Magnetic monopole in induced Einstein-Yang-Mills models

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(Received 22 July 1999; published 24 January 2000)

We show that a regular SO(3) non-Abelian monopole cannot exist in induced gravity models with a variety of symmetry breaking potentials. The no-hair theorem is also shown to be true in these models. We also analyze the global behavior of the gauge field in the presence of a black hole with a nonvanishing cosmological constant. It is shown that the nontrivial non-Abelian monopole solution exists only if the radius of the event horizon is smaller than the characteristic radius of the classical monopole.

PACS number(s): 04.70.Bw, 11.15.Ex, 14.80.Hv

I. INTRODUCTION

The classical behavior of a non-Abelian monopole in a static and spherically symmetric curved space [1–4] has been the focus of many research activities. It is known that the U(1) gauge theory admits a singular magnetic monopole solution [5]. 't Hooft and Polyakov show that a regular spherically symmetric monopole solution exists in a non-Abelian SO(3) gauge theory with a spontaneously symmetry breaking (SSB) Higgs potential [6]. It was shown that the magnetic monopole configuration exists only when the associated second homotopy group is nontrivial. In other words, one needs $\Pi_2(\mathcal{M})\neq 0$ in order for a system to have a nontrivial magnetic monopole solution. Here $\mathcal{M}\equiv\mathcal{G}/\mathcal{H}$ denotes the Higgs vacuum configuration associated with the symmetry breaking process $\mathcal{G}\rightarrow\mathcal{H}$ [7]. The quest for the magnetic monopole has since been an interesting research topic in many different contexts [8,9].

Note that a regular monopole solution has also been found in curved space-time [1–3,10]. In particular, the physical behavior of a magnetic monopole in the presence of a black hole event horizon has also been the focus of research interest [1–4]. On the other hand, the well-known no-hair theorem is a long-standing conjecture in the research of black hole physics [11]. Evidence indicates that only three kinds of physical quantities—the electric charge Q, the gravitational mass M, and the angular momentum J—can be detected outside the event horizon of a black hole [12]. Hence a model independent way of checking the validity of the no-hair theorem is also very important.

In addition, the induced gravity model [13] has also been shown to be related to various fields of interest [14] including its applications in inflationary universe [15]. Note that the gravitational *constant* and cosmological *constant* are proposed to be dynamical variables in the induced gravity model [16]. In fact, it was shown that the behavior of the Higgs scalar field in induced gravity affects the properties of the classical black hole [2,4,17]. The problem of the existence and stability of the magnetic monopole solution has also been studied in induced Einstein-Yang-Mills-Higgs (EYMH) models [4,17].

We will present a more complete analysis of the problem of the existence and global properties of non-Abelian monopoles in an SO(3) induced Einstein-Yang-Mills (EYM) model. We will generalize our result to models with a variety of SSB potentials. In particular, we will show that a regular spherically symmetric monopole solution does not exist in these models. We will also show that the only way to accommodate a regular magnetic monopole solution is to have a magnetic monopole charged black hole. The no-hair theorem will be shown to be valid with the scalar field in these models. In addition, the black hole solution with nontrivial gauge field imposes a number of constraints on the coupling constants. In particular one will show that the radius of the event horizon is smaller than the characteristic radius of the classical monopole. These constraints are listed in this paper and should be helpful for numerical studies.

This paper will be organized as follows: (i) In Sec. II, we will present the field equations of the induced model; (ii) In Sec. III, we will show that a regular spherically symmetric monopole solution cannot exist in the presence of a variety of SSB potentials; (iii) the no-hair theorem and the global behavior of the magnetic monopole charged black hole solution will be studied in Sec. IV; and (iv) finally, we will discuss our results in Sec. V.

II. INDUCED EINSTEIN-YANG-MILLS MODELS

The induced Einstein-Yang-Mills model with a real SO(3) triplet scalar field Φ^a is given by the following action:

$$S=\int d^4x\sqrt{g}\bigg[-\frac{1}{2}\epsilon\Phi^2R-\frac{1}{2}D_\mu\Phi^aD^\mu\Phi^a -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu}-V(\Phi^2)\bigg], \tag{2.1}$$

where R is the scalar curvature and ϵ denotes a dimensionless coupling constant. $a,b,\ldots=1,2,3$ will denote the SO(3) gauge indices. Note that the gauge covariant derivative $D_\mu\Phi^a$ and the field tensor $F^a_{\mu\nu}$ are defined as $D_\mu\Phi^a=\partial_\mu\Phi^a+e\epsilon_{abc}A^b_\mu\Phi^c$ and $F^a_{\mu\nu}=\partial_\mu A^a_\nu-\partial_\nu A^a_\mu+e\epsilon_{abc}A^b_\mu A^c_\nu$, respectively. Here e denotes the dimensionless gauge coupling constant. It is known that the action (2.1) is invariant under a global scale transformation if $V\sim\Phi^4$.

We will focus on the applications of the above induced Einstein-Yang-Mills model coupled with a variety of spontaneously symmetry breaking potentials. In particular, our analysis will be done assuming a static and spherically symmetric pseudo-Riemannian metric given by

$$ds^2 = -B^2(r)C(r)dt^2 + \frac{dr^2}{C(r)} + r^2(d\theta^2 + \sin^2\theta\phi^2), \tag{2.2}$$

in addition to the spherically symmetric 't Hooft–Polyakov magnetic monopole ansatz:

$$\Phi^a = \phi(r)\hat{r}^a, \tag{2.3}$$

$$A_i^a = \epsilon_{aij}\frac{1-w(r)}{er}\hat{r}^j, \tag{2.4}$$

$$A_0^a = 0. \tag{2.5}$$

Note that we will assume that ϕ is positive for simplicity. The result is invariant under the discrete transformation $\phi \to -\phi$ due to the symmetry of the action.

Note that the ansatz for gauge fields is written in Cartesian coordinates. The equations of motion can be shown to be

$$C' = \frac{1}{r}(1-C) - r\rho, \tag{2.6}$$

$$B' = \frac{rB}{2C}(\rho - \tau), \tag{2.7}$$

$$(BCw')' + Bw\left[\frac{1-w^2}{r^2} - e^2\phi^2\right] = 0, \tag{2.8}$$

$$(BCr^2\phi')' + \frac{BCr^2\phi'^2}{\phi} = \frac{Br^2}{(1+6\epsilon)}\left[\partial_\phi V - 4\frac{V}{\phi}\right]. \tag{2.9}$$

Here a prime always denotes differentiation with respect to the argument throughout this paper. In addition, the generalized energy density ρ is defined by

$$\rho = C\left\{\frac{2}{\phi}\left[\phi'' + \left(\frac{C'}{2C} + \frac{2}{r}\right)\phi'\right] + \frac{1}{\phi^2}\left[\left(2 + \frac{1}{2\epsilon}\right)\phi'^2 + \frac{1}{\epsilon}\frac{w'^2}{e^2r^2}\right]\right\} + \frac{1}{\epsilon}\left[\frac{w^2}{r^2} + \frac{1}{\phi^2}\left(\frac{(1-w^2)^2}{2e^2r^4} + V(\phi^2)\right)\right], \tag{2.10}$$

while the generalized radial pressure τ is defined as

$$\tau = C\left[\frac{2\phi'}{\phi}\left(\frac{B'}{B} + \frac{C'}{2C} + \frac{2}{r}\right) - \frac{1}{\epsilon\phi^2}\left(\frac{\phi'^2}{2} + \frac{w'^2}{e^2r^2}\right)\right] + \frac{1}{\epsilon}\left[\frac{w^2}{r^2} + \frac{1}{\phi^2}\left(\frac{(1-w^2)^2}{2e^2r^4} + V(\phi^2)\right)\right]. \tag{2.11}$$

It is known that the trace equation for an induced gravity model will result in a scalar equation (2.9) independent of the gauge field. This equation will be useful in our attempt to show that there is no regular monopole solution and no scalar hair for a variety of scalar potentials.

III. ONLY SINGULAR MONOPOLE FOR A VARIETY OF SCALAR POTENTIALS

The most general boundary conditions of the field variables at the origin $r=0$ are known to be $C(0)=1$, $w(0)=1$, $\phi(0)=0$, $B(0)<\infty$. Therefore, one can expand them as $\phi(r)=\phi_0+k_\phi r+\cdots$, $w(r)=1-k_w r^2+\cdots$, $C(r)=1-k_C r^m+\cdots$, and $B(r)=B_0-k_B r^n+\cdots$ near the origin. Here k_ϕ, k_w, k_C, k_B, and $B_0\equiv B(0)$ are expansion coefficients subjected to constraints from the field equations. Moreover, one can show that $m,n\geq 2$ are both positive integers.

On the other hand, the asymptotic flatness and regularity of the field variables implies that $C(\infty)=B(\infty)=1$, $(\infty)=0$, and $\phi(\infty)=v$ at spatial infinity.

Starting from now, we will be working on a class of potentials such that

$$(\phi\partial_\phi V - 4V)(\phi - v) > 0 \tag{3.1}$$

for all $\phi\neq v$. Here v denotes a constant parameter. We will call this kind of potential a scaling potential in this paper. If fact,the above inequality will imply that $(\phi\partial_\phi V-4V)=0$ when $\phi=v$. This indicates that v is in fact the local minimum of any scaling potential. Note that this inequality in fact tells us that the effective dimension $d_{\text{eff}}\equiv\phi\partial_\phi V/V$ is greater than 4 if $\phi>v$ and vice versa.

For example, one can show that the potentials of the following kinds are scaling potentials: (i) $V=\lambda(\phi^2-v^2)^{2n}$ for $n\geq 1$ and (ii) the renormalized effective potential given by $V=\lambda\phi^4\ln[\phi^4/v^4]-\lambda(\phi^4-v^4)$.

In order to show the nonexistence theorem, one needs to show the following proposition.

Proposition I. ϕ is a monotonically increasing (decreasing) function if $\phi<v$ ($\phi>v$) for any scaling potential.

Proposition I can be shown by noting that ϕ has no maximum (minimum) for $\phi>v$ ($\phi<v$). Indeed, Eq. (2.9) shows that

$$\phi'' = \frac{1}{(1+6\epsilon)C\phi}[\phi\partial_\phi V - 4V] \tag{3.2}$$

at any local extremum of ϕ. Let r_0 be the point of the local minimum such that $\phi'(r_0)=0$. It follows that $\phi''>0$ ($\phi''<0$) at r_0 if $\phi>v$ ($\phi<v$). This means that ϕ is a local minimum (maximum) if $\phi>v$ ($\phi<v$) at r_0. Hence proposition I is proved. It in fact tells us the following.

Corollary I. Φ is monotonically approaching v as r increases independent of its initial data.

Note that one can show that

$$\frac{1}{1+6\epsilon}\int_0^\infty dr Br^2[\phi\partial_\phi V - 4V] = BCr^2\phi'\phi|_{r=\infty} \tag{3.3}$$

from Eq. (2.9). Note that the boundary term $BCr^2\phi'\phi|_{r=0}=0$ since $\phi(0)=0$. Equation (3.3) implies that the left-hand side of Eq. (3.3) is positive (negative) definite if $\phi>v$ ($\phi<v$). On the other hand, corollary I says that $\phi'<0$ ($\phi'>0$) everywhere; hence the right-hand side of Eq. (3.3) is negative (positive) definite. This leads to a contradiction unless $\phi=v$. Note that the regularity of Φ demands that $\phi_0=0$. Therefore, a regular 't Hooft-Polyakov monopole does not exist in this induced EYM model coupled to any scaling potential. Note that the singular $\phi=v$ monopole solution has been studied in Ref. [17]. Our analysis shows that the $\phi=v$ solution is the only possible regular spherically symmetric monopole in these induced models. Hence the only possible way to have a regular monopole solution is to hope that the singularity is hidden inside the event horizon of a charged black hole in these induced gravity models.

IV. NO-HAIR THEOREM AND THE MONOPOLE CHARGED BLACK HOLES

We will assume that a black hole is present with r_H denoting the radius of the event horizon. Note that the boundary conditions at the event horizon are $C(r_H)=0$ and $C'(r_H)\geqslant 0$ while the functions ϕ, ϕ', w, w', and B are finite [18] at the horizon. In addition, the asymptotic conditions are the same as given in Sec. II.

The no-hair theorem of the scalar field can be shown by noting that

$$\int_{r_H}^{\infty} dr B r^2\left[\frac{vC\phi'^2}{\phi}+\frac{1}{(1+6\epsilon)\phi}[\phi\partial_\phi V-4V](\phi-v)\right] = BCr^2\phi'(\phi-v)\Big|_{r_H}^{\infty}. \tag{4.1}$$

This is derived from multiplying Eq. (2.9) by $(\phi-v)$. The right-hand side of Eq. (4.1) vanishes at both boundaries because (i) $C(r_H)=0$ at the lower bound r_H and (ii) $\phi(\infty)\to v$. Therefore one must have $\phi(r)=v$ for all $r>r_H$ due to the fact that the integrand on the left hand side of Eq. (4.1) is positive. Hence the no-hair theorem still holds for the scalar field in these induced gravity models.

Note also that one can show that a regular monopole does not exist if $V=(\lambda/4)\phi^4$. The scalar hair can also be shown to be absent in this scale invariant model. As a result, one is left with a cosmological constant proportional to $V_0=\lambda v^4/4$ if $\phi=v$. Note that v is no longer the minimum of V in this case.

Indeed, one can show that Eq. (2.9) can be integrated to give

$$BCr^2\phi\phi'=k_0, \tag{4.2}$$

with k_0 denoting a constant. Assuming that there is a regular monopole in this model, $\phi(0)=0$ shows that $k_0=0$. The only way to have $BCr^2\phi\phi'=0$ everywhere is that $\phi'=0$ everywhere. This says that $\phi=0$ for all r. This is not acceptable because $\phi=0$ everywhere will introduce an infinite gravitational constant everywhere. In addition, the vanishing scalar field sets the curvature scalar term decoupled from the system. The only solution of w in the system of equations is $w=1$. Therefore, one shows that a scale invariant action will not come along with the regular monopole solution. Note that the requirement $\phi(0)=0$ is not too bad because a vanishing scalar curvature at the origin can make up for this problem.

Therefore, we turn our attention to the charged black hole solution again. Equation (4.2) implies that $k_0=0$ because $C(r_H)=0$ at any black hole event horizon. Therefore one has $\phi'=0$ again and hence $\phi=v$ as promised. The only difference with the regular monopole solution without a black hole is that v need not be 0 anymore. Hence one is left with a cosmological constant coupled with the system of equations in this ϕ^4 theory.

Having solved the scalar equation, one still has

$$\int_{r_H}^{\infty} drB\left[Cw'^2+\frac{w^2}{r^2}(e^2v^2r^2+w^2-1)\right]=BCw'w\Big|_{r_H}^{\infty} \tag{4.3}$$

from Eq. (2.8). A similar argument shows that the right-hand side of Eq. (4.3) vanishes. The left-hand side implies that $w(r)=0$ for all $r>r_H$ provided that r_H is not less than the characteristic radius $1/(ev)$ of the classical monopole. Therefore $r_H<1/(ev)$ is the necessary condition for the existence of a nontrivial monopole black hole solution. Note that this kind of nontrivial charged black hole is in fact a black hole in a monopole [2]. This is possible because that any possible singularity near the origin is hidden inside the event horizon of a charged black hole. One notes that the above result has nothing to do with the form of scalar potential because Eq. (2.8) is independent of the potential V. In fact, only Eq. (2.6) depends on V explicitly.

Note that the field equations reduce to

$$C'=\frac{1}{x}(1-C)-\frac{1}{\epsilon x}\left[Cw'^2+\frac{(1-w^2)^2}{2x^2}+w^2+\frac{x^2V_0}{e^2v^4}\right], \tag{4.4}$$

$$B'=\frac{Bw'^2}{\epsilon x}, \tag{4.5}$$

$$(BCw')'=\frac{Bw}{x^2}(w^2+x^2-1), \tag{4.6}$$

if $\phi=v$. Here we have written $x=evr$, $C=C(x)$, $B=B(x)$, and $w=w(x)$ for simplicity. Note that a cosmological constant term is added for completeness of the analysis even though we know that $V_0\equiv V(v)=0$ for any scaling potential. In other words, the analysis shown from now on will remain valid in the presence of a cosmological constant term if the scalar hair can be shown to be absent by other methods. For example, the ϕ^4 potential has been shown to leave us a nonvanishing cosmological constant.

Note that the solution with $w=0$ is the Reissner-Nordström black hole solution given by $B=1$ and $C=1-m/4\pi\epsilon v^2 r+1/2\epsilon v^2e^2r^2$. Here m is the Arnowitt-Deser-Misner (ADM) mass of the black hole. In addition, one needs the constraint $m\geqslant 4\pi v\sqrt{2\epsilon}/e$ in order to prevent exposure of the naked singularity.

Note that one needs $x_H<1$ in order to have a black hole solution with nontrivial w. Hence we will assume that $x_H<1$ from now on and try to analyze various properties of this kind of charged black hole. One notes that Eqs. (4.4)–(4.6) are similar to those of theories with a minimally coupled SU(2) gauge field. It has been studied in the limit of an infinite Higgs self-coupling constant by Aichelburg and Bizon [4].

Two propositions [4] governing the properties of w will be in need.

Proposition II. w is a monotonically decreasing (increasing) function of x if $w \geqslant 1$ ($w \leqslant -1$).

We will assume $w_H > 0$ for simplicity. Here we have defined $w_H \equiv w(x_H)$. Note that the result will remain valid with $w \to -w$ because the field equations are invariant under this discrete transformation. Note that Eq. (4.6) becomes

$$Cw'' = \frac{w_0}{x_0^2}(w_0^2 + x_0^2 - 1) \tag{4.7}$$

at local extrema where $w'(x_0) = 0$. Here we have written $w_0 \equiv w(x_0)$. If $w \geqslant 1$, one has $w'' > 0$ at the local extrema for all $x_0 > 0$. Hence w_0 is a local minimum if $w_0 \geqslant 1$. Similarly, w_0 is a local maximum if $w_0 \leqslant -1$. Hence any physical solution of w has to be a monotonically decreasing (increasing) function in x as long as $w \geqslant 1$ ($w \leqslant -1$). This proves proposition II. Therefore the boundary condition $w(\infty) \to 0$ implies that there is no local extremum for w in the region $|w| \geqslant 1$. Otherwise, proposition II tells us that w can never turn around and approach 0 at spatial infinity.

Proposition III. w monotonically approaches 0 for all $x \geqslant 1$.

Note that $w_0 w'' \geqslant 0$ at the extrema of w if $x_0 \geqslant 1$. Hence local extrema of w are local minima (maxima) if $w > 0$ ($w < 0$). Hence proposition III is proved because $w(\infty) \to 0$.

On the other hand, Eq. (4.6) reads

$$C'w' = \frac{w_H}{x_H^2}(w_H^2 + x_H^2 - 1) \tag{4.8}$$

at the event horizon of a black hole where $C(x_H) = 0$. Note that $C'(x_H) \geqslant 0$ since C is assumed to be positive definite for all $x > x_H$. Hence $w_H \geqslant 1$ will imply $w'(x_H) > 0$. This is in contradiction to the result of proposition II claiming that $w_H \geqslant 1$ implies that w is a monotonically decreasing function such that $w'(x) < 0$ for all x if $w \geqslant 1$. Hence one concludes that $|w_H| < 1$. Note that the conclusion $-w_H < 1$ comes from the discrete symmetry under $w \to -w$.

On the other hand, one should have $w_H^2 + x_H^2 - 1 \geqslant 0$ if $w'(x_H) \geqslant 0$. In addition, a local maximum of w must exist somewhere at $x > x_H$ in order for w to turn around and approach zero at spatial infinity. And this maximum must show up before w reaches 1. Otherwise w will never be able to turn around according to proposition II. And this maximum also has to show up before x reaches 1 according to proposition III. This is because w will never be able to turn around once $x \geqslant 1$. The conclusion so far can be stated as follows: if $w'(x_H) > 0$, there is a local maximum of w at $x_0 \in (x_H, 1)$ such that $1 \geqslant w_0 > w_H$.

On the other hand, Eq. (4.7) says that $w_0^2 + x_0^2 - 1 < 0$ at the local maximum. Note that this inequality and $w_H^2 + x_H^2 - 1 \geqslant 0$ [since we have assumed that $w'(x_H) > 0$] imply that $w_0 < w_H$. This is not consistent with the result $w_0 > w_H$. Therefore, one shows that $w'(x_H) < 0$ in addition to the result $w_H < 1$ shown earlier.

Note that $w'(x_H) < 0$ and $w_H > 0$ indicate that $1 - w_H^2 > x_H^2$ according to Eq. (4.8). Therefore the necessary condition for the existence of a $w \neq 0$ non-Abelian monopole black hole is $x_H < 1$. This agrees with our earlier result.

Moreover, Eq. (4.4) becomes

$$C'(x_H) = \frac{1}{x_H} - \frac{(1 - w_H^2)^2}{2\epsilon x_H^3} - \frac{w_H^2}{\epsilon x_H} - \frac{x_H V_0}{\epsilon e^2 v^4} \geqslant 0$$

at the event horizon. Hence one has

$$w_H^4 - 2(1 - x_H^2) w_H^2 - 2\epsilon x_H^2 + 1 + x_H^4 \frac{2V_0}{e^2 v^4} \leqslant 0. \tag{4.9}$$

Therefore w_H real implies that

$$\left(1 - \frac{2V_0}{e^2 v^4}\right) x_H^2 - 2(1 - \epsilon) \geqslant 0. \tag{4.10}$$

There are a number of combinations of possible constraints available at this moment. First of all, let us consider the case where $V_0 < e^2 v^4/2$. One can show that (i) $x_H \geqslant \sqrt{2(1-\epsilon)/(1-V_h)}$ if $(1+V_h)/2 < \epsilon < 1$ (note that the first inequality about ϵ comes from the fact that $x_H < 1$), (ii) a magnetically charged black hole cannot exist if $0 < \epsilon \leqslant (1 + V_h)/2$, and (iii) there is no constraint at all for $\epsilon \geqslant 1$. Here we have written $V_h \equiv 2V_0/e^2 v^4$ such that $V_h < 1$.

On the other hand, we can also consider the case where $V_0 > e^2 v^4/2$ such that $V_h > 1$. In that case, one has that (iv) $x_H \leqslant \sqrt{2(\epsilon-1)/(V_h-1)}$ if $\epsilon > (1+V_h)/2$, (v) there is no solution for $\epsilon \leqslant 1$, and (vi) if $1 < \epsilon < (1+V_h)/2$, there is no constraint except $x_H < 1$.

Finally, one can consider the case where $V_0 = e^2 v^4/2$. This will imply that $\epsilon \leqslant 1$.

In addition, one notes that the inequalities $1 - w_H^2 > x_H^2$ and Eq. (4.9) saturate for extremal black holes. This implies that $x_H = \sqrt{2(1-\epsilon)/(1-V_h)}$ if $(1-\epsilon)(1-V_h) > 0$.

In summary, we have shown that (a) w can be an oscillatory function in the domain $x \in (x_H, 1)$, (b) w has to approach 0 monotonically in the domain $x \geqslant 1$, (c) $w_H < 1$, (d) $w'(x_H) < 0$, (e) $x_H \geqslant \sqrt{2(1-\epsilon)/(1-V_h)}$ if $(1+V_h)/2 < \epsilon < 1$ and $V_0 < e^2 v^4/2$, (f) a magnetically charged black hole cannot exist if $0 < \epsilon \leqslant (1+V_h)/2$ and $V_0 < e^2 v^4/2$, (g) there is no constraint at all for $\epsilon \geqslant 1$ and $V_0 < e^2 v^4/2$, (h) $x_H \leqslant \sqrt{2(\epsilon-1)/(V_h-1)}$ if $\epsilon > (1+v_h)/2$ and $V_0 > e^2 v^4 2$, (i) a magnetically charged black hole cannot exist if $\epsilon \leqslant 1$ and $V_0 > e^2 v^4/2$, (j) if $1 < \epsilon < (1+V_h)/2$ and $V_0 > e^2 v^4/2$, there is no constraint except $x_H < 1$, (k) in the case where $V_0 = e^2 v^4/2$ and $\epsilon > 1$, there is no additional constraint, and (l) there is no solution if $V_0 = e^2 v^4/2$ and $\epsilon \leqslant 1$.

Note that these constraints will be very helpful for choosing appropriate initial data for numerical solutions. Note that Eqs. (4.4)–(4.6) can be written as

$$C' = \frac{1}{x}(1-C) - \frac{1}{\epsilon x}\left[Cw'^2 + \frac{(1-w^2)^2}{2x} + w^2\right] - \frac{xV_0}{\epsilon e^2 v^4}, \tag{4.11}$$

$$Cw'' + C'w' + \frac{Cw'^3}{\epsilon x^3} + \frac{w}{x}(1-w^2-x^2) = 0, \tag{4.12}$$

once $B(x)$ is eliminated. Note that one can expand $C(x)$ and $w(x)$ as $C(x) = c(x-x_H)$ and $w(x) = w_H + \omega(x-x_H)$ to linear order in $x-x_H$ near the neighborhood of x_H. Therefore one has

$$c = \frac{1}{x_H} - \frac{(1-w_H^2)^2}{2\epsilon x_H^3} - \frac{w_H^2}{\epsilon x_H} - \frac{x_H V_0}{\epsilon e^2 v^4}, \tag{4.13}$$

$$\omega = -\frac{w_H}{c x_H^2}(1-w_H^2-x_H^2), \tag{4.14}$$

for the leading term. Note that w_H is confined by

$$1 - x_H^2 - x_H\sqrt{x_H^2(1-V_h) - 2(1-\epsilon)} \leqslant w_H^2 \leqslant 1 - x_H^2. \tag{4.15}$$

Note that the first inequality comes from Eq. (4.9) and the second inequality comes from Eq. (4.14) by noting that $\omega < 0$. In addition, one has $x_H = \sqrt{2(1-\epsilon)/(1-V_h)}$ and $w_H = \sqrt{(2\epsilon - 1 - V_h)/(1-V_h)}$ for extremal black holes.

V. CONCLUSION

We have shown that a regular SO(3) non-Abelian 't Hooft–Polyakov monopole solution does not exist in induced Einstein-Yang-Mills theories with a variety of SSB potentials. The only way to obtain a regular monopole solution is to have a charged black hole inside the magnetic monopole. We did show that scalar hair for these models cannot exist in the presence of a black hole. It is also shown that the same conclusion holds for the scale invariant potential.

We also show that a nontrivial monopole-charged black hole imposes a number of constraints on the field parameters ϵ, w_H, $w'(x_H)$ as well as the size of the event horizon x_H. In particular, it is shown that a nontrivial non-Abelian monopole charged black hole solution exists only if the radius of the event horizon, r_H, is smaller than the characteristic radius $1/ev$ of the classical monopole. The global behavior of the monopole function w is also analyzed in the presence of a nonvanishing cosmological constant. The large distance behavior of w is found without solving the field equation directly. These analyses may be helpful for related studies. In particular, it was shown that the presence of the cosmological constant affects the properties of the monopole charged black hole in a very significant way. Our results indicate that this induced gravity model deserves more attention. The effect of the induced theory should have more applications in different areas of interest.

ACKNOWLEDGMENTS

This work was supported in part by the Taiwan NSC under contract NSC88-2112-M009-001.

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