n(\varepsilon_n)=\tilde{A}_{n-1}(\varepsilon_{n-1})$. At the coupler, the probability amplitude for the electrons to maintain their phase coherence is contained in Eq. (19) and also in the recurrence relation for $\tilde{A}_n(\varepsilon_n)$ just before Eq. (19).

The time-averaged current $\langle dj_3\rangle_t$ in the ring is evaluated to be

$$\begin{aligned}\langle dj_3\rangle_t&=-\sum_{n=0}^{N}|D_{-n}(\varepsilon_{-n})|^2+\sum_{n=-N}^{\infty}|C_n(\varepsilon_n)|^2\\&=-N(\varepsilon)[1-(1-\alpha)^{N+1}]+N(\varepsilon)(1-\alpha)^N,\end{aligned} \tag{22}$$

where the first, second, current term is, respectively, the contribution of the clockwise moving, counter-clockwise-moving, part of the electron wave function in the ring. It is worth noting that in the case of large N, the first current term will dominate and its value approaches $-N(\varepsilon)$, which is exactly equal but opposite to the time-averaged current $\langle dj_4\rangle_t$ in Eq. (16). This result implies that the net dc component in the current goes to zero rapidly as the energy of the injected electrons increases beyond $\varepsilon\gg 2\pi\omega$.

In any case, the net dc component in the current due to particles injected from within the $d\varepsilon$ interval in the reservoir is

$$\begin{aligned}\langle dj\rangle_t&=\langle dj_3\rangle_t+\langle dj_4\rangle_t\\&=2(1-\alpha)^N(2-\alpha)f(\varepsilon)d\varepsilon.\end{aligned} \tag{23}$$

The total dc component in the current $\langle j\rangle_t$ then involves an integral over all possible incident energies from particles within the reservoir. The expression for $\langle j\rangle_t$ is

$$\langle j\rangle_t=2(2-\alpha)\int_0^{\infty}(1-\alpha)^N f(\varepsilon)d\varepsilon. \tag{24}$$

Since our major focus in this paper is the effects of incoherent processes on the low-temperature characteristics in mesoscopic rings, it suffices to let the temperature of the reservoir be zero. Thus the expression for $\langle j\rangle_t$ we used in our following numerical examples is given by

$$\langle j\rangle_t=2(2-\alpha)\int_0^{\mu}(1-\alpha)^N\,d\varepsilon, \tag{25}$$

where μ is the chemical potential of the reservoir.

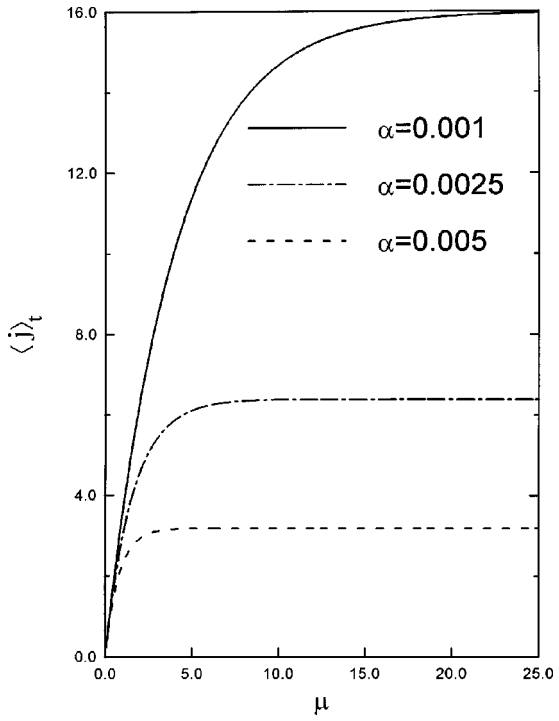

FIG. 2. Total dc component in the current $\langle j\rangle_t$ as a function of μ for $2\pi\omega=0.001$ and for $\alpha=0.001,\ 0.0025$, and 0.005.

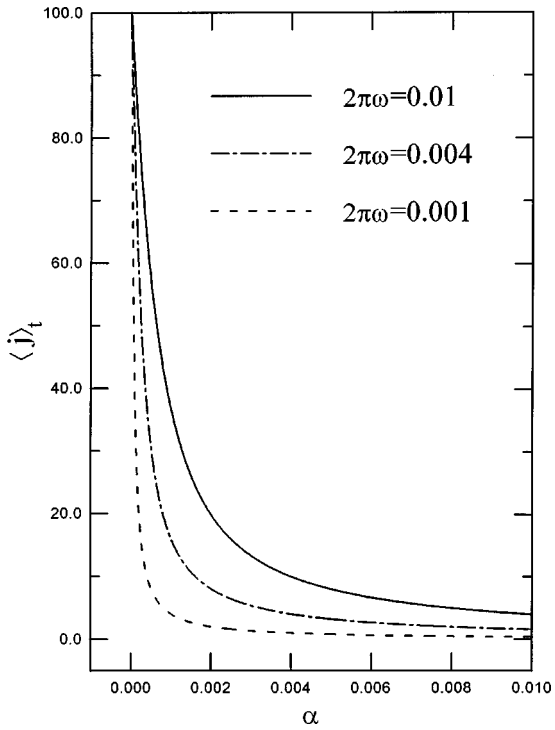

FIG. 4. $\langle j\rangle_t$ as a function of α for $\mu=25$, and for $2\pi\omega=0.01,\ 0.004$, and 0.001.

III. RESULTS

We present in Figs. 2–4 the $\langle j\rangle_t$ characteristics and its dependence on the chemical potential μ of the reservoir, on $2\pi\omega$, and on α, respectively. In these numerical examples, the physical parameters are chosen to be that of a semiconductor ring, with $R^*=150$ nm and $m_e^*=0.067m_e$. According to our choice of units, the induced electromotive force $2\pi R^*F$ in the ring equals 10^{-7} V when $2\pi\omega=0.004$. Since our emphasis is upon the interplay between the coherent nature of the mesoscopic ring and the dissipation in the ring, the numerical examples presented will be in the small α regime.

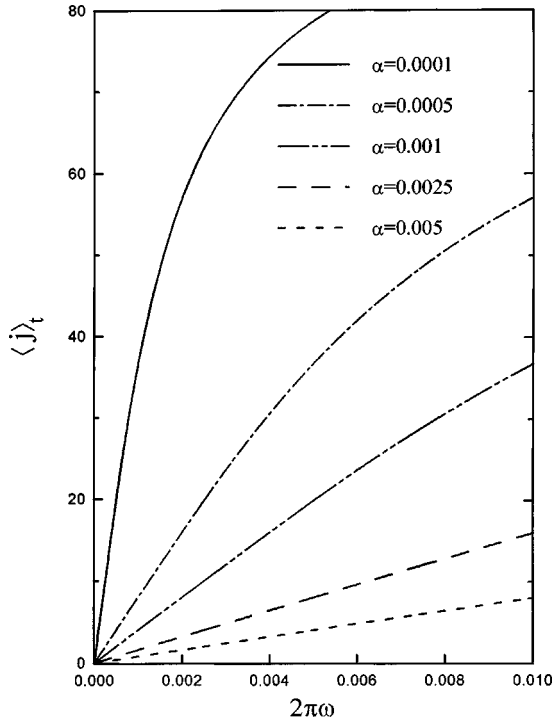

FIG. 3. $\langle j\rangle_t$ as a function of $2\pi\omega$ for $\mu=25$, and for five α values.

In Fig. 2, we present the total dc component in the current $\langle j\rangle_t$ as a function of the chemical potential μ for $2\pi\omega=0.001$ and for $\alpha=0.001,\ 0.0025$, and 0.005. All the three curves show that $\langle j\rangle_t$ increases with μ initially and saturates in the larger μ regions. The saturated value of $\langle j\rangle_t$ depends on α and is larger for smaller α. Furthermore, the curves for larger α have their saturation features occurred earlier, at smaller μ values. This saturation feature, together with the trend that the slope of $\langle j\rangle_t$ decreases monotonically with μ, demonstrates unequivocally that the greater the energies of the electrons that emanate out of incoherent scatterings, the smaller will be their net contribution to $\langle j\rangle_t$. These features can be understood according to the remarks we made in the paragraph before Eq. (22).

A more comprehensive account is summarized in the following. The electrons that have suffered from incoherent scatterings are reinjected back into the system, but they are equally likely to be injected into either clockwise- or counter-clockwise-moving states in the ring. Those electrons that move against the direction of the induced field will keep on moving in the same direction while their *energies* increase continuously. This cannot keep on indefinitely because the electrons will, unavoidably, lose their phase coherence via incoherent scatterings. On the other hand, those electrons that move along the direction of the induced field have their *energies* decrease continuously. Whether these electrons can hit their classical turning point and change their direction of propagation depends on the degree of coherence,

the energy of the reinjected electrons, and the magnitude of the induced field. The electrons will have a greater chance of hitting their classical turning point if they can maintain their coherence longer. Hence, the favorable condition for the electrons to hit their classical turning point and be reflected is to have small α, μ, and large $2\pi\omega$. For those electrons that cannot hit their classical turning point, the corresponding $\alpha N=\alpha[\varepsilon/2\pi\omega]$ is very large. Our results in Eq. (22) show that the contributions to the dc component in the current from both the clockwise- and counter-clockwise-injected electrons will then cancel each other exactly. However, for those electrons that hit their classical turning point and be reflected, they have a net contribution. With this understanding, we then expect the above two physical situations to manifest differently in their dissipation characteristics.

Indeed, two dissipation characteristics are identified in Fig. 2. In the small μ region, $\langle j\rangle_t$ increases with μ, showing that all reinjected electrons contribute. The curvature in the curves show that contributions from electrons of larger injected energies are smaller. This then is the regime when the reinjected electrons can hit their classical turning points. We call this the regime of coherent reflection. In the larger μ region, $\langle j\rangle_t$ becomes saturated, showing that electrons of larger injected energies no longer contribute. This then is the regime when electrons of larger injected energies cannot hit their classical turning point. We call this the saturation regime. In addition, as α decreases, the regime of coherent reflection becomes more favorable such that the saturation feature is pushed to larger μ values and the saturation value of $\langle j\rangle_t$ increases.

In Fig. 3, we present $\langle j\rangle_t$ as a function of the induced electromotive force $2\pi\omega$ for $\mu=25$, and for five values of α, as indicated in the figure. The curves of $\alpha=0.005$, and 0.0025 are linear, showing Ohmic-like behavior. However, the other curves of smaller α values are no longer linear, showing non-Ohmic-like behavior. With the help of Fig. 2, we see that the Ohmic-like behavior corresponds to the saturation regime while the non-Ohmic-like behavior corresponds to the coherent reflection regime. Thus Fig. 3 is another manifestation of these two regimes. The dependence of $\langle j\rangle_t$ on α is presented in Fig. 4 where $\mu=25$, and $2\pi\omega=0.001$, 0.004, and 0.01. The overall trend shown is that $\langle j\rangle_t$ drops as α increases or as $2\pi\omega$ decreases. Again, this trend is consistent with our above understanding.

The approach in this paper also allows us to obtain the analytic expressions for the ac component in the current in the saturation regime and in the small ω regime. Since the current expression obtained from the wave functions in Eqs. (12) and (13) involves a double sum over terms with time dependences of the form $\sin(2\pi m\omega t)$, we look, in particular, at the ac component $m=1$. For electrons injected within the $d\varepsilon$ interval, the net contribution to the ac component j_1 in the ring is obtained, given by

$$j_1=4N(\varepsilon)\sqrt{1-\alpha}\sin(\sqrt{\varepsilon}2\pi)\sin(2\pi\omega t). \tag{26}$$

In obtaining this expression, we have assumed that the electrons cannot reach their classical turning point due to the incoherent processes. This result for j_1 has the added property that the contribution from larger ε is greater, which is different from the dissipation characteristics in the dc component in the current.

As a comparison with an adiabatic point of view, we calculate the persistent current $j_p(a)$ in the ring for a static flux $\Phi=a\Phi^*$. Using similar approach in this paper, we obtain

$$j_p=\alpha N(\varepsilon)[F(\alpha,\varepsilon,a)-F(\alpha,\varepsilon,-a)], \tag{27}$$

where

$$F(\alpha,\varepsilon,a)=\frac{1}{2-\alpha-2\sqrt{1-\alpha}\cos[2\pi(\sqrt{\varepsilon}-a)]}. \tag{28}$$

If we assume the adiabatic viewpoint by allowing $a=\omega t$, then $j_p(\omega t)$ would contain many terms with time dependences of the form $\sin(2\pi m\omega t)$. We find that the $m=1$ term $j_{p,1}$, extracted from Eq. (27) using Fourier expansion, is the same as j_1 in Eq. (26). Thus, we find in this situation that the ac component in the current is consistent with the adiabatic point of view but the dc component is not.

IV. CONCLUSION

In conclusion, we have found interesting dissipation characteristics in a partially coherent flux-driven ring. Two regimes are identified. The first regime corresponds to the situation when the electrons that emanate out of incoherent scatterings, and move along the direction of the induced electric field, have appreciable chance of reaching their classical turning point. In this regime, the dissipation is not Ohmic-like and the dc component in the current increases with the Fermi energy μ. The second regime corresponds to the situation when most of the electrons that emanate out of incoherent scatterings, and move along the direction of the induced electric field, have negligible chance of reaching their classical turning point. In this regime, the dissipation is Ohmic-like while the dc component in the current becomes independent of the Fermi energy. However, in this latter regime, we find that the ac component in the current, with a period of $1/\omega$, is the same as the adiabatic result. All these results are the consequences of the coherence and the topological nature of the ring combined. We expect these dissipation characteristics to manifest also in flux-driven rings containing impurities that are in the regime of significant Zener tunneling.

ACKNOWLEDGMENT

This work was supported in part by the National Science Council of the Republic of China through Contract No. NSC88-2112-M-009-028.

[1] M. Büttiker, Phys. Rev. B **32**, 1846 (1985).
[2] M. Büttiker, Phys. Rev. B **33**, 3020 (1986).
[3] Y. Imry, *Introduction to Mesoscopic Physics* (Oxford University Press, New York, 1997).
[4] M. Büttiker, Y. Imry, and R. Landauer, Phys. Lett. A **96A**, 365 (1983).
[5] Similar method can also be found in the work of D. Lenstra and W. van Haeringen [J. Phys. C **14**, 5293 (1981)] who considered one-dimensional systems of finite length.
[6] D. Lenstra and W. van Haeringen, J. Phys. C **14**, L819 (1981)
[7] D. Lenstra and W. van Haeringen, Physica (Amsterdam) **128B**, 26 (1985).
[8] D. Lenstra and W. van Haeringen, Phys. Rev. Lett. **57**, 1623 (1986); Phys. Scr. **34**, 438 (1986).
[9] R. Landauer, Phys. Rev. B **33**, 6497 (1986).
[10] R. Landauer, Phys. Rev. Lett. **58**, 2150 (1987), comment on Ref. 7; The response of Lenstra and Haeringen is in, *ibid.* **58**, 2151 (1987).
[11] Y. Gefen and D. Thouless, Phys. Rev. Lett. **59**, 1752 (1987).
[12] G. Blatter and D.A. Browne, Phys. Rev. B **37**, 3856 (1988).
[13] L. Gorelik, S. Kulinich, Yu. Galperin, R.I. Shekhter, and M. Jonson, Phys. Rev. Lett. **78**, 2196 (1997).
[14] R. Landauer and M. Büttiker, Phys. Rev. Lett. **54**, 2049 (1985).
[15] A.D. Stone and A. Szafer, IBM J. Res. Dev. **32**, 384 (1988).