# A Microstrip Elliptic Function Filter with Compact Miniaturized Hairpin Resonators

Jen-Tsai Kuo, *Member, IEEE*, Ming-Jyh Maa, and Ping-Han Lu

***Abstract*—A four-pole elliptic function bandpass filter is designed using compact miniaturized microstrip hairpin resonators. It is shown that the filter occupies a very small area. The full-wave simulator IE3D is used to design the resonator and to calculate the coupling coefficients of the three basic coupling structures. Measurement results are compared with the computed responses.**

***Index Terms*—Coupling coefficient, hairpin resonator, microstrip filter, miniaturized circuit.**

## I. INTRODUCTION

RECENTLY, expanding wireless and mobile communication systems have presented new challenges to the design of high-quality miniature RF filters. Planar filters would be preferred since they can be fabricated using printed circuit technology with low cost. Obviously, the size of planar filters with parallel-coupled half-wavelength microstrips [1] is too large to be used in modern systems. Thus, size reduction has been an important issue in developing RF filters.

The hairpin filters [2]–[4] make progress in size reduction from the parallel-coupled lines structure. The whole filter consists of a cascade of U-shaped resonators [2], [3]. If the orientations of the hairpin resonators alternate, the structure is capable of considerable bandwidth; if they do not, the structure has attractive properties for design of compact narrow-band filters [3]. In [4], elliptic function filters are realized by the U-shaped microstrips with different orientations and offsets to establish an adequate coupling combination among the resonators.

Further progress in size reduction is made by the compact miniaturized hairpin resonator filters [5], where the two arms of the U-shaped microstrip are further folded to form a pair of closely coupled lines to enhance the capacitive nature of open-end arms. The area of such a resonator is no more than half of that of a U-shaped resonator. Also of interest are filters with microstrip square open-loop resonators [6], of which the lateral size is only a one-eighth guided wavelength at the midband frequency.

In this letter, we present a design of four-pole elliptic function filters using the compact miniaturized hairpin resonators. It will be shown that each resonator uses about 75% of the area of a square open-loop resonator. The presentation is organized as follows. Section II investigates the design of a single compact miniaturized hairpin resonator. Section III discusses the couplings between two adjacent resonators for the three basic coupling structures. In Section V, measured filter responses are presented and compared with the simulation.

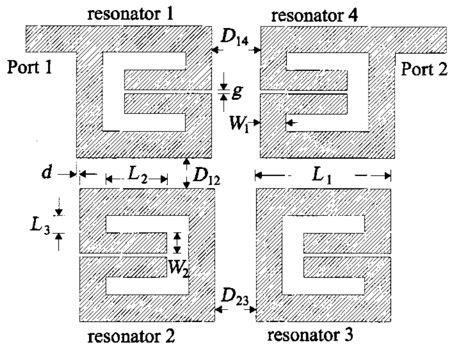

Fig. 1. Layout of the four-pole elliptic function bandpass filter. Geometric parameters for each resonator: $W_1 = 1.187$ mm, $W_2 = 0.91$ mm, $L_1 = 6.124$ mm, $L_2 = 2.56$ mm, $L_3 = 0.8$ mm, and $g = 0.173$ mm.

## II. DESIGN OF A SINGLE RESONATOR

The compact miniaturized hairpin resonators shown in Fig. 1 are evolved from sections of half-wavelength open microstrips, thus the fundamental resonance of each resonator occurs in odd-mode. This means that at the resonance the voltage at the central valley of the folded microstrip is a minimum, and the voltages at both ends of the coupled lines have maximal values with opposite signs. The condition for this fundamental resonance can be derived from the concept of stepped impedance resonators [5].

In designing a single resonator: 1) the width of the microstrip is determined by setting its characteristic impedance to 50 Ω and in making the whole structure compact; 2) the peripheral of each resonator is made a square; 3) the length of the central open-circuited coupled lines is extended to its extremes; and 4) the gap and width of the coupled lines are made as small and large, respectively, as possible. As a result, the lateral size of the resonator is 6.1 mm, which is 13% less than that of a square open-loop resonator reported in [6], for designing a filter with the same center frequency.

## III. BASIC COUPLING STRUCTURES

The filter in Fig. 1 has a 2 × 2 configuration. Significant couplings exist between any two nondiagonally neighboring resonators. In our design, the coupling between resonators 1 and 2 is identical to that between resonators 3 and 4. Thus, there are three basic coupling structures to be investigated. The coupling

---

Manuscript received September 29, 1999; revised January 31, 2000. This work was supported by the National Science Council, Taiwan, R.O.C., under NSC Grant 89-2213-E-009-057.

The authors are with the Department of Communication Engineering, National Chiao Tung University, Hsinchu 300, Taiwan, R.O.C. (e-mail: jtkuo@cc.nctu.edu.tw).

Publisher Item Identifier S 1051-8207(00)03345-6.

1051–8207/00$10.00 © 2000 IEEE

in each structure can be specified by the two dominant resonant frequencies, which are split off from the resonance condition due to the electromagnetic coupling. Let $f_a$ be the lower of the two resonant frequencies, and $f_b$ be the higher one. The coupling coefficient $K_{ij}$ for resonators $i$ and $j$ can be calculated as [6]

$$K_{ij} = \pm \frac{f_b^2 - f_a^2}{f_b^2 + f_a^2} \tag{1}$$

where the upper sign $(+)$ applies to $K_{12}$, $K_{23}$, and $K_{34}$, while the lower sign $(-)$ applies to $K_{14}$. It is to be noted that (1) is applied by separating off two resonators at a time and that $K_{12}$ and $K_{34}$ are also subject to an offset $d$. We use the 3-D full-wave simulator IE3D [8] to design the isolated miniaturized hairpin resonator and to determine the resonant frequencies of each coupling structure. The way to obtain $K_{ij}$ is to model the pair of resonators and compute their transmission response. For sufficiently accurate simulation results, it is found that a discretization of 80 cells per wavelength is required for our particular circuit configuration.

## IV. RESULTS AND MEASUREMENTS

The circuit is designed to be fabricated on a dielectric substrate with $\varepsilon_r = 10.2$ and a thickness of 1.27 mm. Fig. 2 plots the coupling coefficients $K_{12}$, $K_{23}$, and $K_{14}$ against the distances between the resonators. The magnitude of $K_{14}$ is less than those of $K_{12}$ and $K_{23}$. Each $K_{ij}$ value decreases as the corresponding distance is increased, as expected.

A four-pole elliptic function filter response can be realized using proper cross couplings. For the particular 2 × 2 configuration in Fig. 1, the cross couplings give the input signal two paths from the input port to the output. Through the two paths, the signal magnitude and phase are altered differently. Thus, at the output port, the multipath effect may cause attenuation poles at finite frequencies if the couplings are properly designed [4]. A bandpass filter is fabricated to demonstrate the design. The center frequency $f_o$ is 2.46 GHz and the fractional bandwidth $\delta = \Delta f / f_o$ is 4%. The cross couplings can be synthesized using the method described in [7]. The lumped circuit element values of the low-pass prototype filter are found to be $g_0 = 1$, $g_1 = 1.1425$, $g_2 = 2.0558$, $J_1 = -0.0828$, and $J_2 = 1.1307$. The entries of the 4 × 4 coupling coefficient matrix $K$ are found to be $K_{12} = K_{21} = K_{34} = K_{43} = 2.610 \times 10^{-2}$, $K_{14} = K_{41} = -2.899 \times 10^{-3}$, $K_{23} = K_{32} = 2.200 \times 10^{-2}$, and the rest of the eight entries are zero. The specified $K_{23}$ and $K_{14}$ values are used to determine the distances $D_{23}$ and $D_{14}$, respectively, according to Fig. 2. The offset between resonators 1 and 2, $d = (D_{14} - D_{23})/2$, which should be obtained before $D_{12}$ is searched.

Fig. 3 compares the computed and measured filter responses. The measurements are performed with an HP 8720C network analyzer. The measured results agree very well with the computations. The passband insertion loss is about 3 dB, which is mainly resulted from the conductor loss.

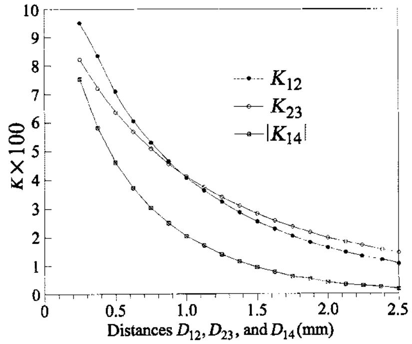

Fig. 2. Simulation coupling coefficients $K_{12}$, $K_{14}$, and $K_{23}$ versus the distances between the resonators.

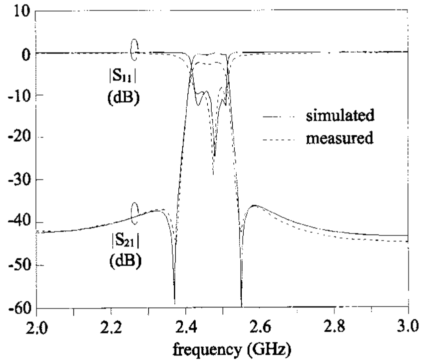

Fig. 3. The measured and simulated $S$-parameters of the filter. $D_{12} = 1.44$ mm, $D_{23} = 1.82$ mm, $D_{14} = 2.22$ mm, and $d = 0.20$ mm.

## V. CONCLUSION

We have presented the design of microstrip cross-coupled bandpass filers with miniaturized hairpin resonators. The design is based on the knowledge of the coupling coefficients of the three basic coupling structures. The measured responses have good agreement with the theoretical predictions. The compactness in circuit size makes the design of cross-coupled filters using the miniaturized hairpin resonators attractive for further developments and applications in modern mobile radio systems.

## ACKNOWLEDGMENT

The authors wish to thank the reviewers for their positive comments which greatly enhanced the quality of this letter.

## REFERENCES

[1] S. B. Cohn, “Parallel-coupled transmission-line resonator filters,” *IRE Trans. Microwave Theory Tech.*, vol. MTT-6, pp. 223–231, Apr. 1958.
[2] E. G. Cristal and S. Frankel, “Hairpin-line and hybrid hairpin-line/half-wave parallel-coupled-line filters,” *IEEE Trans. Microwave Theory Tech.*, vol. MTT-20, pp. 719–728, Nov. 1972.
[3] G. L. Matthaei, N. O. Fenzi, R. J. Forse, and S. M. Rohlfing, “Hairpin-comb filters for HTS and other narrow-band applications,” *IEEE Trans. Microwave Theory Tech.*, vol. 45, pp. 1226–1231, Aug. 1997.
[4] J. S. Hong and M. J. Lancaster, “Cross-coupled microstrip hairpin-resonator filters,” *IEEE Trans. Microwave Theory Tech.*, vol. 46, pp. 118–122, Jan. 1998.
[5] M. Sagawa, K. Takahashi, and M. Makimoto, “Miniaturized hairpin resonator filters and their application to receiver front-end MIC’s,” *IEEE Trans. Microwave Theory Tech.*, vol. 37, pp. 1991–1996, Dec. 1989.
[6] J.-S. Hong and M. J. Lancaster, “Couplings of microstrip square open-loop resonator for cross-coupled planar microwave filters,” *IEEE Trans. Microwave Theory Tech.*, vol. 44, pp. 2099–2108, Nov. 1996.
[7] A. E. Atia and A. E. Williams, “Narrow-bandpass waveguide filters,” *IEEE Trans. Microwave Theory Tech.*, vol. 20, pp. 258–265, Apr. 1972.
[8] Zeland Software Inc., IE3D simulator, Jan. 1997.