Pilot-Aided Multicarrier Wireless Channel Estimation via MMSE Polynomial Interpolation

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***Abstract*—Orthogonal frequency division multiplexing (OFDM) and multiple access (OFDMA) signal receivers often employ pilot-aided channel estimation. For it, an often considered technique is frequency-domain polynomial interpolation, due to its simplicity. However, the performance of polynomial interpolators suffers in channels with large delay spreads due to modeling error. The problem can be remedied by including a linear phase to the interpolator. In this paper, we derive a method to estimate the optimal phase shift that minimizes the mean-square channel estimation error. We further consider adaptive selection of the interpolation order for best performance. As a practical application, we adapt the proposed channel estimation technique to Mobile WiMAX downlink transmission and examine the resulting performance.**

I. INTRODUCTION

Polynomial interpolation is a simple and well-established approach to channel estimation in pilot-aided multicarrier communication [1]. In this approach, one may first estimate the channel responses at the pilot subcarriers by some means and then interpolate between them to obtain the responses at the intervening data subcarriers. Extrapolation is needed if some data subcarriers lie beyond the frequency range covered by the two outermost pilot subcarriers. But we shall refer to the overall approach as interpolation for convenience.

Of all polynomial interpolation schemes, linear (or first-order) interpolation is one of the simplest. However, linear interpolation may suffer great performance loss in poor symbol synchronization [2] or under a large channel delay spread [3]. As a result, some have proposed to alleviate the problem by resorting to high-order interpolation [1]. The way how interpolation order influences the channel estimation performance can be appreciated via its effect in the time domain. Specifically, polynomial interpolation is effectively a linear filtering operation [4]. Since convolution in the frequency domain corresponds to multiplication in the time domain, different orders of interpolation in the frequency domain correspond to different kinds of windowing on the channel impulse response. Fig. 1 is a conceptional illustration of the situation. Linear interpolation (dashed line) has a more tapered window than quadratic interpolation (dash-dot line) and hence results in a greater distortion of the channel response. However, if one can shift the window corresponding to linear interpolation by some amount to better capture the time range of the significant channel response samples (solid line), then the performance could improve. Higher-order interpolation is similar.

Three questions arise. First, how to effect the window shift in pilot-aided channel estimation where, instead of the channel impulse response, one only has some frequency response samples at the pilot subcarriers to work with? To answer, note that time shift of a waveform amounts to imposing a linear phase shift (or rotation) on its frequency spectrum. But to realize the phase rotation would cost one complex multiplication per subcarrier. However, if the required time shift is an integer multiple of samples, then the effect can be achieved equivalently and simply by shifting the received signal circularly in the opposite direction by that amount before feeding it into the discrete Fourier transform (DFT).

The second question is how to obtain the optimal window shift for given interpolation order. Hsieh and Wei [1] adopt the single frequency estimators proposed in [5]. However, for channel estimation this is not optimal in the mean-square error (MSE) sense, as later derivation will show. We will derive the optimal window shift that achieves minimum MSE (MMSE) in channel estimation for arbitrary polynomial order.

Thirdly, subsequent analysis will show that the MMSE in channel estimation depends on the amount of channel noise. In high channel noise, higher-order interpolation may be affected so more adversely than lower-order interpolation as to yield worse channel estimates. Therefore, it should be desirable to make the interpolation order adaptive according to the channel condition to attain the best possible channel estimation performance. We will propose a scheme for this.

In what follows, we first consider how to find the MMSE window or phase shift under given polynomial order in Section II. Then, as a practical application, in Section III we adapt the proposed technique to suit Mobile WiMAX downlink specifications and examine the resulting performance. Adaptive selection of interpolation order is investigated in this context.

II. MMSE WINDOW/PHASE SHIFTING

Consider a generic discrete-time, equivalent lowpass multipath Rayleigh wireless channel with its impulse response and (normalized) frequency response given by

$$h(t)=\sum_{l=1}^{L}\alpha_l\delta(t-\tau_l),\quad H(f)=\sum_{l=1}^{L}\alpha_l W^{\tau_l f}, \qquad (1)$$

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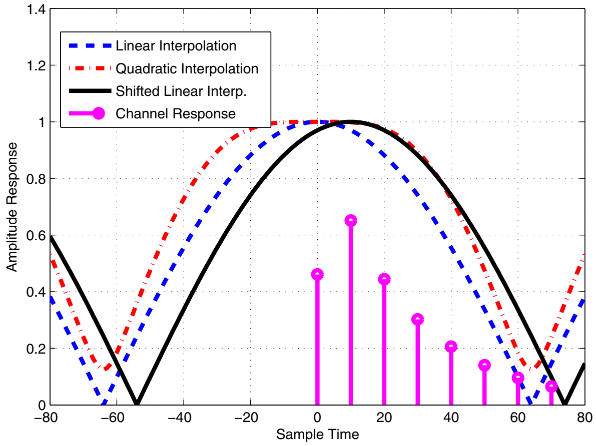

Fig. 1. Comparison of different ways of interpolation in terms of equivalent time-windowing effects.

where L is the number of paths, α_l is the complex Gaussian gain of the lth path, τ_l is the associated path delay, and $W = \exp(-j2\pi/N_s)$ with N_s being the DFT size used in multicarrier transformation. Both t and τ_l are integers whose unit is the sampling period. And $H(f)$ has period N_s.

Consider comb-type pilot allocation and let the mth pilot subcarrier be located at frequency $p_0 + mF$ where F is the spacing of pilot subcarriers. Let $\hat{H}(p_0 + mF)$ be some estimated channel responses at the pilot locations; how they are estimated is of no concern at this point. Then conventional polynomial interpolation between two pilot locations p and $p + F$ can be written as

$$\hat{H}(p+k) = \sum_{n=0}^{N} C_{n,k}\,\hat{H}(p+x_n) \tag{2}$$

where $0 < k < F$, N is the interpolation order, x_n defines the nth pilot location used in interpolation, and $C_{n,k}$ is the corresponding interpolation coefficient. The interpolation coefficients are real and are well-known [6] to be given in the Vandermonde form by

$$\begin{aligned}\underline{C}_k &\triangleq [C_{0,k}\ \ C_{1,k}\ \ \cdots\ \ C_{N,k}]\\ &= \underbrace{\left[1\ \ k\ \ \cdots\ \ k^N\right]}_{\triangleq \underline{V}(k)} \underbrace{\begin{bmatrix} 1 & x_0 & \cdots & x_0^N \\ 1 & x_1 & \cdots & x_1^N \\ \vdots & \vdots & \ddots & \vdots \\ 1 & x_N & \cdots & x_N^N \end{bmatrix}^{-1}}_{\triangleq X}\end{aligned} \tag{3}$$

or in the Lagrange form by

$$C_{n,k} = \prod_{m=0, m\neq n}^{N} \frac{k - x_m}{x_n - x_m}. \tag{4}$$

Now consider phase-shifted interpolation corresponding to τ_c samples of window shift. The interpolator coefficients are given by $W^{\tau_c(k-x_n)}C_{n,k}$, yielding

$$\begin{aligned}\hat{H}(p+k) &= \sum_{n=0}^{N}\left[W^{\tau_c(k-x_n)}C_{n,k}\right]\hat{H}(p+x_n)\\ &= W^{\tau_c(p+k)}\sum_{n=0}^{N} C_{n,k}\left[W^{-\tau_c(p+x_n)}\hat{H}(p+x_n)\right].\end{aligned} \tag{5}$$

Interpreted, the right-hand side (RHS) of (5) outlines a procedure for channel estimation under a given τ_c as follows: 1) obtain channel estimates at the pilot subcarriers by some means, 2) phase-rotate the above channel estimates by an amount corresponding to time advance by τ_c by carrying out the computation indicated by the last bracket in (5), 3) perform conventional polynomial interpolation as indicated by the last summation in (5), and 4) phase-derotate the channel estimates to undo the earlier time advance by premultiplication with $W^{\tau_c(p+k)}$. As indicated previously, when τ_c is an integer, a simple time-domain equivalent can be used to replace steps 2 and 4. But the theory is unaffected. Further, though we have limited the treatment to comb-type pilot allocation, the results can be readily extended to noncomb-type pilot allocation. But the equations become somewhat cumbersome.

Now we turn to the key issue of determining τ_c. For this we first find how the channel estimation MSE depends on it.

A. Dependence of Channel Estimation MSE on Window Shift

For convenience, let $\bar{H}(f)$ be the frequency response associated with the τ_c-early channel, i.e.,

$$\bar{H}(f) \triangleq W^{-\tau_c f}H(f) = \sum_{l=1}^{L}\alpha_l W^{\Delta\tau_l f} \tag{6}$$

where $\Delta\tau_l = \tau_l - \tau_c$, and let

$$\hat{\bar{H}}(f) = W^{-\tau_c f}\hat{H}(f). \tag{7}$$

Then

$$\hat{\bar{H}}(p+k) = \sum_{n=0}^{N} C_{n,k}\hat{\bar{H}}(p+x_n). \tag{8}$$

The MSE in $\hat{\bar{H}}(p+k)$ is the same as that in $\hat{H}(p+k)$ as they are related by mere phase rotation. Ignore the effect of channel noise and consider only the modeling error for now, so that $\hat{\bar{H}}(p+x_n) = \bar{H}(p+x_n)$. The Nth-order Taylor series expansion of the τ_c-advanced channel response at frequency $p+k$ about a pilot subcarrier p is given by

$$\bar{H}(p+k) = \sum_{n=0}^{N}\frac{\bar{H}^{(n)}(p)k^n}{n!} + R_N(p+k) \tag{9}$$

where $\bar{H}^{(n)}(p)$ is the nth derivative of $\bar{H}(p)$ and $R_N(p+k)$ is the remainder term which has an integral form given by

$$\begin{aligned}R_N(p+k) &= \int_0^k \frac{\bar{H}^{(N+1)}(p+u)}{N!}(k-u)^N du\\ &= \sum_{l=1}^{L}\alpha_l\left(\frac{-j2\pi\Delta\tau_l}{N_s}\right)^{N+1} W^{\Delta\tau_l p}\int_0^k \frac{W^{\Delta\tau_l u}}{N!}(k-u)^N du.\end{aligned}$$

If k is small enough compared to the coherence bandwidth such that $W^{\Delta\tau_l u} \approx 1$ for $0 \leq u \leq k$, then

$$\int_0^k \frac{W^{\Delta\tau_l u}}{N!} (k-u)^n \, du \approx \frac{k^{N+1}}{(N+1)!} \tag{10}$$

and thence

$$R_N(p+k) \approx \frac{k^{N+1}}{(N+1)!} \underbrace{\sum_{l=1}^{L} \alpha_l \left(\frac{-j2\pi\Delta\tau_l}{N_s}\right)^{N+1} W^{\Delta\tau_l p}}_{\triangleq \xi_N(p)}. \tag{11}$$

Therefore, the channel estimation error at $p+k$ is given by

$$\begin{aligned} &\bar{H}(p+k) - \underline{V}(k)X^{-1}\underline{\hat{\bar{H}}}(p) \\ &\quad = R_N(p+k) - \underline{V}(k)X^{-1}\underline{R}_N(p) \\ &\quad \approx \frac{1}{(N+1)!}\left[k^{N+1} - \underline{V}(k)X^{-1}\underline{x}\right]\xi_N(p) \end{aligned} \tag{12}$$

where

$$\begin{aligned} \underline{\hat{\bar{H}}}(p) &= \left[\hat{\bar{H}}(p+x_0), \hat{\bar{H}}(p+x_1), \cdots, \hat{\bar{H}}(p+x_N)\right]^T, \\ \underline{R}_N(p) &= \left[R_N(p+x_0), R_N(p+x_1), \cdots, R_N(p+x_N)\right]^T, \\ \underline{x} &= \left[x_0^{N+1}, x_1^{N+1}, \cdots, x_N^{N+1}\right]^T. \end{aligned} \tag{13}$$

It can be shown that

$$G(k) \triangleq k^{N+1} - \underline{V}(k)X^{-1}\underline{x} = \prod_{n=0}^{N}(k-x_n). \tag{14}$$

Consequently, the desired relationship between the MSE and τ_c is given by

$$\begin{aligned} \sigma_e^2(k) &= \frac{1}{[(N+1)!]^2} G^2(k) \langle\!\langle |\xi_N(p)|^2 \rangle\!\rangle \\ &= \frac{1}{[(N+1)!]^2} G^2(k) \underbrace{\frac{(2\pi)^{2N+2}}{N_s^{2N+2}} \sum_{l=1}^{L} |\alpha_l|^2 \Delta\tau_l^{2N+2}}_{\triangleq \sigma_\xi^2(N,\tau_c)} \end{aligned} \tag{15}$$

where $\langle\!\langle\cdot\rangle\!\rangle$ denotes averaging over all pilot subcarriers.

B. Estimation of Optimal Window/Phase Shift

From the above, the channel estimation MSE is minimized by minimizing $\sigma_\xi^2(N,\tau_c)$. For this, note that since

$$\bar{H}^{(N+1)}(f) = \sum_{l=1}^{L} \alpha_l (-j2\pi\Delta\tau_l/N_s)^{N+1} W^{\Delta\tau_l f}, \tag{16}$$

by Parseval's theorem we get

$$\sigma_\xi^2(N,\tau_c) = \langle |\bar{H}^{(N+1)}(f)|^2 \rangle \tag{17}$$

where $\langle\cdot\rangle$ denotes averaging over all frequencies. Thus the minimization of $\sigma_\xi^2(N,\tau_c)$ can be replaced by that of $\langle|\bar{H}^{(N+1)}(f)|^2\rangle$. While $\bar{H}^{(N+1)}(f)$ is usually not available, either, it can be approximated by the $N+1$st-order difference.

For example, by using the $N+1$st forward difference, it can be shown that the cost function is approximately given by

$$\sigma_\xi^2(N,\tau_c) \approx \frac{1}{F^{2N+2}} \left[P_N R_0 + 2I_N(\theta)\right] \tag{18}$$

where $P_N = 2^{N+1}(2N+1)!!/(N+1)!$, $\theta = 2\pi F\tau_c/N_s$, $R_n = \langle\!\langle \hat{\bar{H}}(p+nF)\hat{\bar{H}}^*(p)\rangle\!\rangle$, and

$$I_N(\theta) = \Re\left\{\sum_{m=1}^{N+1} (-1)^m A_{N,m} e^{-jm\theta} R_m\right\}, \tag{19}$$

with $m!! = m(m-2)(m-4)\cdots x$ where $x=2$ for even m and $x=1$ for odd m and

$$A_{N,m} = \frac{2^{N+1}(2N+1)!!(N+1)!}{(N+1-m)!(N+1+m)!}. \tag{20}$$

Hence an estimate of the optimal window shift is

$$\hat{\tau}_c = N_s \cdot \arg\min_\theta I_N(\theta)/2\pi F. \tag{21}$$

For linear, quadratic and cubic interpolators, we have

$$\begin{aligned} I_1(\theta) &= \Re\{e^{-j2\theta}R_2 - 4e^{-j\theta}R_1\}, \\ I_2(\theta) &= \Re\{-e^{-j3\theta}R_3 + 6e^{-j2\theta}R_2 - 15e^{-j\theta}R_1\}, \\ I_3(\theta) &= \Re\{e^{-j4\theta}R_4 - 8e^{-j3\theta}R_3 + 28e^{-j2\theta}R_2 - 56e^{-j\theta}R_1\}. \end{aligned}$$

One good way to find $\hat{\tau}_c$ is to search over $[0, 2\pi F\tau_{\max}/N_s]$ for value of θ that minimizes $I_N(\theta)$.

III. Application to WiMAX Downlink

The Mobile WiMAX system [7], [8] furnishes an interesting example to test the performance of the proposed technique. In its downlink (DL), the subcarriers in an orthogonal frequency-division multiple access (OFDMA) symbol are divided into "clusters" that contain 14 consecutive subcarriers each. Alternating patterns of pilot subcarriers are placed in temporally successive symbols. Fig. 2 illustrates the cluster structure in successive symbols, where the dark circles indicate pilot subcarriers. For convenience, in this section let $H(s,k)$ denote the channel response at the kth subcarrier of some cluster in symbol s. We now tailor the proposed channel estimation technique to suit the WiMAX DL signal structure and investigate the resulting channel estimation performance, assuming perfect carrier frequency and symbol timing synchronization. We also propose a method to adaptively select the interpolation order for minimizing the channel estimation MSE.

A. Channel Estimator Design

There are only two pilot subcarriers per cluster and their spacing varies with symbol index. In a typical system with 10 MHz bandwidth and $N_s = 1024$, the adjacent subcarriers are approximately 10 kHz apart. Considerations over coherence bandwidth and interpolator property lead to the following method of channel estimation. For convenience, we describe it for symbols 2 and 3 (enclosed in the dashed box in Fig. 2) together. To save computation, we estimate $\hat{\tau}_c$ only once per DL subframe based on the preamble symbol that begins the subframe [7], using the method of Section II-B. In addition, experience shows that an accuracy in $\hat{\tau}_c$ of 2 to 6 samples can be enough. Therefore, we let $\hat{\tau}_c$ be an integer.

1) Circularly shift the signal samples in symbols 1 to 4 by $-\hat{\tau}_c$ before taking their DFT. This is equivalent to having a channel with phase-rotated frequency response

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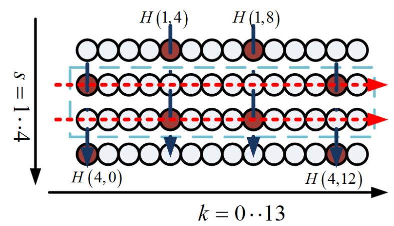

Fig. 2. Cluster structure in Mobile WiMAX downlink and corresponding channel estimation method.

$\bar{H}(s,k) = W^{-\hat{\tau}_c f(k)} H(s,k)$ where $f(k)$ denotes the normalized frequency of subcarrier k. (We have used an adapted version of the notations in (6) here.)

2) Do LS channel estimation for pilot subcarriers in symbols 1 to 4 (which simply takes the ratio of the received signal value at each pilot subcarrier to the known BPSK pilot value there). This yields, for each pilot with time-frequency index (s,k), a phase-rotated channel estimate $\hat{\bar{H}}(s,k) = W^{-\hat{\tau}_c f(k)} \hat{H}(s,k)$. (We have used an adapted version of the notations in (7) here.)
3) Linearly interpolate along the time axis to obtain $\hat{\bar{H}}(2,4)$, $\hat{\bar{H}}(2,8)$, $\hat{\bar{H}}(3,0)$ and $\hat{\bar{H}}(3,12)$.
4) Perform conventional interpolation along the frequency axis to obtain channel estimates for the remaining subcarriers in symbols 2 and 3. This corresponds to the operation described in (8), or equivalently, that in the last summation in (5). To minimize the modeling error, in Nth-order interpolation we use the $N+1$ nearest pilots of each data subcarrier for it.

Note that we do not have to phase-derotate the result as indicated by the premultiplication with $W^{\tau_c(p+k)}$ in the RHS of (5), for the circular shifting in step 1 makes the phase-rotated channel the target of estimation. In other words, the phase-rotated channel is all that is needed for signal detection. Note also that the temporal linear interpolation in step 3 results in four reference data points per cluster. Hence in step 4 we may employ an interpolation order up to three.

B. Performance Analysis

Four factors contribute to the channel estimation error. They are, in order of their appearance in the channel estimation steps: 1) suboptimality in the estimated window shift $\hat{\tau}_c$, 2) the channel noise (introduced in step 2), 3) modeling error due to time-domain interpolation (introduced in step 3), and 4) modeling error due to frequency-domain interpolation (introduced in step 4). Item 1 is difficult to analyze but, fortunately, constitutes a minor contribution in the total MSE. Thus only the other three factors need to be analyzed. We assume that these three kinds of error are uncorrelated.

First, consider the channel noise, assumed additive white Gaussian (i.e., AWGN). It enters the channel estimator computation through the LS channel estimation conducted at the pilot subcarriers. Since the pilots are BPSK-modulated, let σ_n^2 denote the estimation noise variance at any pilot. This estimation noise propagates to other subcarriers in subsequent temporal and frequency interpolations. Via the temporal interpolation, it contaminates $\hat{\bar{H}}(s,k)$ where $(s,k) \in \{(2,4),(2,8),(3,0),(3,12)\}$. Since $\hat{\bar{H}}(s,k) = [\hat{\bar{H}}(s-1,k) + \hat{\bar{H}}(s+1,k)]/2$ where $\hat{\bar{H}}(s-1,k)$ and $\hat{\bar{H}}(s+1,k)$ contain independent noise, the noise variance in $\hat{\bar{H}}(s,k)$ is given by

$$\sigma_w^2(s,k) = \left(\frac{1}{2}\right)^2 \sigma_n^2 + \left(\frac{1}{2}\right)^2 \sigma_n^2 = \frac{1}{2}\sigma_n^2. \tag{22}$$

Via the frequency interpolation, it results in an estimation noise variance

$$\sigma_w^2(s,k) = \sum_{n=0}^{N} C_{n,k}^2 \sigma_w^2(s,x_n) \tag{23}$$

where $s \in \{2,3\}$, $k \in \{1,2,3,5,6,7,9,10,11,13\}$, and $x_n \in \{0,4,8,12\}$, with $C_{n,k}$ indexed similarly to (2). Hence the average noise variance over all data subcarriers is given by

$$\sigma_{w,1}^2 = 0.5130\sigma_n^2,\ \sigma_{w,2}^2 = 0.6699\sigma_n^2,\ \sigma_{w,3}^2 = 0.8121\sigma_n^2,$$

where the second subscript to σ denotes interpolation order.

Secondly, consider the modeling error in temporal interpolation. Its mean-square value is defined by

$$\sigma_D^2(s,k) = E\left|\bar{H}(s,k) - \frac{1}{2}\left[\bar{H}(s-1,k) + \bar{H}(s+1,k)\right]\right|^2$$

for $(s,k) \in \{(2,4),(2,8),(3,0),(3,12)\}$. Assume Rayleigh faded paths. Then [3], [9]

$$\begin{aligned}\sigma_D^2(s,k) &= \sum_{l=1}^{L} E|\alpha_l|^2 \left[\frac{3}{2} - 2J_0(2\pi f_l) + \frac{1}{2}J_0(4\pi f_l)\right] \\ &\approx \frac{3\pi^4}{2}\sum_{l=1}^{L} f_l^4\, E|\alpha_l|^2 \triangleq \sigma_D^2 \end{aligned} \tag{24}$$

where f_l is the peak Doppler shift of path l times the OFDMA symbol period, $J_0(\cdot)$ is the Bessel function of the first kind of order 0, and the approximation is obtained by expanding the Bessel function into a second-order Taylor series. Assume that the channel responses at different subcarriers are uncorrelated. Then a relation similar to (23) exists concerning the propagation of the temporal modeling error in the frequency domain via channel estimation step 4, and we get the average MSE over all data subcarriers as

$$\sigma_{d,1}^2 = 0.4531\sigma_D^2,\ \sigma_{d,2}^2 = 0.5577\sigma_D^2,\ \sigma_{d,3}^2 = 0.6525\sigma_D^2,$$

where the second subscript to σ again gives the interpolation order.

Finally, consider the modeling error in frequency interpolation. From (15), straightforward numerical calculation yields the following average MSE over data subcarriers:

$$\sigma_{i,1}^2 = k_1\,\sigma_\xi^2(1,\tau_c^o),\ \sigma_{i,2}^2 = k_2\,\sigma_\xi^2(2,\tau_c^o),\ \sigma_{i,3}^2 = k_3\,\sigma_\xi^2(3,\tau_c^o),$$

where $k_1 = 2.6458$, $k_2 = 12.8125$, $k_3 = 93.0820$, and the second subscript to σ once more gives the interpolation order.

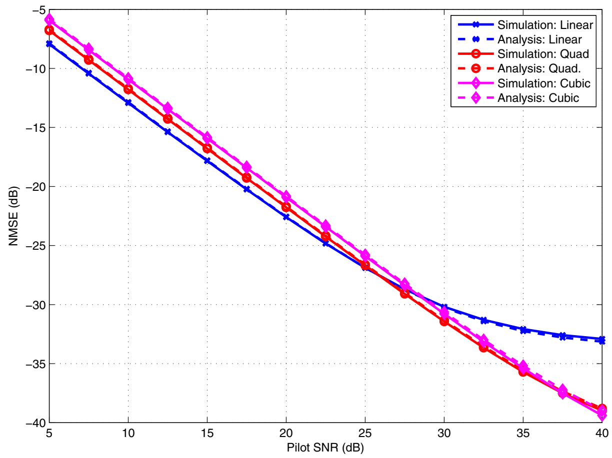

Fig. 3. NMSE of channel estimation in WiMAX DL transmission over SUI-4 channel at 100 km/h mobile speed with different orders of interpolation.

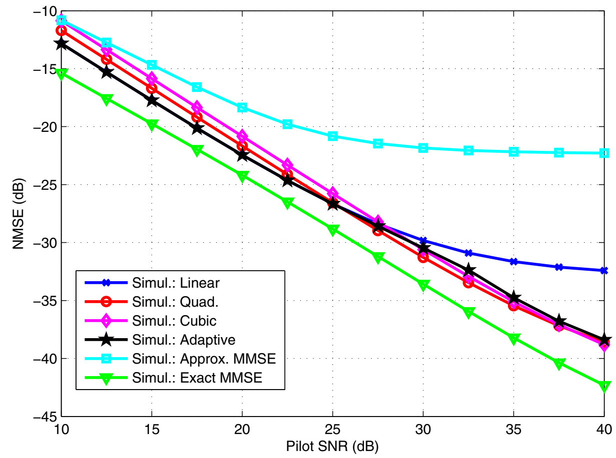

Fig. 4. Comparison of adaptive interpolation with fixed interpolation over the SUI-4 PDP under 100 km/h mobile speed.

Putting all together, we obtain the overall average channel estimation MSE as

$$\sigma_{H,j}^2 = \sigma_{w,j}^2 + \sigma_{d,j}^2 + \sigma_{i,j}^2 \qquad (25)$$

for interpolation order $j \in \{1, 2, 3\}$.

C. Simulation Results

We simulate a system with carrier frequency = 2.5 GHz, bandwidth = 10 MHz, DFT size = 1024, and cyclic prefix length = 128. And we let a DL subframe contain 24 OFDMA symbols following the preamble. Below we present some results under the SUI-4 power-delay profile (PDP) [10] with block-static fading at a rate corresponding to 100 km/h of mobile speed. The channel power profile is $[0, -4, -4]$ (in dB) and the delay profile is $[0, 14, 36]$ (in sample periods).

Fig. 3 show some results. The approximate analysis matches the simulation results very closely. (For channels with larger delay spread, the approximate analysis will be less accurate, but will still follow the general behavior of the simulation results.) Compared to quadratic and cubic interpolations, linear interpolation exhibits a higher MSE floor (due to frequency interpolation) in high pilot SNR, but it performs somewhat better in low SNR where the AWGN effect is more prominent.

D. Adaptive Selection of Interpolation Order

That lower-order interpolators incur smaller error due to AWGN whereas higher-order ones have smaller modeling error suggests adaptive selection of the interpolator order. This can be accomplished if we can estimate σ_n^2, $\sigma_\xi^2(N, \tau_c^o)$, and σ_D^2 from the preamble symbol and use the results to predict the MSE via (25) and determine the interpolation order.

It is relatively easy to estimate σ_n^2 using the null subcarriers in the preamble symbol. Further $\sigma_\xi^2(N, \tau_c^o)$ can be estimated based on (18), with R_0 estimated by $\langle\langle |\hat{H}(f)|^2 \rangle\rangle - \hat{\sigma}_n^2$ where $\hat{\sigma}_n^2$ is the estimate of σ_n^2. (Note that $P_1 = 6$, $P_2 = 20$, and $P_3 = 70$.) But it is hard to estimate σ_D^2, which depends on the time-variation of the channel response, using only the preamble symbol. Fortunately, it is the least dominating term of all and is thus disregarded.

Fig. 4 compares the MSE performance of adaptive interpolation with that of fixed interpolation in the SUI-4 channel. The performance of adaptive interpolation is nearly optimal. For comparison, we also plot the performance of 4-tap MMSE channel estimation obtained with the exact channel correlation function (marked "exact MMSE") and that with the correlation function corresponding to a uniform PDP of length equal to the cyclic prefix (marked "approx. MMSE"). The exact MMSE estimator performs better than polynomial interpolation, but is impractical. On the other hand, the adaptive interpolation scheme outperforms the approximate MMSE estimator.

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