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Symbolic path-based protocol verification

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Abstract

The principal problem in protocol verification is state explosion problem. In our work (W.C. Liu, C.G. Chung, Path-based Protocol Verification Approach, Technical Report, Department of Computer Science and Information Engineering, National Chiao-Tung University, Hsin-Chu, Taiwan, ROC, 1998), we have proposed a "divide and conquer" approach to alleviate this problem, the *path-based approach*. This approach separates the protocol into a set of concurrent paths, each of which can be generated and verified independently of the others. However, reachability analysis is used to identify the concurrent paths from the Cartesian product of unit paths, and it is time-consuming. Therefore, in this paper, we propose a simple and efficient checking algorithm to identify the concurrent paths from the Cartesian product, using only Boolean and simple arithmetic operations. © 2000 Elsevier Science B.V. All rights reserved.

Keywords: Protocol verification; Reachability analysis; Path-based approach

1. Introduction

Communication protocols are concurrent software to maintain, coordinate and govern the interactions of concurrent processes in a distributed system. As protocols become increasingly large-scale and complex, the designs of correct protocols are becoming challenging and difficult tasks. To deal the complexity and difficulty of protocol design, protocol verification is introduced. It is a process to verify protocols which are modeled as finite state machines with respect to the crucial logical properties, such as deadlock, livelock, channel overflow, etc. Most verification approaches to date are based on *reachability analysis* (or known as *state enumeration*) to enumerate all the reachable states from an initial one and to check whether all reachable states can satisfy the necessary properties [2–4]. Although such technique is simple, automatic and effective, it suffers from the "*state explosion problem*" [4–6]. This problem asserts that the number of states grows exponentially with the complexity of the protocol. Quantitatively speaking, a protocol has at most $|q|^n((|m|+1)^h))^{nn}$ states, where q is the number of local states for each unit, m the number of message types, n the number of units in the protocol, and h the channel capacity, the number of messages in a channel [7]. Although this amount is the number of syntactically reachable states and is several orders of magnitude larger than that of semantic ones, it still grows exponentially with the complexity of protocol. All reachable states must explicitly or implicitly be memorized in a *reachability graph* (RG) to avoid generating duplicate states and to exclude the infinite exploration. Due to the limited memory capacity, on the basis of Ref. [3], the limit of the fully-search reachability analysis is 10^5 states, and that of the controlled partial search or known as *relief strategy* [3] is 10^8 states, which intends to reduce the number of global states necessary to be explored. Although the technique of BDD [8] has mad much progress in reducing the number of states [9], the efficient use of BDD depends on the problem domain, and the conventional state enumeration technique outperforms the BDD-based technique in some cases [10] and is still the most general approach for protocol verification.

The state explosion problem raises two issues: *large memory space* and *long verification time*. For the first issue, we have proposed the *path-based approach* [1] underlying the concept of *concurrent path* to represent the execution behavior of the protocol into a partial-order representation as a set of unit paths [11,12]. This approach treats the protocol as a set of concurrent paths so that each concurrent path can be generated and verified independently of the others. Thus, the memory required to store reachable global states depends on the complexity of a concurrent path rather than the whole protocol, and the memory space issue is alleviated. However, the issue of long verification time does not have a very good solution. Since, in our approach,

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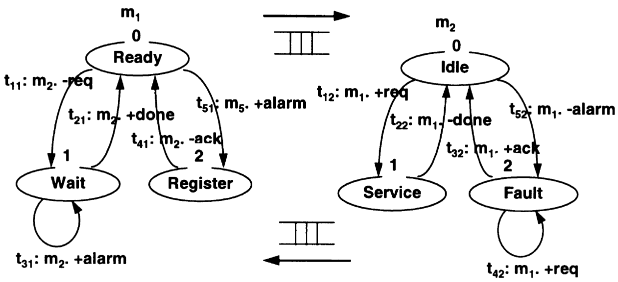


Fig. 1. A simple protocol.

the concurrent path is generated via the Cartesian product of unit paths, each product member (denoted as *concurrent path candidate*) is an arbitrary combination of unit paths and is not always a concurrent path. We use reachability analysis to identify the concurrent paths from the candidates by checking whether a candidate corresponds to a real execution behavior in the protocol. Although, we use several techniques to avoid the unnecessary check, reachability analysis is still a time-consuming technique to perform such a check.

In Ref. [1], reachability analysis is used to identify the last reachable global state with respect to a concurrent path candidate. As long as the last state is identified, we can classify the candidates as valid or invalid by checking if the last reachable global state is a real blocking state in the system. If it is not, this candidate is identified as invalid. However, for a communication finite state machine (CFSM) model, the execution completely depends on the message types and orders of sending and receiving. Thus, in this paper, we propose an approach to compute the last reachable global state using such relations. The computation uses simple Boolean and arithmetic operations, and the checking speed has improved a lot. The remaining context is organized as follows: In Section 2, we first overview the underlying protocol model, the CFSM model and the path-based approach [1]. Then in Section 3, we illustrate the new method to compute the last reachable state. Finally, we give a conclusion about our work.

2. Concepts of path-based protocol verification

2.1. CFSM model

The underlying protocol model in this paper to prescribe a protocol is the *Communication Finite State Machine* (CFSM) model, which is a collection of modules communicating with each other via messages.

Definition 1. A protocol P in the CFSM model, denoted as a *CFSM system* or shortly a *system*, is 5-tuple:

$$P = \left(\langle S_i\rangle_{i=1}^n, \langle M_{ij}\rangle_{ij=1}^n, \langle O_i\rangle_{i=1}^n, \langle Z_i\rangle_{i=1}^n, \langle \Delta_i\rangle_{i=1}^n\right),$$

where n is the number of modules, i.e. $m_1, \ldots, m_n$, S_i the set of states of m_i and $S_i \cap S_j = \phi$ for $i \neq j$ ("ϕ" denotes an empty set), M_{ij} the set of messages that can be sent from m_i to m_j, M_{ii} the empty for each i, and $M_{ij} \cap M_{pq} = \phi$ for $i \neq p$ or $j \neq q$, and O_i and Z_i represent the initial and terminal states of m_i that range over S_i, respectively, Δ_i is a partial mapping function: $S_i \times I_i \rightarrow S_i$, and $\Delta_i(s, x)$ is the state that entered after the m_i receives the message x in state s, for each i. ($I_i = \bigcup_{j=1}^n M_{ji}$ is the set of messages that can be received by m_i.)

Each module m_i in the CFSM system P is a *Finite State Machine* (FSM) composed of states and transitions defined by S_i and Δ_i, respectively. Every two modules m_i and m_j are connected by a dedicated communication channel to transmit the message in M_{ij} from m_i to m_j, which is modeled by a FIFO queue with channel capacity h_{ij} limiting the number of messages in the channel.

Fig. 1 shows a simple CFSM system with two modules in a graphical form. The label attached to the transition t in m_i is of the form $m_j.-msg$, or $m_j.+msg$ for the *sending* and *receiving transitions*, respectively, where m_j $(1 \leq j \leq n)$ is the module sending or receiving the message *msg* ($msg \in M_{ij}$ or M_{ji}, respectively). (When there are only two modules, the label of m_i always denotes the other module and can be omitted.) The transition t is defined by the function $\beta(t) = \Delta_i(\alpha(t), \lambda(t))$, where $\alpha(t)$ and $\beta(t)$ are referred to as the heading and tail states of t, respectively, and $\lambda(t)$ denotes the message to be sent or received in transition t.

The status of a CFSM system, at any moment of execution, is depicted by the global state of the system recording the states of the constituent modules and the contents of the communication channels.

Definition 2. A *global state* g of the CFSM system P is a pair $g = \langle S, C\rangle$, where S is a n-tuple of states $\langle s_1, \ldots, s_n\rangle$ (s_i represents the current state of module m_i), and C is a n^2-tuple $\langle c_{11}, \ldots, c_{1n}, c_{21}, \ldots, c_{nn}\rangle$. ($c_{ij}$ is a sequence of messages ranging over M_{ij} whose length is denoted as $|c_{ij}|$. The message sequence c_{ij} represents the contents of the communication channel from module m_i to m_j. Note that every c_{ii} is empty since M_{ii} is empty.) (The brackets around S and C could be combined without confusion so that g is in the form of $\langle s_1, \ldots, s_n, c_{11}, \ldots, c_{1n}, c_{21}, \ldots, c_{nn}\rangle$.)

The system stays at a global initial state when it is initialized and finally reaches a global terminal state on normal execution.

Definition 3. The *global initial state* of P is a global state, denoted as G_0, $G_0 = \langle\langle O_i\rangle_{i=1}^n, \langle \varepsilon\rangle_{i,j=1}^n\rangle$ (ε denotes an empty message sequence). The *global terminal state* of P is a global state, denoted as G_{T}, $G_{\mathrm{T}} = \langle\langle Z_i\rangle_{i=1}^n, \langle \varepsilon\rangle_{i,j=1}^n\rangle$.

The execution behavior of a CFSM system is defined by the relation (called *global transitions* to differentiate with module transitions) between the global states.

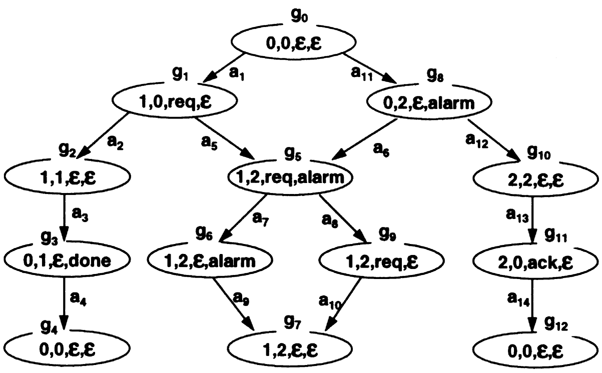


Fig. 2. The reachability graph of the protocol in Fig. 1.

Definition 4. A global transition is defined as a binary relation " $\Rightarrow$ " on global states of P (meaning that P at one global state can be transferred to the other in one step of execution): $g \Rightarrow g'$ iff there exist i, k and x satisfying $s_i' = \Delta_i(s_i, x)$ in either of the following two conditions:

$$c_{ki} = xc'_{ki} \qquad \text{and} \qquad x \in M_{ki} \tag{1}$$

or

$$c'_{ik} = \begin{cases} c_{ik}x & \text{if} \quad x \in M_{ik} \quad \text{and} \quad |c_{ik}| < h_{ik} \\ c_{ik} & \text{if} \quad x \in M_{ik} \quad \text{and} \quad |c_{ik}| = h_{ik} \end{cases}. \tag{2}$$

where $g = \langle S, C\rangle$, $g' = \langle S', C'\rangle$, $S' = \langle s_i'\rangle_{i=1}^n$ and $C' = \langle c'_{ij}\rangle_{i,j=1}^n$.

The first and the second conditions denote a blocking receiving and a non-blocking sending (when the channel reaches the channel capacity, the next sending action can be executed, but its messages are ignored and the channel content remains the same), respectively. The transition associative with $\Delta_i(s_i, x)$ is referred to as the global transition from g to g'.

Definition 5. A global state g' is reachable from g if $g \Rightarrow^* g'$, where " $\Rightarrow^*$ " is the reflexive and transitive closure of " $\Rightarrow$ ". g' is said to be reachable with respect to the system if $g = G_0$.

Definition 6. If g' is reachable from g, then all the global states traversed from g to g' constitute a *subsequence* of the system. If $g = G_0$, then the subsequence from g to g' is an *execution sequence*.

One simple but effective method to enumerate all reachable global states and execution sequences, is *reachability analysis* which demonstrates the interactions among the modules is a total order manner and which constructs the RG as the one shown in Fig. 2. In the RG, every node denotes the global state $g_i = \langle c\rangle$, and every trail from the starting node, such as node g_0 in Fig. 2, to a sink node, such as nodes g_4, g_7 and g_{12}, is an execution sequence, where g_i is the name of the node (global state) and c is the text in the ellipse. For example, there are six trails in Fig. 2, each of which corresponds to an execution sequence as follows:

$$x_1 : g_0 \Rightarrow g_1 \Rightarrow g_2 \Rightarrow g_3 \Rightarrow g_4,$$

$$x_2 : g_0 \Rightarrow g_1 \Rightarrow g_5 \Rightarrow g_6 \Rightarrow g_7,$$

$$x_3 : g_0 \Rightarrow g_1 \Rightarrow g_5 \Rightarrow g_9 \Rightarrow g_7,$$

$$x_4 : g_0 \Rightarrow g_8 \Rightarrow g_5 \Rightarrow g_6 \Rightarrow g_7,$$

$$x_5 : g_0 \Rightarrow g_8 \Rightarrow g_5 \Rightarrow g_9 \Rightarrow g_7, \quad \text{and}$$

$$x_6 : g_0 \Rightarrow g_8 \Rightarrow g_{10} \Rightarrow g_{11} \Rightarrow g_{12}.$$

Among them, the execution sequences x_1 and x_6 are correct execution sequences, whereas the sequences x_2, x_3, x_4, and x_5 are faulty ones (they deadlock at g_7.)

However, reachability analysis suffers from the state explosion problem because the size of the RG increases exponentially with the complexity of protocol. As a result, reachability analysis and its variations are unsuitable for analyzing complex protocols.

2.2. The path-based approach

Inspecting reachabiltiy analysis and its variations, the major hurdle to a successful verification is the enormous size of the RG and the necessity to memorize the complete graph. However, as the properties to be verified depend on the global states and the execution sequence (i.e. the safety properties and some liveness properties), if all execution sequences (thus including all global states) are generated separately without constructing the RG, we can completely verify the system by examining every execution sequence and its global states rather than the whole RG. The memory requirement to store the global states is limited by the length of an execution sequence rather than the complete RG, and the state explosion problem may be alleviated from such divide and conquer approach.

An execution sequence can be classified as *terminated*, *non-terminated* and *infinite*. First of all, a terminated execution sequence is a finite one ranging from the global initial state to the global state in which every module reaches its corresponding terminal state, such as x_1 and x_6 in Fig. 2. If we project the sequence of transitions in a terminated execution sequence onto the set of transitions of a module, say m_l, we obtain a sequence of m_ls transitions and this sequence will be a path of m_l.

Definition 7. Within a system P, a path p_a in module m_a is defined as follows: $p_a = [s_1, \ldots, s_i, s_{i+1}, \ldots; t_1, \ldots, t_i, \ldots]$, where $s_1 = O_a$, and t_i is the transition from s_i to s_{i+1}.

If we perform such projection of an execution sequence with respect to every module, we can get a set of path each of which belongs to a distinct module. For example, sequence x_l in Fig. 2, if we project the transitions of x_1, i.e. [a_1,a_2,a_3,a_4] onto the transition of m_1 and m_2, we can get the path [t_{11},t_{21}] of m_1 and [t_{12},t_{22}] of m_2 because $a_1 = t_{11}$, $a_2 = t_{12}$, $a_3 = t_{22}$, and $a_4 = t_{21}$.

Second, a non-terminated execution sequence is a finite sequence with at least one module not reaching its terminal state. If we perform the projection of such execution sequence, we obtain the sequence of subpaths, and the behavior of the non-terminated execution sequence can be represented by this set of subpaths.

Definition 8. An subpath (or infix) u_a of p_a of module m_a is defined as $u = [s_i, s_{i+1}, \ldots, s_{j+1}; t_i, t_{i+1}, \ldots, t_j]$ if its length is $j-i+1 > 0$ or $[s_i;]$ if it is empty, where $1 \le i \le j \le k$.

Definition 9. A prefix x_a of p_a is a subpath whose heading state is the heading state of p_a and it is said to be *included* by p_a, i.e. $p_a = x_a \cdot u_a$, where "·" is the subpath concatenation operator, and u_a is a subpath of p_a. If u_a is empty, x_a is denoted as a pure prefix.

However, any subpath must be included by at least one path provided every state is reachable from the initial one. (This requirement should be satisfied for any correct system; otherwise, there must be dead code.) Since the execution blocks at the tailing state of the subpath, the additional transitions in the path but not in the subpath are not executable. Thus, the behavior of such non-terminated execution sequence can also be represented by a set of paths.

Third, an infinite execution sequence must imply that there exists a cyclic structure in it, as the number of global states is finite and there exists a last global state (unless the structure of the path is similar to the infinite decimal which should be rare). Thus, we can also classify the infinite execution sequence into terminated or non-terminated according to the last global state reached, and it can also be represented by a set of paths. (These paths may have loops.)

Definition 10. Within a system P, a nonempty cyclic path p_a in module m_a is defined as $p_a = [s_1, \ldots, (s_i, \ldots, s_j)^*, \ldots s_{k+1}; t_1, \ldots, (t_i, \ldots, t_{j-1})^*, \ldots, t_k]$, where $s_1 = O_a, s_{k+1} = Z_a$ and t_i is the transition from s_i to $s_{i+1} (i \ge 1)$. The subpath $[s_i, \ldots, s_j; t_i, \ldots, t_{j-1}]$ is the loop of p_a.

Therefore, we can use a set of module paths to represent the behavior of the execution sequence:

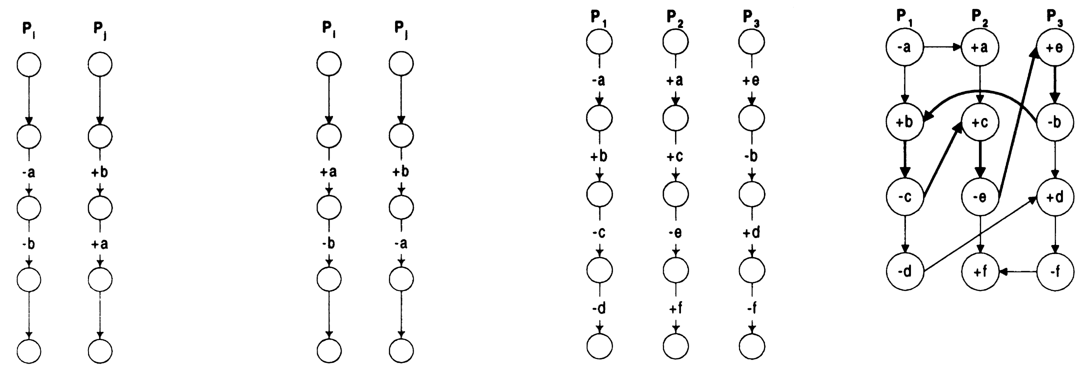
Definition 11. Within a system P, a *concurrent path* is defined as an ordered set of paths $\{p_1, p_2, \ldots, p_n\}$ (or denoted as $\{p_i\}_{i=1}^{n}$ for short), where p_i is a path of m_i.

The concurrent path represents the execution behavior in a partial-order manner, whose ordering relationship is implicitly defined by the precedence of sending and corresponding receiving transitions, and explicitly by the sequential ordering among the transitions in a path [12]; the execution sequence does the same execution behavior in a total-order manner, whose ordering relationship is explicitly defined by the relation of global states. It should be noted that both notations exhibit the same "happen-before" relation of the transitions' execution [13]. In Ref. [1], we have shown that the behavior of all execution sequences within a RG can be equivalently represented by a set of concurrent paths and we can completely verify the protocol using this set.

The advantage of using the notion of concurrent path instead of that of execution sequence is three-fold: (1) all concurrent paths can be generated automatically and within low space complexity; (2) the system is separated into a set of concurrent paths, each of which is much smaller then the original system; and (3) the verification can be performed in parallel. The first advantage results from the representation of concurrent path, i.e. a set of module paths. The complete behavior of protocol is represented by the set of all concurrent paths, which are included by the Cartesian product of the sets of all module paths. The Cartesian product can be generated easily, but each member of the product is an arbitrary combination of module paths and obviously not all members are concurrent paths. Thus, we have to identify the concurrent paths from Cartesian product. Each member of Cartesian product is a simplified CFSM system and can be executed. If it is a concurrent path, its execution finally reaches the terminal global state or is blocked at an erroneous state that no transitions in the original system can release this blocking situation. Otherwise, it cannot be executed normally; its execution must be blocked at an intermediate global state that the execution of the complete system will not be blocked at and that is an anomaly of error. Thus, we can determine whether a member is a concurrent path via its execution within the complexity of a concurrent path rather than the whole system.

The second advantage is also obvious. The original system is now divided into a set of concurrent paths, each of which denotes partial behavior of, and is smaller than the original system. As stated before, each of them can be executed and thus verified independently using the algorithm of reachability analysis to enumerate the potential execution sequence(s). Then, these potential execution sequences can be checked for the required properties. As each concurrent path is smaller than the original system, the memory requirement of reachability analysis is also much smaller. Therefore, the state explosion problem is alleviated.

Furthermore, the conventional parallel verification algorithms such as in Ref. [14] rely on the message passing or shared memory mechanisms to build the image of the whole reachability graph. They occasionally have to exchange newly generated global states among the parallel computers to maintain the RG consistency. Thus, a great communication cost is required. Since each concurrent path is verified independently in our approach, it is naturally done in parallel. All the information to be exchanged is the verification



(a) Mismatch of Sending (b) Mismatch of Receiving (Cyclic Waiting) (c) Another Cyclic Waiting (d) Dependency Graph

Fig. 3. Unsuccessful execution examples.

result of each concurrent path and the communication cost is very low.

3. The symbolic method

3.1. General ideas

One of the principal problems of the path-based approach [1] concerns the time to check whether a concurrent path candidate is valid or not, and the bottleneck of this approach's performance depends on the checking method. In Ref. [1], we use reachability analysis upon candidates, but the analysis is time-consuming. Thus, in this section, we propose a more efficient candidate checking method rather than the reachability analysis.

Given a concurrent path candidate $\{p_i\}_{i=1}^{n}$, we can classify it into the following categories *valid* and *invalid*. A candidate is said to be *valid*, if there is at least one execution sequence corresponding to it; otherwise, it is *invalid*. The valid can be further classified as correct and erroneous. If its corresponding execution sequences contain errors (such as deadlocks, livelocks), it is erroneous; otherwise, it is correct. We can identify the category of a candidate as follows.

3.1.1. Valid (correct)

A candidate is correct means that all transitions must be executed successfully; that is, all messages are sent and received correctly, and the execution has no infinite loop so that the transitions after the loop cannot be executed. Thus, a candidate is correct if and only if the following three conditions are satisfied:

1. match of sending;
2. match of receiving; and
3. no infinite loop.

The first condition says that if a message is to be received successfully, it must be sent first; that is, the type of message and the order of sending and receiving must be matched. Thus, to check such a condition, we can separate the path p_i into a sending sequence a_{ij} and a receiving sequence b_{ij}, and check the equivalence of a_{ij} and b_{ij} for every i and j, where a_{ij}/b_{ij} is the sending/receiving message sequence of p_i with respect to p_j and is defined as follows.

Definition 12. Given a concurrent path candidate $\{p_i\}_{i=1}^{n}$ of the system P, for every path p_i in module m_i, the sequence of the sending/receiving messages is a_{ij}/b_{ij} with respect to module m_j, with $a_{ij} = \mu(p_i, j)$, $b_{ij} = \nu(p_i, j)$, and the projection function μ/ν, being defined as

$$\mu(p_i, j) = \begin{cases} (\lambda(t_i), \mu(p'_i, j)) & \text{if } \lambda(t_i) \in M_{ij} \\ \mu(p'_i, j) & \text{otherwise} \end{cases},$$

and

$$\nu(p_i, j) = \begin{cases} (\lambda(t_i), \nu(p'_i, j)) & \text{if } \lambda(t_i) \in M_{ji} \\ \nu(p'_i, j) & \text{otherwise} \end{cases}$$

where $p_i = \{t_i\}\cdot p'_i$ and p'_i is a subpath of p_i.

If a_{ij} and b_{ij} are not equal, there are unmatched messages that cannot be received successfully. One such example is shown in Fig. 3(a), where $a_{ij} =$ "*ab*" but $b_{ji} =$ "*ba*". In our model, a receiving action can only be executed if the required message is at the head of the channel. Thus, in Fig. 3(a), the message sequence that module p_j wants to receive is "*ba*,", but that sent by module p_i, the channel content is "*ab*" where "*a*" is at the head of the channel. Thus, the action in p_j to receive message "*b*", i.e. "$+b$" cannot be executed and p_j is blocked. In general, let a'_{ij} and b'_{ji} denote the longest prefixes of a_{ij} and b_{ji} that are equivalent. Then a'_{ij} and b'_{ji} contain the maximally possible transitions to be executed successfully, i.e. messages sent are received correctly. The transitions after b'_{ji} are impossible to be executed since the execution will be blocked at the first transition after b'_{ji} due to incorrect sending from a_{ij}. As for a'_{ij}, the transitions after a'_{ij} may be possibly executed.

The second condition reflects the fact that a message cannot be received successfully before its sending and it also cannot be received unless it is at the head of the communication channel. One such example is shown in Fig. 3(b). This is a typical case of cyclic waiting, i.e. the first transition of every path is waiting for the second transition in another path to send the required message. To examine such a situation, we can construct a directed graph of dependencies as follows.

Add a vertex denoting every transition in the candidate. Add an edge for each dependent relation, denoting that a transition has to be executed after another. [1] (All transitions in the same path must be executed sequentially; and a sending transition must be executed before its corresponding receiving transition.) Thus, if this graph has any cycle, there exists cyclic waiting. The dependency graph of Fig. 3(c) is shown in Fig. 3(d), and the heavy line shows there exists a cycle in the graph and it is the situation of receiving mismatch. As for the third condition, if there are infinite loops in the behavior of a concurrent path candidate, there is at least one path with a loop. (If the candidate with infinite loop behavior has only one path with a loop, the transitions in the loop must be sending transitions.[2]) To simplify the discussion, assume all p_i involve in the infinite loop behavior, and each p_i includes a loop, i.e. $p_i = p'_i{\cdot}(c_i)^*{\cdot}p''_i$, where p'_i, c_i, and p''_i are subpaths of p_i, and c_i is the loop corresponding to the cyclic behavior. Since it renders the infinite loop, the global states before entering and after leaving the loop must be the same; that is, $\{p'_i\}_{i=1}^n$ and $\{p'_i{\cdot}(c_i)^{k_i}\}_{i=1}^n$ (k_i denotes the corresponding number of loop cycles in the cyclic behavior[3]) will reach the same state. To examine such a situation, we have to identify the global states before and after entering the loop. To do so, we can treat $\{p'_i\}_{i=1}^n$ and $\{p'_i{\cdot}(c_i)^{k_i}\}_{i=1}^n$ as concurrent path candidates and use the technique to check the previous two conditions to determine whether they can execute the last transition and the last global state they reached. Therefore, if a candidate can pass the above three conditions, i.e. $a_{ij} = a'_{ij}$ and $b_{ij} = b'_{ij}$ for the first condition; there are no cycles in the dependency graph of a_{ij} and b_{ij}; and there are no infinite loops, it is a valid (correct) concurrent path candidate.

3.1.2. Valid (erroneous) or invalid

Since the candidate is erroneous or invalid, it must block at some global state due to disobeying the conditions mentioned above. However, as described in Section 2.2, we have to distinguish these two types of blocking by checking whether the blocking global state is an anomaly or not.

To do so, we have to identify the blocking global state. Using the above checking method, we can generate the dependency graph of a candidate and the cycle(s) in the graph. For each path, if there are transitions involved in the cycle and independent with any other transition(s) within the cycle and of the path, it is the blocking transition. If there is no such transition, the blocking transition is the receiving transition depending (directly or indirectly) on the blocking or non-executable (all transitions succeeding the blocking transitions are non-executable) transitions of the other path. We can identify the blocking global state via these blocking transitions. Then, if the blocking global state is also a blocking one within the original system, the candidate is erroneous; otherwise, it is invalid.

Therefore, in summary, if the concurrent path does not include any loop, we can identify the category of a concurrent path candidate by the following steps: first check the equivalence of a_{ij} and b_{ji}, construct the dependency graph of a'_{ij} and b'_{ji}, use a cycle detection algorithm [15] to determine the last reachable global state, and use this global state to identify its category. Otherwise, we have to check if the global state before and after the loop in the path is the same. If they are, this is a valid (infinite loop) concurrent path. The basic algorithm to verify a concurrent path is given below:

verify_concurrent_path$(p_1, p_2, \ldots, p_n)$
begin/* check all possible cycles in the concurrent path */

  for $i = 1$ **to** n

    if p_i has a loop and $p_i = p'_i{\cdot}(c_i)^*{\cdot}p''_i$, **then**
      $q_i = p'_i$
    else
      $q_i = p_i$

  entering_state = *simple_check*$(q_1, q_2, \ldots, q_n)$
      /* *simple_check* determines the last global sate using the method show above. It will be replaced by *simple_check*2 and *simple_check*3 method using the symbolic technique and thus omitted here.*/
  if $\{q_1, q_2, \ldots, q_n\}$ is in invalid/erroneous candidate

    report $\{p_1, p_2, \ldots, p_n\}$ as invalid/erroneous candidate
    break

  K_set = empty set
  Repeat

[1] Since dependent relations are transitive, they can be classified as direct or indirect. For any dependent relation, say t_i depends on t_j, if there exists a transition t_k such that t_i depends on t_j, this dependent relation is indirect; otherwise, it is direct. In our dependency graph, only the direct dependent relations are included.

[2] If only one path has a loop and any receiving transitions is in the loop, the loop is impossible to be executed infinitely because the corresponding module has no loop and can provide only finite number of messages for that receiving transition. After all messages are consumed, the loop stops at the receiving transition.

[3] The value of k_js can be any value that is smaller than the threshold of the module m_i. This threshold denotes the maximum possible number of loop times in that module. It can be determined by the protocol designer or follows the recommendation of [1], i.e. only one time of loop is enough to cover most cyclic behavior. It should also be noted that it is unnecessary that the values of all k_is are the same.

```
for i = 1 to n
    if p_i has a loop then
        Let p_i = p'_i·(c_i)*·p''_i
        for j = 1 to max_k /* max_k denote the maximal possible number of loops
                              and is specified by users */
            Let k_i = j
            if the combination (k_1, k_2, …, k_n) is not in the K_set then
                Add (k_1, k_2, …, k_n) into K_set
                Let q_i = p'_i·c_i^{k_i}
                Let q'_i = q_i·p''_i
                break
    else
        q_i = p_i
        q'_i = q_i
    /* Obtain the global states after the cycle to determine the errors of infinite loop */
    leaving_state = simple_check(q_1, q_2, …, q_n)
    if {q_1, q_2, …, q_n} is in invalid/erroneous candidate
        report {p_1, p_2, …, p_n} as invalid/erroneous candidate
        continue
    if entering_state = leaving_state then
        report {p_1, p_2, …, p_n} as valid(infinite_loop)
    /* Check the complete concurrent path */
    simple_check(q'_1, q'_2, …, q'_n)
    if {q'_1, q'_2, …, q'_n} is in invalid/erroneous candidate
        report {p_1, p_2, …, p_n} as invalid/erroneous candidate
    continue
  until (k_1, k_2, …, k_n) = (max_k, max_k, …max_k)
end cycle_check
```

3.2. Verification of two modules

Although the approach mentioned above is more efficient than the approach using reachability analysis in Ref. [1], the algorithm to detect a cycle in a directed graph is still time-consuming. To make further improvement, we provide a more efficient checking method to determine the last reachable global state. To ease the discussion, we first present the verification of only two modules in this subsection and show that of more than two in next subsection. We first assume the concurrent path candidate to be checked is $\{p_1, p_2\}$ in the case of two modules only; and let $a_{12} = \mu(p_1, 2)$, $a_{21} = \mu(p_2, 1)$, $b_{12} = \nu(p_1, 2)$, and $b_{21} = \nu(p_2, 1)$.

As described above, there are two reasons of blocking: mismatches of sending and receiving, and the former can be checked by the equivalence of a_{12} and b_{21}; a_{21} and b_{12}. Such check is quite easy (by the Boolean exclusive or operation) and we can use it as the first check to determine the maximal possible reached transitions. Let a'_{12}, ,b'_{21}, a'_{21} and b'_{12} be the maximal equivalent prefixes of a_{12}, b_{21}, a_{21} and b_{12}, respectively. The transitions behind b'_{21} and b'_{12} are impossible to be executed due to the incorrect order of sending.

Then, with respect to b'_{21} and b'_{12}, we have to check whether the concurrent path will be blocked due the incorrect order of receiving. Instead of using the dependency graph, we can identify the blocking due to incorrect receiving order with the help of the sequence of sending or receiving, denoted as *i/o* sequence of p_i.

Definition 13. The i/o sequence io_{ij} of a path p_i of module m_i with respect to module m_j is defined by the function o, i.e. $io_{ij} = o(p_i, j)$, with $o(p_i, j)$ being defined as

$$o(p_i, j) = \begin{cases} (+, o(p'_i, j)) & \text{if } t_i \text{ is a receiving transition w.r.t module } m_j \\ (-, o(p'_i, j)) & \text{if } t_i \text{ is a receiving transition w.r.t module } m_j, \\ o(p'_i, j) & \text{otherwise} \end{cases}$$

where $p_i = \{t_i\}\cdot p'_i$ and t_i is a transition and p'_i a subpath of p_i.

With these two i/o sequences $io_{12} = o(\bar{p}_1, 2)$ and $io_{21} = o(\bar{p}_2, 1)$ ($\bar{p}_1/\bar{p}_2$are the subpaths of p_1/p_2 corresponding to b'_{12} and b'_{21}), we can use two counters q_1 and q_2 to denote the number of messages in the channel to determine whether a transition can be executed. Since the order of sending and receiving are matched with respect to $\bar{p}_1$ and $\bar{p}_2$, a message sent to the channel will be received successfully. If the value of q_1 or q_2 is zero, it means the channel is currently empty and no receiving is possible. When both counters have the values of zeros and the next transitions to be executed are both receiving transitions, then a blocking situation occurs and the last reachable global state can be identified accordingly.

We can perform such check using the algorithm below: At first q_1 and q_2 are reset to zeros and io_{12} and io_{21} are examined sequentially. When the inspection reaches a "−" in io_{12} or io_{21}, q_1 or q_2 are, respectively, increased by one; when a "+" is regarded in io_{12} or io_{21},q_2 or q_1 are, respectively, decreased by one provided the value of q_2 or q_1 does not become negative. If it would become negative, check the other sequence to see whether it can be further advanced to next transition. If it cannot (due to the fact that its counter would become negative as well), it means that both are blocked at the receiving transitions that will make its counter negative. This algorithm is formally described as follows:

*simple_check*2(p_1, p_2)

$a_{ij} =$
the sending message list of p_i with respect to p_j
$b_{ij} =$
the receiving message list of $p_i$with respect to p_j
/* The following two statements compute sending match */
xor_1 = The position of the element in p_1 corresponding to the first occurrence of non-zero value in $a_{21} \oplus b_{12}$
/* Compute the first non-identical element in a_{21} and b_{12} */
xor_2 = The position of the element in p_2 corresponding to the first occurrence of non−zero value in $a_{12} \oplus b_{21}$
/* Compute the first non-identical element in a_{12} and b_{21} */
$io_1 = o(\bar{p}_1, 2)$, $\bar{p}_1$ the prefix of p_1 with the length of xor_1
$io_2 = o(\bar{p}_1, 1)$, $\bar{p}_2$ the prefix of p_2 with the length of xor_2
$q_1 = 0$; /* The number of messages in the channel from p_1 to p_2 */
$q_2 = 0$; /* The number of messages in the channel from p_2 to p_1 */
$x_1 = 0$; /* Next transition in p_1 to be inspected */
$x_2 = 0$; /* Next transition in p_2 to be inspected */
/* compute the receiving match */
Repeat
  progress = False
  if ($io_1[x_1]$ = "−")
  /* current transition is sending transition */
    x_1^{++}
    q_1^{++}
    progress = True
  else if $q_2 > 0$ **then**
  /* current transition is receiving transition and the message channel is not empty */
    x_1^{++}
    q_{2--}
    progress = True
  if ($io_2[x_2]$ = "−")
  /* current transition is sending transition */
    x_2^{++}
    q_2^{++}
    progress = True
  else if $q_1 > 0$ **then**
  /* current transition is receiving transition and the message channel is not empty */
    x_2^{++}
    q_{1--}
    progress = True
until progress = False /* no more transition to be executed */
/* identify the last global state */
for $i = 1$ to 2
    $s_i = \alpha$ (the x_ith transition in p_i)
for $i = 1$ to 2
  for $j = 1$ to 2
    $c_{ij} = revert(a_{ji} - b_{ij})$ /* *revert* converts the string to another in the reverse order */
    /* "−" is the quotient operator on strings and "$a_{ji} - b_{ij}$" returns a substring whose element belongs to a_{ji} but not b_{ij}. */
return the global state $(s_1, s_2, c_{11}, c_{12}, c_{21}, c_{22})$
end simple_check2

3.3. Verification of more than two

In Section 3.2, we explained how to use simple Boolean and arithmetic operations to compute the blocking transitions and global states in the case of two modules. The similar concept can be applied to the case of more than two modules using the sending, receiving and i/o sequences of paths. However, we have to extend the variables used in the algorithm *simple_check*2 as follows:

The main extension concerns the i/o sequence, which now, for the case of p_i, becomes a set of i/o sequences, each of which, say io_{ij} corresponds to communication between two modules m_i and m_j. Then, $io_{ij} = o(\bar{p}_i, j)$, where $\bar{p}_i$ is a prefix of p_i, corresponds to the shortest b'_{ji} for all j, and a'_{ij} and b'_{ji} denote the longest prefixes which are common in a_{ij} and b_{ji}. Since the i/o sequence of a path is now separated into a set of i/o sequences, we cannot know the original position of each sending and receiving action in the path, and we have to record the original position in the path p_i for each i/o action. Thus, the i/o sequence for the concurrent path candidate becomes $(act_i^{pos_i})_{i=1}^{l}$, where act_i is "−" or "+" denoting a sending or receiving transition, pos_i denotes the original position of this action in the path, and l is the number of elements in the sequence. For example, the i/o sequences for the paths in Fig. 4 are $io_{12} = (-^1, +^3)$ and $io_{13} = (+^2, -^4)$ for p_1, $io_{21} = (+^1, -^2)$ and $io_{23} = (-^3, +^4)$ for p_2, and $io_{31} = (-^2, +^3)$ and $io_{32} = (+^1, -^4)$ for p_3.

In addition, since there are different communication channels between a module and its neighbors, each channel between module m_i and m_j corresponds to a counter q_{ij} to record the number of messages in the channel. Two types of additional counter are also required, the counter x_i of path p_i to identify the next transition to be inspected and e_{ij} of the i/o sequence io_{ij} to record the next element to be inspected.

Then for each action in the i/o sequence, if its original position is identical to the counter q_i, it means this action should be executed at this moment. If it is a sending action; or receiving action and the corresponding channel counter is larger than zero, it can be executed and the related counters

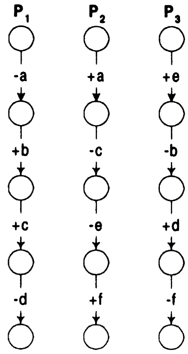


Fig. 4. Non-blocking example of concurrent path candidate.

are updated accordingly. If the result of the last step shows the path counter exceeds the length of the path, all transitions in this candidate can be executed; otherwise, there is a blocking situation. We show inspection steps of a non-blocking (as shown in Fig. 4) and blocking examples (as shown in Fig. 3(c)) in Fig. 5(a) and (b), respectively.

In the example of Fig. 5(a), the result of the last step (step 13) showing that the path counter exceeds the length of the path and the fact that every channel is zero, identify this candidate as correct. In that of Fig. 5(b), if there are transitions that cannot be executed, the execution blocks at these transitions. The detailed algorithm is as follows:

*simple-check*3$(p_1, p_2, \ldots p_n)$
{

a_{ij} = the sending message list of p_i with respect to p_j
b_{ij} = the receiving message list of p_i with respect to p_j
xor_{ij} = The position of the element in p_i corresponding to the first occurrence of non-zero value in $a_{ji} \oplus b_{ij}$

/* compute the sending match */
$y_i = \min(\prod_{j=1.n} xor_{ij})$
io_{ij} = *i/o* sequence of p_i with respect to p_j before the (y_j+1)th transition

/* $io_{ij}(i) \cdot p$ = the *i*th message is the *p*th transition in the original path */
/* $io_{ij}(i) \cdot x$ = *"−" or "+" denoting a sending or receiving action* */
$e_{ij} = 0$ /* the current position in io_{ij} */
$q_{ij} = 0$ /* the message number of the channel $p_j \rightarrow p_i$ */
$x_i = 0$ /* the next transition to be executed in p_i */
progress = True /* flag to exit the check */
/* compute the receiving match */
Repeat

Step \ Path / Channel Counters	p_1	p_2	p_3
1	$z_1 = (0, 0)^1 \Rightarrow -^1$	$z_2 = (0, 0)^1$	$z_3 = (0, 0)^1$
2	$z_1 = (0, 0)^2$	$z_2 = (1, 0)^1 \Rightarrow +^1$	$z_3 = (0, 0)^1$
3	$z_1 = (0, 0)^2$	$z_2 = (0, 0)^2 \Rightarrow -^2$	$z_3 = (0, 0)^1$
4	$z_1 = (1, 0)^2$	$z_2 = (0, 0)^3 \Rightarrow -^3$	$z_3 = (0, 0)^1$
5	$z_1 = (1, 0)^2$	$z_2 = (0, 0)^4$	$z_3 = (0, 1)^1 \Rightarrow +^1$
6	$z_1 = (1, 0)^2$	$z_2 = (0, 0)^4$	$z_3 = (0, 0)^2 \Rightarrow -^2$
7	$z_1 = (1, 1)^2 \Rightarrow +^2$	$z_2 = (0, 0)^4$	$z_3 = (0, 0)^3$
8	$z_1 = (1, 0)^3 \Rightarrow +^3$	$z_2 = (0, 0)^4$	$z_3 = (0, 0)^3$
9	$z_1 = (0, 0)^4 \Rightarrow -^4$	$z_2 = (0, 0)^4$	$z_3 = (0, 0)^3$
10	$z_1 = (0, 0)^5$	$z_2 = (0, 0)^4$	$z_3 = (1, 0)^3 \Rightarrow +^3$
11	$z_1 = (0, 0)^5$	$z_2 = (0, 0)^4$	$z_3 = (0, 0)^4 \Rightarrow -^4$
12	$z_1 = (0, 0)^5$	$z_2 = (0, 1)^4 \Rightarrow +^5$	$z_3 = (0, 0)^5$
13	$z_1 = (0, 0)^5$	$z_2 = (0, 0)^5$	$z_3 = (0, 0)^5$

Note:
$z_1 = (q_{12}, q_{13})^{x_1}$ $z_2 = (q_{21}, q_{23})^{x_2}$ $z_3 = (q_{31}, q_{32})^{x_3}$
$z_i \Rightarrow act^{Pos}$ means the transition corresponding to the act^{Pos} in i/o sequence is executed.

(a) I/O Sequence Inspection of the Example in Figure 4

Step \ Path / Channel Counters	p_1	p_2	p_3
1	$z_1 = (0, 0)^1 \Rightarrow -^1$	$z_2 = (0, 0)^1$	$z_3 = (0, 0)^1$
2	$z_1 = (0, 0)^2$	$z_2 = (1, 0)^1 \Rightarrow +^1$	$z_3 = (0, 0)^1$
3	$z_1 = (0, 0)^2 \not\Rightarrow +^2$	$z_2 = (0, 0)^2 \not\Rightarrow +^2$	$z_3 = (0, 0)^1 \not\Rightarrow +^1$

Note:
$io_{12}=(-^1, +^3)$, $io_{21}=(+^1, -^2)$, $io_{31}=(-^2, +^3)$, $io_{13}=(+^2, -^4)$, $io_{23}=(-^3, +^4)$ and $io_{32}=(+^1, -^4)$
$z_i \not\Rightarrow act^{Pos}$ means the transition corresponding to act^{pos} cannot executed.

(b) I/O Sequence Inspection of the Example in Figure 3 (c)

Fig. 5. Inspection by i/o sequences.

For $i = 1$ to n

For $j = 1$ to n

$progress$ = false
if $io_{ij}(e_{ij}) \cdot p = x_i$ **then**
/* the corresponding transition of $io_{ij}(e_{ij})$ should be executed */

if $io_{ij}(e_{ij}) \cdot x =$ "−" **then** /* a sending transition */

e_{ij}^{++} /* increase the counter of io_{ij} */
q_{ji}^{++} /* increase no. of msg in channel $p_i \rightarrow p_j$ */
x_i^{++} /* increase the counter of p_i */
$progress$ = true
else /* $io_{ij}(e_{ij}) \cdot x =$ "+" a receiving transition */

if $x_i(j) > 0$ **then**

e_{ij}^{++} /* increase the counter of io_{ij} */
q_{ij}^{--} /* decrease no. of msg in channel $p_j \rightarrow p_i$ */
x_i^{++} /* increase the counter of p_i */

$progress$ = true
Until $progress$ = False /* no more transition to be executed */
/* identify the last global state */
for $i = 1$ to n
$s_i = \alpha$(the x_ith transition in p_i)
for $i = 1$ to n

for $j = 1$ to n

$c_{ij} = revert(a_{ji} - b_{ij})$/* *revert* converts the string to another in the reverse order */

return the global state $(s_1, s_2, \ldots, s_n, c_{11}, c_{12}, \ldots, c_{nn})$

end simple_check3

4. Conclusion

The "state explosion problem" in protocol verification raises two issues: large memory requirement and long verification time. For the former issue, we have proposed the path-based approach to separate the protocols into a set of concurrent paths. Each one can be generated and verified independently of the others [1]. Thus, the memory to store reachable global states depends on the complexity of a concurrent path rather than the whole protocol, and the memory space issue is alleviated.

As for the later issue, in this paper, we show a performance improvement technique of the path-based approach to compute the last reachable global state of every concurrent path candidate more efficiently. Both algorithms (simple_check2 and simple_check3) linearly check every transitions in the paths with the time complexity of $O(n^2 * l)$[4], where n is the number of modules and l is the average of a path. For each iteration of check, only the exclusive-or, increment, and decrement operations are required. Therefore, the long verification time issue is also alleviated.

Although the checking method shown in this paper is efficient in identifying the concurrent paths from the Cartesian product of the module paths, the main limitation results from the model. Since the channel between two modules is independent of the others in the underlying CFSM model, the i/o sequences of paths can thus be inspected independently of the others. There is the situation that the incoming channels from different modules share a common queue [16]. In this case, the receiving of a message does not only depend on the match of sending and receiving of the corresponding module, but also the execution speeds of all the modules that may send messages at this moment. When a receiving transition is waiting to receive a message from a module but another module executes faster and sends a message in advance, it occupies the head of the queue, and this transition cannot be executed successfully. Thus, our algorithm to determine whether a message can be received must check every possible combination in the contents of common queue resulting from different execution speeds of modules.

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[4] To check every transitions, we need $n * l$ iterations. However, in each iteration, additional iteration is necessary to determine which transitions can be executed among n modules and the average value in $n/2$. Thus, the number of iterations in average is $n^2 * l/2$.

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