Optical Chaotic AM Demodulation by Asymptotic Synchronization

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***Abstract*—Synchronization between two identical chaotic systems (drive and response) with different initial conditions can be achieved provided that the conditional Lyapunov exponents are all negative. Adding an AM signal to the drive system, the two systems become asymptotically synchronized. The AM signal can be recovered by utilizing the property of the asymptotic synchronization. Chaotic AM noise is present as a direct result of the chaotic AM demodulation. Signal-to-chaotic-AM-noise ratio is calculated as a function of the signal amplitude.**

***Index Terms*—Asymptotic synchronization, chaotic AM noise, optical chaotic AM demodulation, self-pulsating laser diodes.**

I. INTRODUCTION

PECORA and Carroll have pointed out that a subsystem of a chaotic system (the drive system) can be synchronized with a separate chaotic system (the response system) provided that the conditional Lyapunov exponents (CLE) for the drive and response systems are all negative [1], [2]. Although these criteria can be affected by the external additive noise [3] or parameter mismatch [4], the ability to design synchronizing systems has opened opportunities for applications of chaos to private communications [5].

In optical chaotic communication which have the potential for high-speed commnication, class B lasers (master–slave model) have been used to achieve the synchronization and signal transmission [6]. In this letter, a drive-response model according to Pecora and Carroll's theory is taken using self-pulsating laser diodes [7]. Synchronization can be achieved for optical simplex and duplex transmissions. Further, optical chaotic AM demodulation is investigated by the asymptotic synchronization. When an AM signal is added to a drive system, the two systems are no longer identical and become asymptotically synchronized. Utilization of asymptotic synchronization can lead to optical chaotic AM demodulation.

A drive system described by a three-dimensional rate equation is given by

$$\begin{aligned}\frac{dN_1}{dt} &= F_1(N_1, N_2, S) + a + b\sin 2\pi f t + \delta\left(\hat{S} - S\right)\\ \frac{dN_2}{dt} &= F_2(N_1, N_2, S)\\ \frac{dS}{dt} &= F_3(N_1, N_2, S)\end{aligned} \tag{1}$$

and the response system $(\hat{\ })$ is given by

$$\begin{aligned}\frac{dN_1}{dt} &= \hat{F}_1\left(N_1, \hat{N}_2, \hat{S}\right) + \hat{a} + \hat{b}\sin 2\pi \hat{f} t + \hat{\delta}\left(S - \hat{S}\right)\\ \frac{d\hat{N}_2}{dt} &= \hat{F}_2\left(\hat{N}_1, \hat{N}_2, \hat{S}\right)\\ \frac{d\hat{S}}{dt} &= \hat{F}_3\left(\hat{N}_1, \hat{N}_2, \hat{S}\right)\end{aligned} \tag{2}$$

where

- S photon density;
- N_1 electron density in the active region;
- N_2 electron density in the saturable absorption region;
- $I = a + b\sin 2\pi f t + \delta\left(\hat{S} - S\right)$ injection current;
- δ coupling coefficient.

Note that I is normalized by a factor of eV_1, where V_1 is the active layer volume. The nonlinear functions F_1, F_2, and F_3, which describe the self-pulsating laser diodes are written by [8]

$$\begin{aligned}F_1 &= -\frac{k_1\xi_1}{V_1}(N_1 - N_{g1})S - \frac{N_1}{\tau_s} - \frac{N_1 - N_2}{T_{12}}\\ F_2 &= -\frac{k_2\xi_2}{V_2}(N_2 - N_{g2})S - \frac{N_2}{\tau_s} - \frac{N_2 - N_1}{T_{21}}\\ F_3 &= [k_1\xi_1(N_1 - N_{g1}) + k_2\xi_2(N_2 - N_{g2}) - G_{th}]S\\ &\quad + C\frac{N_1V_1}{\tau_s}\end{aligned} \tag{3}$$

where

- τ_s carrier lifetime;
- ξ confinement factor;
- G_{th} threshold gain level;
- T carrier time diffusion constant between the two layers;
- k linear approximation constant for the gain curve;
- N_g transparent level of electron density;
- C coupling ratio between the spontaneous field and the lasing mode.

The subscripts 1 and 2 describe each term in the active and absorption layers, respectively. Table I lists all the parameters of

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TABLE I
THE PARAMETERS USED FOR THE SIMULATION OF THE SELF-PULSATING LASER DIODES

parameters	value	unit
k_1	3.08×10^{-12}	m^3/S
k_2	1.232×10^{-11}	m^3/S
ξ_1	0.2034	-
ξ_2	0.1449	-
N_{g1}	1.4×10^{24}	m^{-3}
N_{g2}	1.6×10^{24}	m^{-3}
V_1	72	μm^3
V_2	102.96	μm^3
T_{12}	2.65	ns
T_{21}	4.452	ns
G_{th}	3.91×10^{11}	S^{-1}
C	1.573×10^{-23}	μm^{-3}
τ_s	3	ns

the self-pulsating laser diodes obtained from [8] used in the simulation. When $a = 30$ mA is injected, the corresponding f_0 is 2.28 GHz. This is used in our calculations throughout.

By the injection a sinusoidal carrier into a self-pulsating laser diode ($a = 30$ mA, $b = 9$ mA, and $f = 0.6 f_0$, the optical chaotic light can be obtained [7]. By choosing a proper coupling coefficient ($\hat{\delta} = 45$ and $\delta = 0$), negative CLE's can be obtained. CLE's are calculated from the real parts of the eigenvalues of A^{-1} in the difference system $dE(t)/dt = AE(t)$, where $E_{N_1} = N_1 - \hat{N}_1$, $E_{N_2} = N_2 - \hat{N}_2$ and $E_S = S - \hat{S}$. If all of the CLE's are negative ($\lim_{t\to\infty} E(t) = 0$), the two identical systems will be synchronized independent of initial conditions.

In the context of synchronization, an AM signal is added to the amplitude of the carrier and the resulting injection current is $I = a + (b + \Delta b \sin 2\pi\Delta f t) \sin 2\pi f t + \delta(\hat{S} - S)$, where Δf is the AM signal frequency, and Δb is the AM signal amplitude. In the response system, $\hat{b}$ remains a constant current. Thus, the two chaotic systems are no longer identical. If the Δb is small, the two chaotic systems become asymptotically synchronized. This asymptotic synchronization can be described by

$$\lim_{t\to\infty} |S(t) - \hat{S}(t)| = \lim_{t\to\infty} |E_S(t)| = k\left[\frac{|\epsilon|}{(\delta + \hat{\delta})}\right] \quad (4)$$

where k is the proportional constant and ϵ is the difference term between the drive and response systems without coupling. When the AM signal is added, ϵ is given by

$$|\epsilon| = |\Delta b \sin 2\pi\Delta f t \sin 2\pi f t|. \quad (5)$$

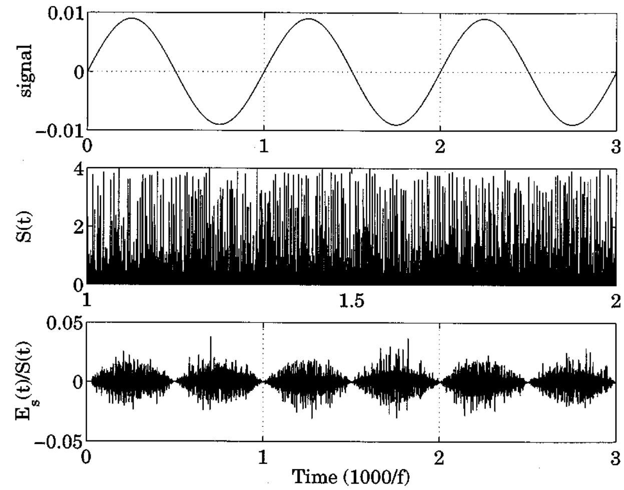

Fig. 1. The waveforms of the signal, transmitted chaotic light $S(t)$, and normalized asymptotic synchronization term $(S(t) - \hat{S}(t))/S(t)$. The parameters are $\hat{\delta} = 45$, $\delta = 0$, $f = \hat{f} = 0.6 f_0$, $b = \hat{b} = 9$ mA, $a = \hat{a} = 30$ mA, and $(\Delta f/f) = (\Delta b/b) = 10^{-3}$.

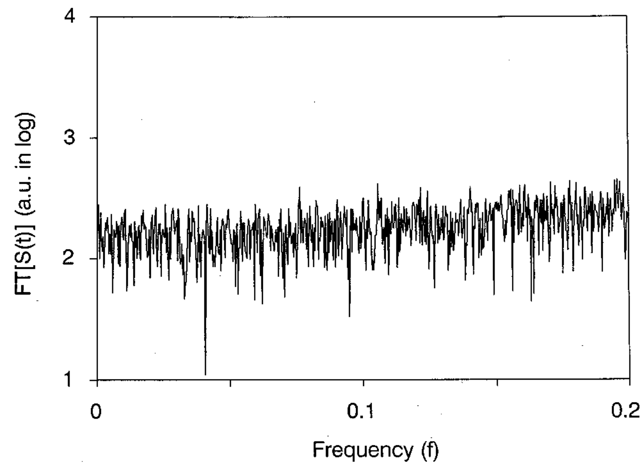

Fig. 2. The frequency spectrum of the $S(t)$ for $(\Delta f/f) = (\Delta b/b) = 10^{-3}$.

Note that without the AM signal, $\lim_{t\to\infty} |E_S(t)| = 0$. Equations (4) and (5) can then be used as a baseline principle for optical chaotic AM demodulation.

When an AM signal is added to the drive system, the signal can be recovered by measuring the difference term $E_S(t)$ in the receiving end. Fig. 1 shows the waveforms of the signal, transmitted chaotic light $S(t)$, and normalized difference term $E_S(t)/S(t)$, where $\Delta f/f = \Delta b/b = 10^{-3}$. The carrier frequency, $f = 0.6 f_0 = 1.368$ GHz, and the AM signal are at an intermediate frequency ($\Delta f = 1.368$ MHz). The frequency spectrum of the transmitted chaotic light $S(t)$ which has a complex and unpredictable pattern is shown in Fig. 2. The signal frequency Δf is masked by the chaotic carrier, and the signal can not be recovered merely by low-pass filtering the $S(t)$. Thus, the purposes of the private communication can be achieved. However, the difference term shown in Fig. 1 clearly recovers an AM envelope with a frequency of Δf. Within this envelope, there is a high frequency term corresponding to the carrier frequency f.

In the frequency domain, $\Delta b \sin 2\pi\Delta f t \sin 2\pi f t$ gives the sum frequency $(f + \Delta f)$ and the difference frequency $(f - \Delta f)$. Fig. 3 shows the frequency spectrum of the normalized

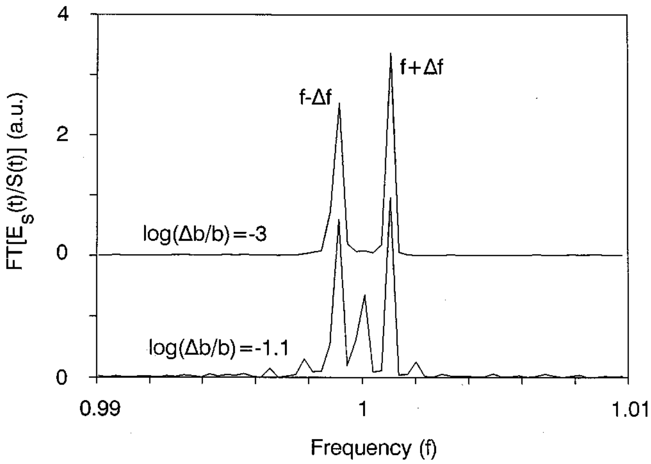

Fig. 3. The frequency spectrum of the normalized $E_s/S(t)$ for $(\Delta b/b) = 10^{-3}$ and $(\Delta b/b) = 10^{-1.1}$.

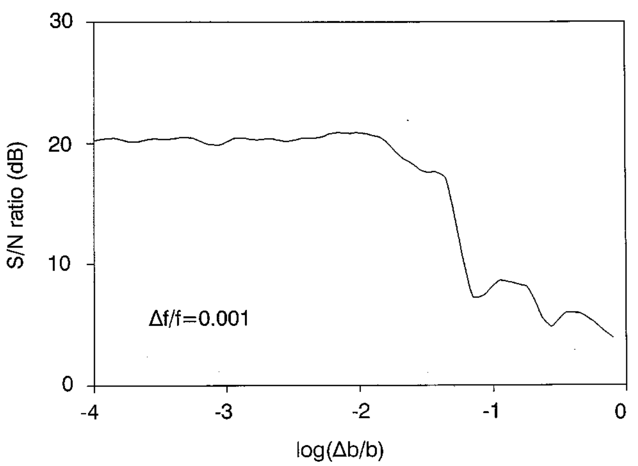

Fig. 4. The signal to chaotic AM noise power ratio as a function of $(\Delta b/b)$ when $(\Delta f/f) = 10^{-3}$.

$E_s/S(t)$ for $\Delta b/b = 10^{-3}$ and $\Delta b/b = 10^{-1.1}$. Two distinct signal peaks are found at $(f + \Delta f)$ and $(f - \Delta f)$ for $\Delta b/b = 10^{-3}$. The signal, Δf, can then be recovered by the conventional down-conversion techniques (for example, multiplied by the carrier frequency). Note that there is only one peak at f for the transmitted chaotic light $S(t)$. By carrying out the down-conversion techniques on the transmitted chaotic light alone, the signal will not be recovered in this case. Any other peak is unwanted energy that is present as a direct result of the chaotic AM demodulation. This correlated noise is called chaotic AM noise. In addition, for a conventional AM there is one more peak at the carrier frequency f. In this chaotic AM, one significant advantage is that no power is wasted in the carrier. As Δb increases ($\Delta b/b = 10^{-1.1}$), chaotic AM noise become significant around the signal peaks. It is expected that (4) is valid only for small signal amplitudes. In this case, chaotic AM demodulation will be difficult.

To characterize the effect of chaotic AM noise, one can calculate the signal to chaotic AM noise power ratio, as shown in the Fig. 4. The power level of the chaotic AM noise is the average noise level from Fig. 3. For $\Delta b/b = 10^{-3}$, the signal-to-noise ratio exceeds 20 dB (a feasible number for practical applications). As Δb increases, the drive and response systems become less correlated. Therefore, the chaotic AM noise level increases and signal level decreases. The turning point occurs at –1.5 to –1.1 of $\log(\Delta b/b)$.

In conclusion, drive and response systems become asymptotically synchronized when an AM signal is added to the drive system. The AM signal can be recovered by calculating the normalized difference term $E_S(t)/S(t)$. Two distinct signal peaks $(f + \Delta f)$ and $(f - \Delta f)$ appear in the frequency spectra. One type of correlated noise (called chaotic AM noise) is present as a direct result of chaotic AM demodulation. The signal to chaotic AM noise ratio is shown to be a function of the signal amplitude.

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