

Fault-tolerant gamma interconnection networks by chaining

C.W.Chen, N.P.Lu, T.F.Chen and C.P.Chung

Abstract: The authors propose two single-fault-tolerant gamma interconnection networks. The first is a partially chained gamma interconnection network (PCGIN) with two disjoint paths between any source-destination pair. A PCGIN has the characteristics of one fault tolerance and destination tag routing, but backtracking may be necessary when a fault occurs. To eliminate the backtracking penalties of a PCGIN, a fully chained gamma interconnection network (FCGIN); that can at least tolerate one link or switch fault at each stage without backtracking, is also proposed. FCGIN has the advantages of destination tag routing, lower hardware costs than a PCGIN, low fault penalty, and strong reroutability.

1 Introduction

A multiprocessor system consists of several processors and memory modules interconnected by a network. Designing a suitable interconnection network has become an important issue in multiprocessor systems because overall system performance relies on the interconnection network. Multistage interconnection networks are very suitable for communication among tightly coupled system components, and offer a good balance between cost and performance. The assurance of high reliability is a significant task for complex systems, thus fault-tolerance is crucial for MINs serving the communication needs of large-scale multiprocessor systems [1].

In a gamma interconnection network (GIN) [2], there are multiple paths between any source and destination pair except when source and destination are the same. To overcome this difficulty, several schemes have been introduced to improve the GIN's fault-tolerant capability, such as Extra Stage Gamma Network [3], B-network [4], and CGIN [5]. Extra Stage Gamma Network provides at least two disjoint paths between any source and destination pair by adding one more stage to the GIN, but the delay time and routing conflicts will increase [3]. B-network changes the direction of a non-straight link of the GIN's switch as a backward link [4]. When a fault exists in the routing path, the packet will go backward to the previous stage and find

another path. But if a link fault occurs between stage 0 and stage 1, no alternate path can be found. Moreover, B-network cannot satisfy the requirement for strong reroutability [6]. Strong reroutability means that a packet can find an alternate path at each stage whenever a fault is encountered. CGIN [5] is a revised GIN which provides at least two disjoint paths, however its routing algorithm is complicated. When a fault occurs, the system must use a backtracking scheme to tolerate the fault. CGIN does not satisfy strong reroutability need either.

In this paper, we first introduce two new destination tag routing functions *borrow* and *carry*, which can be used to find disjoint paths in GIN when the difference between source and destination indices is odd. Then we propose a partially chained gamma interconnection network (PCGIN). By applying these two destination tag routing functions, the PCGIN provides at least two disjoint paths from any source to any destination. When a fault occurs in the routing path, if backtracking is used, packets are guaranteed to find an alternate path to reach the destination in the PCGIN. To achieve strong reroutability, we also propose a fully chained gamma interconnection network (FCGIN). Chain links are added between nodes belonging to a neighbouring group at the same stage. When a link fault occurs at a stage in a FCGIN, the chain link is taken.

2 Gamma interconnection networks

2.1 Topology

A GIN of size $N = 2^n$ consists of $n + 1$ stages labelled from 0 to n , and each stage has N switches [2]. Every switch at the intermediate stages is a 3×3 crossbar, while the first and last stages are composed of switches of sizes 1×3 and 3×1 , respectively. Switch number j at stage i has three output links to switches at stage $(i + 1)$ based on the plus-minus- 2^i function. That is, the j^{th} switch at stage i has three output links to switches $[(j - 2^i) \bmod N]$, j , and $[(j + 2^i) \bmod N]$ at the successive stage. Fig. 1 illustrates the network of size $N = 8$.

In the GIN, an n -digit tag determines the path connecting the source to its destination. Each tag digit can be 1, 0, or $\bar{1}$. An n -digit tag T represents the difference between destination D and source, S , i.e. $T = D - S \pmod{N}$. Digit

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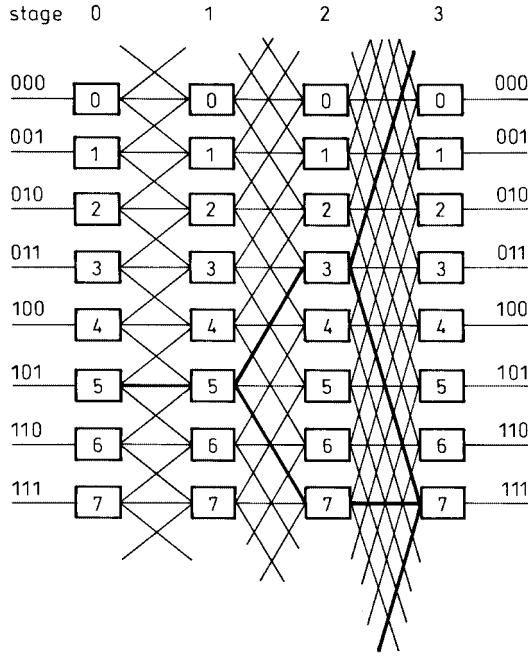


Fig. 1 Gamma interconnection network with $N=8$ and three paths between nodes 5 and 7

d_i is used at stage i in such a way that the lower (upper) connection is taken when d_i is equal to 1 ($\bar{1}$), and the straight connection is taken when d_i is 0. A non-zero tag T has multiple representations, that is, there are multiple paths between S and D when $S \neq D$. For example, if S is 5 and D is 7, then the tag T can be 010 or 0 $\bar{1}$ 1, or 0 $\bar{1}$ $\bar{1}$ (shown in Fig. 1).

2.2 Destination tag routing in a GIN

The routing algorithm discussed above is distance tag routing, that is, we must compute the tag before the packet is sent. Otherwise the packet must include the source and destination tags so that the switch can make routing decisions by comparing the source and destination tags at each stage. To avoid preprocessing overhead and switch hardware complexity, two destination tag routing functions $c(j, t_i)$ and $c'(j, t_i)$ have been proposed in [7].

A switch j at stage i in a GIN is an even switch if $j_i = 0$, or an odd switch if $j_i = 1$, where $j_0 j_1 j_2 \dots j_{n-1}$ is the n -bit binary representation of j , and j_{n-1} is the most significant. Let T denote a destination tag where $t_0 t_1 t_2 \dots t_{n-1}$ is the binary representation of t and t_{n-1} is the most significant. Two functions $c(j, t_i)$ and $c'(j, t_i)$ are defined as follows:

Definition 1:

$$c(j, t_i) = \begin{cases} j + 2^i & \text{if } j \text{ is an even switch, and } t_i \text{ is } 1 \\ j - 2^i & \text{if } j \text{ is an odd switch, and } t_i \text{ is } 0 \\ j & \text{otherwise} \end{cases}$$

$$c'(j, t_i) = \begin{cases} j + 2^i & \text{if } j \text{ is an odd switch, and } t_i \text{ is } 0 \\ j - 2^i & \text{if } j \text{ is an even switch, and } t_i \text{ is } 1 \\ j & \text{otherwise} \end{cases}$$

When a message is routed the $c(j, t_i)$ function will only change j_i to t_i , and the other digits will not be changed. But $c'(j, t_i)$ will change j_i to t_i and $j_0 j_1 j_2 \dots j_{i-1}$ are unchanged. $j_{i+1} j_{i+2} \dots j_{n-1}$ may be changed when $j_i \neq t_i$.

As discussed in [7], $c(j, t_i)$ and $c'(j, t_i)$ in a GIN are destination tag routing functions.

When a nonstraight link is taken from stage i to stage $(i+1)$, the bit i of switch index at stage $(i+1)$ will be the same as t_i , no matter whether an up or down non-straight link is taken. However, $c(j, t_i)$ and $c'(j, t_i)$ functions have a drawback, that is, the paths generated by $c(j, t_i)$ and $c'(j, t_i)$ functions may not be disjoint after the two paths are separated. For example, when S is 4 and D is 5 with $N=8$, the path generated by $c(j, t_i)$ goes through switches 4, 5, 5, and 5 and the path generated by $c'(j, t_i)$ is via switches 4, 3, 5, and 5. These two paths will join at stage 2. To overcome this drawback, we define two destination tag routing functions $carry(j, t_i)$ and $borrow(j, t_i)$, which are derived from $c(j, t_i)$ and $c'(j, t_i)$:

Definition 2:

$$carry(j, t_i) = \begin{cases} j + 2^i & \text{if } (j_i = 0 \text{ and } t_i = 1) \\ & \text{or } (j_i = 1 \text{ and } t_i = 0) \\ j & \text{otherwise} \end{cases}$$

$$borrow(j, t_i) = \begin{cases} j - 2^i & \text{if } (j_i = 0 \text{ and } t_i = 1) \\ & \text{or } (j_i = 1 \text{ and } t_i = 0) \\ j & \text{otherwise} \end{cases}$$

By Definition 2, we realise that $carry(j, t_i)$ and $borrow(j, t_i)$ are destination tag routing functions: Only using t_i can decide routing from the switch at stage i to the switch at stage $i+1$. The behavior of $carry$ and $borrow$ functions is depicted in Fig. 2. The $carry$ function goes straight or downward, while the $borrow$ function goes upward or straight. With $carry$ and $borrow$ functions if the two paths generated by $carry$ and $borrow$ functions are separated at stage i , these two paths will not join again until at the destination node. If these paths are separated at stage 0, as a result, we have two disjoint paths from the source to the destination.

Theorem 1: When $(S-D) \bmod 2 = 1$, there are at least two disjoint paths in GIN.

Proof: If $(S-D) \bmod 2 = 1$, the source node S is not equal to the destination node D , and the two nonstraight links, down link and up link, will be taken by applying $carry$ and $borrow$ functions respectively at stage 0. The two paths will not join until the distance between $carry$ and $borrow$ paths that have been traversed is over 2^n .

Let maximum distance from source to stage i by $carry$ and $borrow$ functions be C_{\max} and B_{\max} , respectively.

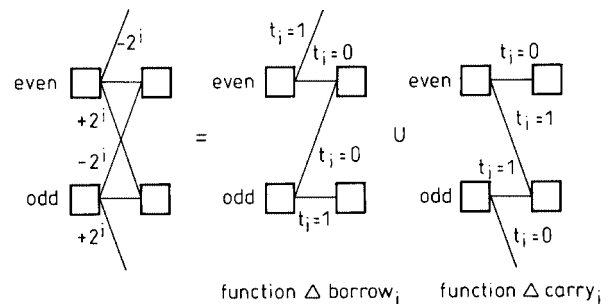


Fig. 2 Switching by $\Delta borrow_i$ and $\Delta carry_i$ functions at stage i

When the two *carry* and *borrow* paths meet at stage i , the following equation should hold.

$$C_{\max} + B_{\max} = \sum_{j=0}^{i-1} (1+1)2^j \geq 2^n \quad (1)$$

Solving eqn. 1, we obtain $i \geq n$, where $n = \log_2 N$. From the result, we conclude that the two paths will not intersect before stage n . Hence, the two paths generated by carry and borrow routing functions are disjoint when the difference of source and destination is odd.

Theorem 1 states that the GIN has two disjoint paths when $(S-D) \bmod 2 = 1$. But when $(S-D) \bmod 2 = 0$, multiple paths between source and destination are not disjoint because the carry and borrow functions take the same straight link from stage 0 to stage 1.

3 Partially chained gamma interconnection network

In order to derive disjoint paths when $(S-D) \bmod 2 = 0$, the topology of the GIN must be modified, which motivates us to propose a partially chained gamma interconnection network (PCGIN).

3.1 Topology

The PCGIN is modified from the GIN by adding a new link between switches at stage 0 and removing redundant nonstraight links between the last two stages shown in Fig. 3. The switches at stage 0 are replaced with 2×4 switches, those at stage $n-1$ with 3×2 switches, and those at the final stage with 2×1 switches. The PCGIN can provide at least two disjoint paths between any source and destination pair. As a result, a PCGIN can tolerate one link or switch fault.

The routing scheme in a PCGIN is similar to what we have described in Section 2, except at stages 0 and $n-1$. Let the destination node be $D = d_0 d_1 d_2 \dots d_{n-1}$. In an even (odd) switch at stage 0, when d_0 is 0(1), either the straight or chained link can be taken: if the *carry* function is applied, the straight link is taken; otherwise the chained link is chosen. At stage $n-1$, both the up and down links are connected to the same switch, and the *borrow* function takes the unique nonstraight link when the up link is

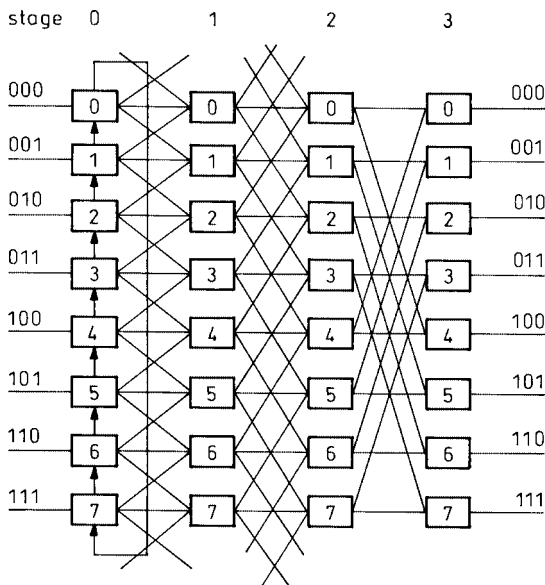


Fig. 3 Partially chained gamma interconnection network with $N=8$

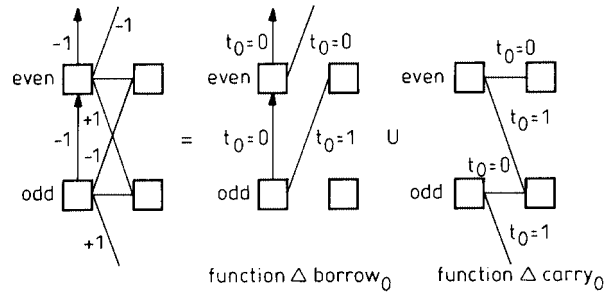


Fig. 4 The behaviour of borrow and carry functions at stage 0 in PCGIN

supposed to be taken. The behavior of *carry* and *borrow* functions in a PCGIN is similar to that in Definition 2 except for the *borrow* function at stage 0, as shown in Fig. 4.

Definition 3: Carry and borrow functions in PCGIN

Stage 0 :

$$borrow(j, t_0) = \begin{cases} (j-1) \bmod N \text{ at stage 0} \\ \quad \text{if } (j_i = 0 \text{ and } t_0 = 0) \\ \quad \text{or } (j_i = 1 \text{ and } t_0 = 1) \\ (j-1) \bmod N \text{ at stage 1} \\ \quad \text{if } (j_i = 0 \text{ and } t_0 = 1) \\ \quad \text{or } (j_i = 1 \text{ and } t_0 = 1) \end{cases}$$

Stage $n-1$:

$$borrow(j, t_{n-1}) = \begin{cases} (j+2^i) \bmod N \text{ at stage } n \\ \quad \text{if } (j_i = 0 \text{ and } t_{n-1} = 1) \\ \quad \text{or } (j_i = 1 \text{ and } t_{n-1} = 0) \\ j \text{ at stage } n \\ \quad \text{if } (j_i = 0 \text{ and } t_{n-1} = 0) \\ \quad \text{or } (j_i = 1 \text{ and } t_{n-1} = 1) \end{cases}$$

Other stages use the same functions as defined in Definition 2.

Theorem 2: The PCGIN has two disjoint paths from any source node to any destination node.

Proof: Two cases must be considered: the first is $(S-D) \bmod 2 = 1$, and the other is $(S-D) \bmod 2 = 0$. The first case has been proved in Theorem 1, and we only have to prove the case of $(S-D) \bmod 2 = 0$.

When $(S-D) \bmod 2 = 0$, the first bit of the source tag is the same as the first bit of the destination tag. The straight link is taken by applying *carry* function, and the chained link is taken by a *borrow* function. The distance of switch indices by *carry* and *borrow* functions at stage 0 is 1. And at stage 1, the distance will be 2, because the *borrow* function will take the nonstraight link due to the chained link property. At stage i the maximum distance from the source node by *borrow* function will be $1 + 1 + 2^1 + \dots + 2^{i-1}$, and by *carry* function it will be $0 + 2^1 + 2^2 + \dots + 2^{i-1}$. If the sum of these two paths in a vertical direction is not over 2^n , the proof is shown. Consider the worst case:

1. The distance at the 0th stage, where the chained link is taken by the *borrow* function, is 1.
2. The distance at the first stage is 2, since $(S-D) \bmod 2 = 0$, *carry* function takes the straight link and the *borrow* function selects the nonstraight link.

3. Hence, the maximum distance at the second stage is $2 + 2*2^1$.

4. The maximum distance at the $(n - 1)$ th stage is $2 + 2*2^1 + \dots + 2*2^{n-2} = 2^n - 2$, which is less than 2^n .

This indicates that the two paths will be disjoint from source to stage $n-1$. And the straight and nonstraight links will be taken by the two routing functions, respectively, from stage $n-1$ to stage n (final stage). Hence, the two routing paths will not intersect at intermediate switches before they reach their destination. \square

3.2 Routing schemes

The routing scheme of a PCGIN is similar to what has been described in [7]. Each switch is initially set to behave as an odd or even switch. Each switch contains a state flag, which can be dynamically set to one of the two logical states *borrow* and *carry*. And destination tag routing is used. When a packet is to be routed from stage i via a nonstraight link and the link is blocked, the switch can dynamically toggle its state, i.e., the other alternative nonstraight link can be taken. If the packet takes straight from stage i , and the link is blocked, the packet needs to be routed back all the way until a nonstraight link is used in prior routing. At that point, the switch can change the state, and then, the other nonstraight link is taken. If a packet is backtracked to stage 0 by a straight link, the chained link can be taken to find an alternate path.

4 Fully chained gamma interconnection network (FCGIN)

When a backtracking mechanism is used in a PCGIN, the system performance will be degraded. To avoid backtracking penalties, we propose a fully chained gamma interconnection network (FCGIN). The FCGIN supports distributed control, and dynamic rerouting; moreover a FCGIN has multiple paths between any source-to-destination pair to provide better fault tolerance capability.

4.1 Topology

The FCGIN changes one of the original nonstraight links at each stage to a chained link, except at the final stage, as shown in Fig. 5. There are many kinds of FCGINs, for example, *C*-function, *C'*-function, *carry*-function, and *borrow*-function based FCGINs. In this paper, we present the *borrow*-function based FCGIN only. In a FCGIN, each switch is augmented with a chain-in link and a chain-out link. $2*3$ switches are needed at stage 0, final stage still uses $2*1$ switches, and other stages require $3*3$ switches. Such a network can tolerate a single link fault at each stage.

The naming scheme for FCGINs is as follows: the stages are labelled in a sequence from 0 to $(\log_2 N)$, with 0 being the leftmost source-side stage. The chaining scheme in a FCGIN is that switch j is chained to switch $(j - 2^i) \bmod 2^n$, where i is the stage number, and $n = \log_2 N$. For example, at stage 0, the chain-out link of switch 1 is connected to the chain-in link of switch 0.

4.2 Routing Scheme

The routing scheme is destination tag routing. Similar to the PCGIN, the first step is to initialise the switch to be even or odd.

Routing in the absence of faults: When faults are absent, routing in an FCGIN is through the *borrow* function. For example, a packet has a source tag $s_0s_1 \dots s_{n-1}$ and a

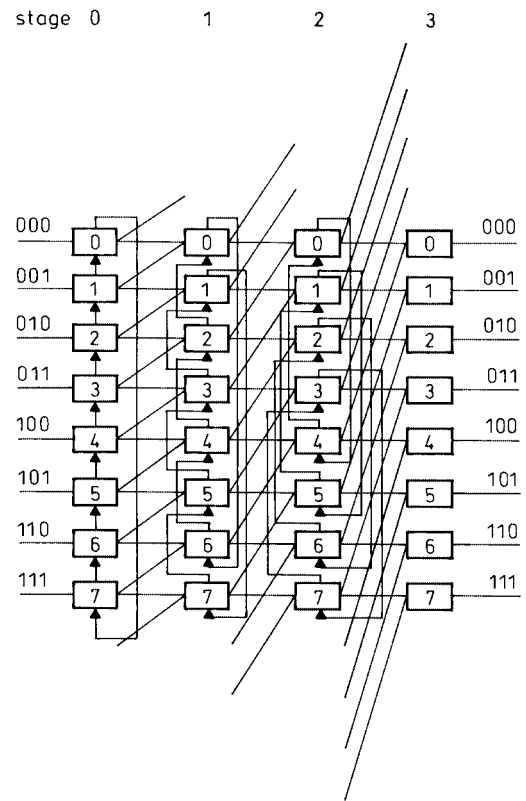


Fig. 5 Fully chained gamma interconnection network with $N=8$

destination tag $d_0d_1 \dots d_{n-1}$. If $d_i = 0$ in an odd switch at stage i , the packet is routed to an up link; and if $d_i = 1$, the packet is routed to the straight link by *borrow* routing function. And when a switch is even, the routing is the opposite.

Routing under link faults: When faults occur, the chain links provide alternate paths. Assume a packet at the switch $j = j_0j_1 \dots j_i \dots j_{n-1}$ of stage i , where j_0 is the least significant. If the link from stage i to stage $i+1$ is faulty, the packet should be routed via the chain link at stage i to switch $(j - 2^i) \bmod 2^n$, that is, the bit j_i will be changed ($0 \rightarrow 1, 1 \rightarrow 0$). Then, the packet can be routed to the next stage $i+1$ from this switch $(j - 2^i) \bmod 2^n$, for $0 \leq i \leq n-1$ where $n = \log_2 N$. With this routing, the FCGIN has the ability to tolerate one link fault at each stage. So the FCGIN can tolerate $\log_2 N$ link faults that are distributed evenly among all stages. We can say that therefore the FCGIN has link strong reroutability.

Definition 4: Switches j and k at stage i , $0 \leq i \leq \log_2 N$, belong to the same partition if $j_i = k_i, \forall 0 \leq l \leq i-1$, where $j_0j_1 \dots j_{n-1}$ and $k_0k_1 \dots k_{n-1}$ are the binary representations of j and k , respectively.

From Definition 4, all switches belong to the same partition at stage 0, and at stage 1, the switches 0, 2, 4, \dots , $N-2$ constitute a partition. Hence, stage i has 2^i partitions for $0 \leq i \leq \log_2 N$. If there is no link that can be used except for the chain link, the packet can still be routed via the chain link to another switch at the same stage and be routed to the next stage. If the link to the next stage is still faulty, the packet is repetitively routed via another chain link until $N/2^i$ switches have been traversed. Hence FCGINs have strong link reroutability and can tolerate multiple link faults at the same stage.

Theorem 3: An FCGIN can tolerate one fault except at stage 0 and n .

Proof: There are two parts in the proof. One is with one link fault, and the other is with one switch fault. For the one link fault case, because the FCGIN has strong link reroutability, it can tolerate at least one link fault. We only prove the case with one switch fault.

If a packet is routed at stage i to stage $i + 1$, and the switch at stage $i + 1$ is faulty, the packet will be routed via a chained link to find another link to stage $i + 1$ in its partition (let the partition be H). By Definition 4, there are $N/2^i$ switches in the partition H at stage i , and there are $N/2^{i+1}$ legal switches which H can reach at stage $i + 1$ for the destination. If there is a switch fault, there are still $N/2^{i+1} - 1$ switches being sent to the destination at stage $i + 1$, for $0 \leq i \leq n - 2$. Hence, a packet can be routed to stage $n - 1$, and then the destination node is reached successfully if there is only one switch fault. \square

5 Performance comparison and analysis

5.1 Analysis

Table 1 shows the characteristics of these five networks. The hardware costs in these five networks are almost equal except at stage 0 or stage n . As for fault-tolerance, the CGIN, the PCGIN and the FCGIN are single-fault tolerant from stage 0 to stage n , whereas the B-network is one fault tolerant between stage 1 and stage n . When a fault occurs, the CGIN and the PCGIN use the backtracking mechanism to find alternate paths, but the B-network and the FCGIN can find an alternate path without backtracking. In the B-network, if there is a fault between stages i to $i + 1$, the backward link to stage $i - 1$ will be taken, and then a nonstraight link to stage i [8]. If the link to stage i is faulty, the backward link to stage $i - 2$ will be taken, and so on. In the FCGIN, when a fault occurs, the chain link will be taken, and the penalty is only one more link, but in the B-network the penalty is two links to back up to stage i under a fault.

In the PCGIN, the probability that the fault occurs at a straight link, or that the switch that is reached via a straight link is 50%; and the probability of a straight link fault is 33.33%. Hence, there is 16.67% probability of backtracking, but at stage $n - 1$ the probability is 50% because there is no redundant link available. Let the backtracking penalty at stage i be $T(i)$:

$$\begin{aligned} T(i) &= \frac{1}{2} * (1 + \frac{1}{2}(T(i-1) + 1) + 1) \\ &= \frac{3}{4} + \frac{1}{4}T(i-1), T(0) = 1, T(1) = 5/2 \\ T(i) &= \frac{13}{7} - \frac{6}{7}\left(\frac{-3}{4}\right)^n \end{aligned}$$

Table 1: Comparison of GIN, CGIN, B-network, PCGIN, and FCGIN.

Network	Single-fault tolerance	Strong reroutability	Routing complexity	One fault penalty
GIN	no	no	distance tag routing	$\frac{n-1}{3}$
Cyclic GIN	yes	no	distance tag routing	$\frac{n-1}{3}$
B-network	no	no	destination tag routing	2
Partially Chained GIN	yes	no	destination tag routing	$\frac{1}{3}\left(\frac{13}{42}n + \frac{79}{147} - \frac{17}{49}\left(\frac{-3}{4}\right)^{n-1}\right)$
Fully Chained GIN	yes	yes	destination tag routing	1

Average penalty is

$$\begin{aligned} &\frac{1}{n}\left(\frac{1}{6}\sum_{i=0}^{n-2}T(i) + \frac{1}{2}T(n-1)\right) \\ &= \frac{1}{n}\left(\frac{13}{42}n + \frac{79}{147} - \frac{17}{49}\left(\frac{-3}{4}\right)^{n-1}\right), \end{aligned}$$

for $n \geq 2$

In the CGIN, the average penalty is

$$\frac{1}{3n} * \sum_{i=0}^{n-2} 2i \frac{n-1}{3}$$

In the PCGIN, it is enough to route the packet only with the destination tag. But in the CGIN, the packet may require both a destination tag and a rerouting tag. The last column of Table 1 summarises the one fault penalties of these networks.

5.2 Simulation results

In addition to qualitative comparison, we also use simulation to compare these networks quantitatively. The performance metrics include *arrival rate* and *fault penalty*. ‘Arrival rate’ is the arrival packets divided by the total packets. ‘Fault penalty’ indicates the average time taken by a packet meeting faulty link from its source to its destination. The time is counted from the packet entering the network to leaving the network. The assumptions of our simulation are as follows:

1. Packets arriving at each network input follow a uniform distribution.
2. The destinations of generated packets are uniformly distributed across all network output.
3. The switch has no buffer for routing conflicts. If any conflict occurs, the packet will be dropped or routed to an alternate switch.
4. A network output can accept one packet per network cycle; an accepted packet is removed from the network immediately.

The routing and rerouting schemes list is as follows: the B-network, the PCGIN and the FCGIN use destination tag routing, and the GIN and CGIN use distance tag routing, i.e. the packet must have a routing and rerouting tag. When a fault or conflict occurs, the B-network and FCGIN use a backward and chained link to reroute packets, respectively. But, the PCGIN, GIN, and CGIN use a backtracking scheme when a fault occurs and drop packets when conflict happens. In the PCGIN, the backtracking is used only when the fault link is straight, because the other state can be used when the fault is a nonstraight link.

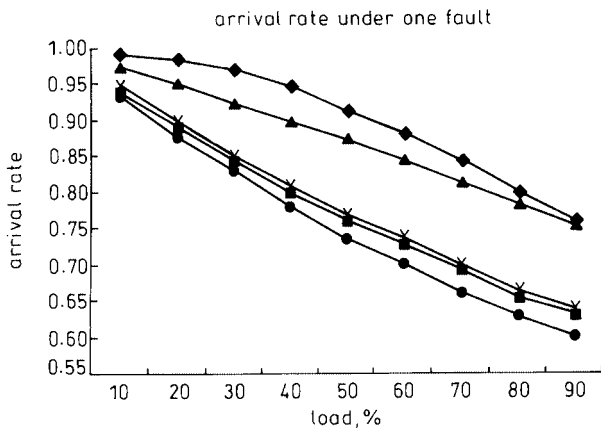


Fig. 6 Arrival rate with one fault used on fault and conflict packet with $N=16$

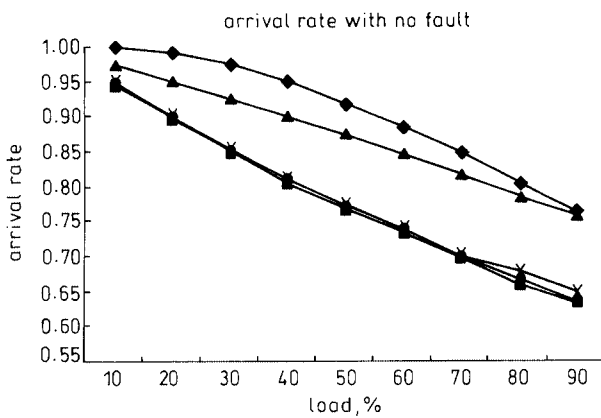
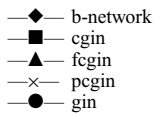


Fig. 7 Arrival rate with no fault used on fault and conflict packet with $N=16$

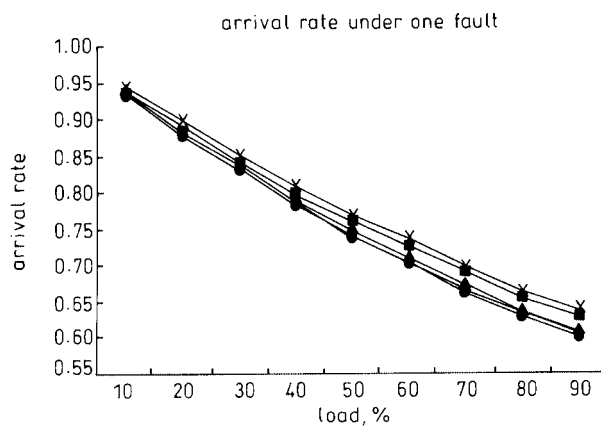
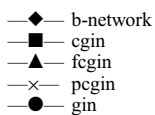
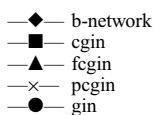


Fig. 8 Arrival rate with one fault used on fault packet and conflict with $N=16$



The simulations are run under the load 0.1 to 0.9 with network $N=16$. Figs. 6 and 7 show the arrival rates with no fault and with one fault. The FCGIN and the B-network give better performance than the PCGIN, GIN, and CGIN, because the FCGIN and B-network have an alternate path when a conflict occurs. Furthermore, the B-network is better than the FCGIN because the switch (3×1) conflict at the last stage can be resolved by backward links in the B-network rather than the FCGIN. Figs. 8 and 9 show the results when the alternate paths are used only under a fault occurring. The PCGIN, GIN and CGIN are better because the utilisation of each link is smoother than in the B-network and FCGIN. To count the fault penalty, we assume that the conflict situation is as low as possible, i.e. the load is 1% (the arrival rate is more than 98%). The results shown in Fig. 10 are the fault penalties of the packets. The FCGIN and B-network are almost 1 and 2, respectively, and the CGIN and GIN are higher because of the backtracking penalty. However, in the PCGIN, the penalty is low enough because the backtracking scheme is used when the straight link is faulty.

To sum up, the FCGIN and B-network have better performance, but the B-network cannot guarantee one fault tolerance. In fault penalty, the PCGIN and FCGIN perform better than the others.

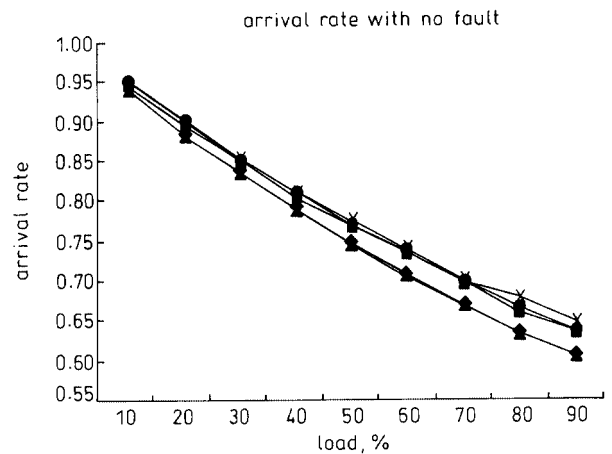


Fig. 9 Arrival rate with no fault used on fault packet and conflict with $N=16$

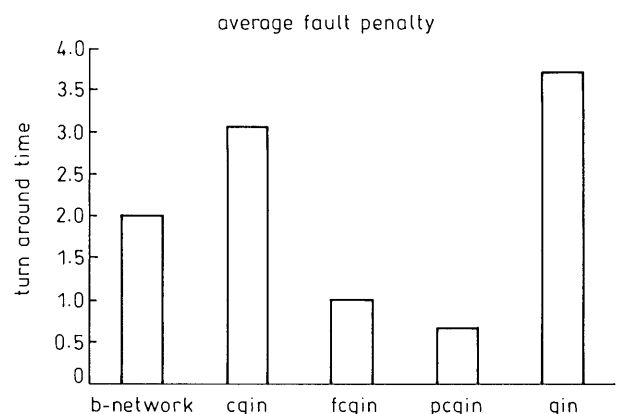
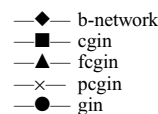


Fig. 10 Average fault penalty under low load with $N=16$

6 Concluding remarks

In this paper, we propose two single-fault-tolerant gamma interconnection networks. The first is a partially chained gamma interconnection network (PCGIN), which employs chain links at stage 0 to provide disjoint paths between any source and destination pair. In the PCGIN, when a fault occurs at stage 0, the chain link provides an alternate path. However, when a fault occurs at the other stages, backtracking to find an alternate path may be necessary. To avoid backtracking, we also propose a fully chained gamma interconnection network (FCGIN). The FCGIN has one fault-tolerance and strong reroutability. The complexities of switches in the FCGIN and the routing algorithm are lower than those of the PCGIN. To demonstrate the effectiveness of the proposed networks, we also conducted simulations to compare the five gamma-related networks in terms of cost, fault tolerance and average penalty. Simulation results show that the B-network and FCGIN give better overall performance, and that the PCGIN has a lower fault penalty.

7 References

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