Loss-Less Pulse Intensity Repetition-Rate Multiplication Using Optical All-Pass Filtering

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***Abstract*—Schemes based on optical all-pass filtering techniques for multiplying the intensity repetition-rate of an optical pulse train are examined. These methods are in principle without energy loss and the multiplication factor can be any given integer. Practical implementation using cascaded side-coupled ring resonators or multireflection filters is proposed and analyzed for the first time.**

***Index Terms*—Optical all-pass filtering, pulse repetition-rate.**

I. Introduction

With the advance of fiber communication technology, terabit fiber communication links have been demonstrated with the combined use of WDM and TDM techniques and are going to serve as the backbone of the new-generation information networks. In present implementation of these systems, the bit-rate per channel ranges from several tens of gigahertz to a few hundred gigahertz [1], which has created a growing demand for stable optical sources with ultrahigh pulse repetition-rates. Although there has been a lot of research progress on directly generating a high repetition-rate pulse train from semiconductor lasers or fiber lasers, it is always very advantageous to have simple ways for further increasing the pulse rate outside the laser cavity. Among the several approaches that have been reported in the literature, multiplexing the pulses in the time domain [1], [2] is the most commonly used method, although it requires the provision of proper time delays and in principle suffers from a minimum 3-dB optical loss when the multiplication factor is a power of 2. In this letter, by taking advantages of optical all-pass filtering techniques [3], we hereby propose and examine new schemes that in principle can losslessly produce a uniform pulse train with a multiplied pulse intensity repetition-rate. Practical implementation of the scheme using optical dispersion [4], [5], cascaded side-coupled ring resonators, or multireflection filters [6] is then discussed and analyzed.

II. Theory

The principle of repetition-rate multiplication schemes based on optical all-pass filtering can be explained in the following way. The envelope function of an optical pulse train with a repetition-rate of f can be expressed in terms of its Fourier series as follows:

$$E(t) = \sum_n E_n \exp\{j(2\pi nft + \phi_n)\} \tag{1}$$

If, in frequency-domain, we divide the Fourier series into N groups as shown in Fig. 1, then in the time domain each individual frequency-domain group corresponds to a pulse train with a repetition-rate of Nf. For simplicity let us assume that the original pulses are un-chirped ($\phi_n = 0$ for all n) and the pulse-duration is sufficiently short so that the assumption of $E_n = 1$ for all n can be applied. Under these two assumptions,

TABLE I
Example Solution Sets of (3): (a) From (4);
(b) Solutions With Only Four Discrete Phase-Shifts

(a)

N	Relative phase factors for each mode group
2	{0, π/2}
3	{0, 2π/3, 2π/3}
4	{0, π/4, π, π/4}
5	{0, 2π/5, 8π/5, 8π/5, 2π/5}
6	{0, π/6, 2π/3, 3π/2, 2π/3, π/6 }
7	{0, 2π/7, 8π/7, 4π/7, 4π/7, 8π/7, 2π/7}
8	{0, π/8, π/2, 9π/8, 0, 9π/8, π/2, π/8}
9	{0, 2π/9, 8π/9, 0, 14π/9, 14π/9, 0, 8π/9, 2π/9}
10	{0, π/10, 2π/5, 9π/10, 8π/5, π/2, 8π/5, 9π/10, 2π/5, π/10}
11	{0, 2π/11, 8π/11, 18π/11, 10π/11, 6π/11, 6π/11, 10π/11, 18π/11, 8π/11, 2π/11}
12	{0, π/12, π/3, 3π/4, 4π/3, π/12, π, π/12, 4π/3, 3π/4, π/3, π/12}
13	{0, 2π/13, 8π/13, 18π/13, 6π/13, 24π/13, 20π/13, 20π/13, 24π/13, 6π/13, 18π/13, 8π/13, 2π/13}
14	{0, π/14, 2π/7, 9π/14, 8π/7, 25π/14, 4π/7, π/2, 4π/7, 25π/14, 8π/7, 9π/14, 2π/7, π/14}
15	{0, 2π/15, 8π/15, 6π/5, 2π/15, 4π/3, 4π/5, 8π/15, 8π/15, 4π/5, 4π/3, 2π/15, 6π/5, 8π/15, 2π/15}
16	{0, π/16, π/4, 9π/16, π, 25π/16, π/4, 17π/16, 0, 17π/16, π/4, 25π/16, π, 9π/16, π/4, π/16}

(b)

N	Relative phase factors for each mode group
2	{0,π/2}
4	{0, 0, π,0}
8	{0, 3π /2, 3π /2, π /2, 0, π /2, 3π /2, 3π /2}
16	{0, π, π /2, 3π /2, π /2, π /2, 3π /2, 3π /2, π ,0, π /2, 3π /2, 3π /2, 3π /2, 3π /2, 3π /2}

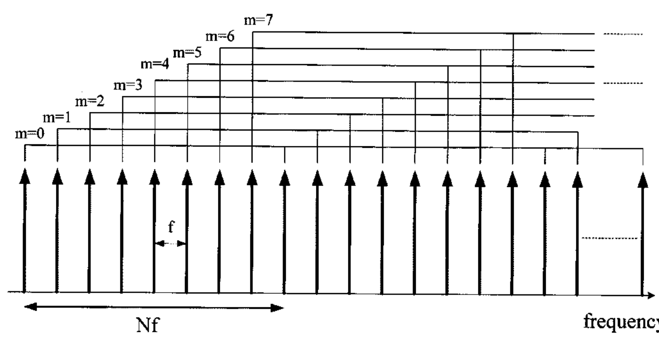

Fig. 1. Schematic of the frequency mode grouping with $N = 8$.

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the relative phases of the kth pulse in the mth group can be generally expressed by the following expression:

$$\theta_{mk} = \exp\left\{ j\frac{(2\pi m * k)}{N} \right\}, \quad m, k, \in \{0, 1, 2, \ldots, N-1\} \tag{2}$$

After writing out all the pulse phases in each group, we then try to introduce appropriate relative phase shifts of the N groups in the frequency domain and sum them up again so as to obtain a uniform optical pulse train in the time domain with a multiplied intensity repetition-rate Nf. The required phase adjustment factors δ_m for each group are found by solving the following equation:

$$\left| \sum_{m=0}^{N-1} e^{j\delta_m} \times e^{j\frac{2\pi m * k}{N}} \right|^2 = N \quad \text{for } k = 0, 1, 2, \ldots, N-1 \tag{3}$$

Equation (3) has multiple solutions for each N. One set of analytical solutions are given by:

$$\begin{aligned} \delta_m &= \frac{\pi}{N} m^2 \quad \text{if } N \text{ is even} \\ &= \frac{2\pi}{N} m^2 \quad \text{if } N \text{ is odd} \end{aligned} \tag{4}$$

For convenient reference, these phase shifts are tabulated in Table I(a) for $N = 2$ to 16. We also find that when N is of powers of 2, the required phase-shifts can be chosen among only four discrete values: $\{\pm 1, \pm j\}$, which have been shown in Table I(b) for $N = 2$, 4, 8, and 16. These are another set of solutions for (3).

To prove the validity of our analyses, Fig. 2 shows the numerical simulation for cases of $N = 4$ and $N = 8$ with a suitable input pulse duration. It can be seen that indeed a uniform pulse train with a multiplied pulse rate can be losslessly generated and the output pulse duration is almost the same as the original pulse duration.

III. Implementation

There are several ways to practically implement the repetition-rate multiplication schemes based on optical all-pass filtering. The required phase-shifts in (4) are quadratic in m and thus can be readily implemented by using the second-order dispersion of the optical fiber. Recently such an approach has been experimentally demonstrated [4], [5] and its principle is analogous to the fractional Talbot effects observed during beam propagation of periodic light patterns. In this letter, we propose another method for practical implementation of the scheme by using cascaded side-coupled ring resonators as shown in Fig. 3(a). This new approach has the advantage of opening up the possibility of fabricating integrated devices for lossless pulse repetition-rate multiplication.

An ideal side-coupled ring resonator is an optical all-pass filter with its phase response given by: [3]

$$\Phi = \text{Arg}\left[\frac{\sqrt{1-\kappa} - e^{j\phi}}{1 - \sqrt{1-\kappa}\, e^{j\phi}} \right]. \tag{5}$$

Here, κ is the cross power coupling ratio of the directional coupler and $\phi = 2n\pi L/\lambda$ is the round-trip phase change of the

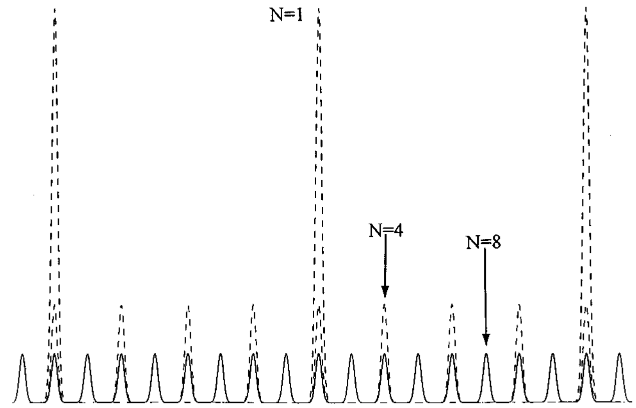

Fig. 2. Numerical simulation of the ideal output pulse trains with $N = 4$ and $N = 8$. The original pulse train is also plotted for comparison.

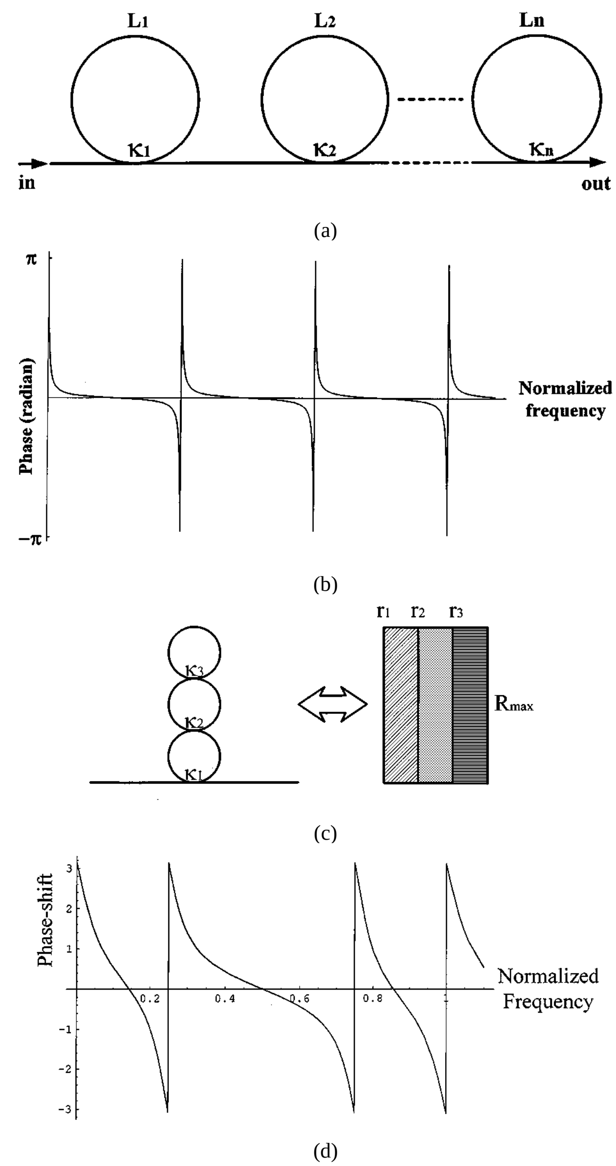

Fig. 3. (a) Schematic of cascaded side-coupled ring resonators; (b) Phase spectrum of a side-coupled ring resonator, $\kappa = 0.1$; (c) Schematic of a multiring resonator and a multireflection filter; (d) Phase spectrum of a three-loop (three-layer) multiring resonator (multireflection filter): $\kappa_1 = 0.8$, $\kappa_2 = 0.854$, $\kappa_3 = 0.8$.

ring loop. This phase spectrum is plotted in Fig. 3(b) for a typical κ value and it is clear that there is a π phase-shift at every resonance frequency. By careful adjustment of the equivalent path lengths of the ring loops and the coupling ratios of the couplers in Fig. 3(a), we can properly bias each ring resonator to selectively induce the required phase adjustment for every frequency-domain group, as long as the Q factors of the ring resonators are large enough to suppress the undesired crosstalk phase-shifts seen by the other mode groups. For this kind of implementation, the solution set in Table I(b) may be more advantageous compared to those in Table I(a), since there are only four levels of phase shifts involved. The number of required resonators is at most $N-1$, but in practice it can be much less. From Table I(b), for example, for $N=4$, only one resonator is needed to create a π phase shift.

One may be concerned with the impact of the undesired crosstalk phase-shifts caused by the finite line-width of side-coupled ring resonators during practical implementation. For the case of $N = 4$, when the cross power coupling ratio is set to be 5%, the pulse-to-pulse intensity variation will be around $\pm$5% from the simulation. This indicates that in practice, resonators with a very high Q-factor are required if one wants to obtain a highly uniform pulse train using the scheme. A possible solution to this crosstalk problem is to use a multiring resonator or equivalently a multireflection filter [6] as shown in Fig. 3(c). The phase spectrum of a particular three-loop (or three-layer) design for the case of $N = 4$ is shown in Fig. 3(d), which is calculated according to the following formula:

$$\Phi = \mathrm{Arg}[F(r_1, F(r_2, F(r_3, 1)))]. \tag{6}$$

Here

$$F(r_a, r_b) = \frac{r_a - r_b e^{j\phi}}{1 - r_a r_b e^{j\phi}} \tag{7}$$

and $r_i = \sqrt{1-\kappa_i}, i = 1, 2, 3$. The derivation of the formula is not difficult to be carried out and thus will be omitted here due to limited space. One can see that in principle the crosstalk phase-shifts can be totally suppressed for the case of $N = 4$ (i.e., the phase shift at 0, 0.25 and 0.75 normalized frequency are exactly π while it is exactly 0 at 0.5 normalized frequency).

This above multiloop design also can greatly reduce the sensitivity to optical loss when compared with the case of a single-loop. This is because one now does not need to use a very high Q resonator. From our simulation, for the case in Fig. 3(d), the pulse-to-pulse intensity variation will be less than $\pm$4%, if the optical loss per loop is 0.5 dB. One may also concern the performance sensitivity to the resonance frequency mismatch. From our simulation, again for the case in Fig. 3(d), the pulse-to-pulse intensity variation will be around $\pm$4% if the deviation of resonance frequencies is 0.005 free spectral range. These numbers indicate the stringent requirements imposed by the scheme if a very uniform pulse train is necessary. Nevertheless, it should be possible to meet these requirements with today's technology, even though one may need to use techniques like active cavity length control and/or optical amplification. For some pulse rates the cascaded multireflection filter may be a more convenient way to implement the scheme. It should also be emphasized that the impact of cavity loss and mismatch are only on the pulse intensity deviation, not on the pulse jitter. This is the main advantage of the frequency-domain approach proposed here compared to the usual time-domain multiplexing approach.

IV. Conclusion

We have theoretically studied schemes that can losslessly multiply the intensity repetition-rate of a steady pulse train by using optical all-pass filtering techniques. New way of practical implementation using cascaded side-coupled ring resonators or multireflection filters is proposed and analyzed for the first time. These schemes should be useful for practically generating highly repetitive optical pulse trains to be used in future ultrahigh-capacity communication systems.

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