Optimal Diversity Multiplexing Tradeoff of Constrained Asymmetric MIMO Systems

Hsiao-feng (Francis) Lu
Department of Communications Engineering
National Chiao Tung University
Hsinchu, Taiwan
Email: francis@cc.nctu.edu.tw

***Abstract*—In a MIMO downlink channel it is often that the number of transmit antennas is strictly larger than the number of receive, and such channel is termed asymmetric MIMO channel. To employ simple decoding techniques in this channel, such as zero-forcing or sphere decoding, the number of active transmit antennas must be constrained to be no larger than the number of receive, and the resulting system is coined "constrained asymmetric MIMO system." For the case of two receive antennas and for any number of transmit antennas, an optimal transmission scheme is presented in this paper and is shown to achieve the same performance as the unconstrained ones in terms of the diversity-multiplexing tradeoff. The construction of optimal constrained codes is also provided.**

I. Introduction

Consider a MIMO communication channel that consists of n_t transmit and n_r receive antennas. When $n_t > n_r$, i.e. when the number of transmit antennas is strictly larger than the number of receive, such $(n_t \times n_r)$ MIMO channel is commonly referred to as the *asymmetric MIMO channel*, and it can often be found in the MIMO downlink communication.

Assuming that all the n_t transmit antennas are active during signal transmission, let $\underline{x}$ be the length-n_t code vector sent from the transmitter to the receiver and let H be the corresponding $(n_r \times n_t)$ channel matrix. The length-n_r signal vector $\underline{y}$ received at the receiver end is given by

$$\underline{y} \;=\; H\underline{x} + \underline{w}, \tag{1}$$

where $\underline{w}$ is a length-n_r vector used to capture the effects of additive white Gaussian noise. Entries of the channel matrix H and the noise vector $\underline{w}$ are modeled as i.i.d. complex Gaussian random variables with zero mean and unit variances in this paper. Further, the code vector $\underline{x}$ is assumed to satisfy the following power constraint

$$\mathrm{Tr}\left(\mathbb{E}\underline{x}\underline{x}^\dagger\right) \;\le\; \mathrm{SNR}, \tag{2}$$

where by $\dagger$ we mean the Hermitian transpose of a vector.

When the channel matrix H is known completely to the receiver but not the transmitter, Telatar [1] first showed that the ergodic channel capacity of such $(n_t \times n_r)$ MIMO channel approximates $\min\{n_t, n_r\}\log_2 \mathrm{SNR}$ at high SNR regime, regardless of the relation between n_t and n_r. Furthermore, it was shown that such capacity can be achieved by using i.i.d. complex Gaussian random vectors $\underline{x}$ having covariance matrix $K_X = \frac{\mathrm{SNR}}{n_t} I_{n_t}$. On the other hand, assuming that the transmitter communicates at rate

$$R \;=\; r\log_2 \mathrm{SNR} \qquad \text{(bits/channel use)}, \tag{3}$$

where r, $0 \le r \le \min\{n_t, n_r\}$, is termed the *multiplexing gain*, Zheng and Tse proved in their landmark paper [2] that given r, the smallest bit error probability that can be achieved by all possible coding schemes is given by

$$P_{\mathrm{e,min}}(\mathrm{SNR}) \;\doteq\; \mathrm{SNR}^{-d^*(r)}, \tag{4}$$

meaning

$$\lim_{\mathrm{SNR}\to\infty} \frac{\log P_{\mathrm{e,min}}(\mathrm{SNR})}{\log \mathrm{SNR}} \;=\; -d^*(r). \tag{5}$$

The negative exponent $d^*(r)$ is termed the *diversity gain* and is given by a piecewise linear function connecting the points

$$\{(k, (n_t-k)(n_r-k)) \;:\; k = 0, 1, \cdots, \min\{n_t, n_r\}\}. \tag{6}$$

$d^*(r)$ represents an optimal tradeoff between the multiplexing gain r and the diversity gain, and is thus termed the optimal *diversity-multiplexing tradeoff* (DMT). It is also proved in [2] that $d^*(r)$ can be achieved by using i.i.d. length-n_t complex Gaussian random vectors, provided that the asymmetric MIMO channel is quasi-static and the channel matrix H remains fixed for $T \ge n_t + n_r - 1$ channel uses.

This remarkable result has spurred a considerable amount of research activities of constructing coding schemes [3], [4], [5], [6], [7] to achieve the optimal tradeoff (6). In particular, Elia *et al.* [6] have provided a sufficient condition for having deterministic DMT optimal codes. Furthermore, for any n_t, using a cyclic division algebra (CDA) with degree n_t^2 over the center $\mathbb{Q}(\imath)$, where $\imath = \sqrt{-1}$, an algebraic construction of $(n_t \times n_t)$ matrix codes meeting this sufficient condition is proposed in [6] for all $T \ge n_t$.

While all the coding schemes mentioned above, including the Gaussian random codes and the CDA-based codes, are DMT optimal, it should be noted that they require that all the n_t transmit antennas are active during each channel use. Such requirement can lead to some unavoidable difficulty in decoding. To see this, note that the channel matrix H is of size $(n_r \times n_t)$ with $n_t > n_r$. Therefore H has no left multiplicative matrix inverse, and it is impossible to use zero-forcing (ZF) decoder to decode the code. Similarly, the same requirement

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again forbids the possibility of using sphere decoder which relies on the QR decomposition of the matrix H. For the minimum-mean square error (MMSE) detector, due to the number of observations, n_r, in each channel use, is strictly less than the number of unknowns, which is n_t in this case, the error performance resulting from the use of MMSE decoding technique cannot be good.

In order to use the ZF decoder, sphere decoder, or the MMSE decoder to reduce the decoding complexity, the number of active transmit antennas in each channel use must not be larger than n_r. With this additional constraint, the resulting system is termed *constrained asymmetric MIMO system* in this paper, and coding schemes meeting this requirement are coined *constrained asymmetric space-time codes*. Similarly, codes without this constraint will be termed *unconstrained codes.*

In [8], Hollanti and Ranto focused on the case of 4 transmit and 2 receive antennas, i.e. $n_t = 4$ and $n_r = 2$, and proposed a block-diagonal coding method for constructing the constrained asymmetric space-time codes. The construction first partitions the 4 transmit antennas into two groups, say $\{T_1, T_2\}$ and $\{T_3, T_4\}$, and then performs a joint-encoding between the two groups by making use of the multi-block space-time codes proposed by the author [9]. Specifically, let $\mathcal{X}$ be a (2×4) multi-block space-time code where the coding is applied over 2 consecutive (2×2) independent fading blocks, and let H_1 (resp. H_2) denote the channel matrix corresponding to the transmit $\{T_1, T_2\}$ (resp. $\{T_3, T_4\}$) and the receive antennas. Given the transmitted code matrix $X = [X_1\, X_2] \in \mathcal{X}$, where the submatrix X_i is of size (2×2), the resulting received signal matrix is

$$Y_i \;=\; H_i X_i + W_i, \qquad i = 1, 2, \tag{7}$$

where W_i is the (2×2) noise matrix. Clearly we have the original (2×4) channel matrix $H = [H_1\, H_2]$ and given the desired multiplexing gain r, it can be easily shown by using results in [9] that the resulting diversity gain $d(r)$ achieved by $\mathcal{X}$ is given by a piecewise linear function connecting the points $(k, 2\,(2-k)\,(2-k))$, for $k = 0, 1, 2$. From Fig. 1 it can be seen that the DMT performance achieved by $\mathcal{X}$ is far from being optimal compared to $d^*(r)$ in (6) .

In this paper, we will investigate the optimal DMT of the constrained asymmetric MIMO systems, and in particular, focus on the case of $n_r = 2$, which represents a very common scenario in the MIMO downlink communications. This paper is organized as follows. In Section II, we will present a DMT optimal transmission scheme for any constrained asymmetric MIMO systems with $n_t > n_r = 2$ and show that the resulting DMT equals $d^*(r)$, meaning that there is no performance loss with the additional constraint on the number of transmit antennas used in each channel use if the code is properly designed. The proposed DMT optimal transmission scheme is basically a selection pattern of the transmitted antennas used, and the corresponding DMT optimal coding schemes that follow this selection pattern will be briefly discussed in Section III. Finally, in Section IV, we conclude the paper.

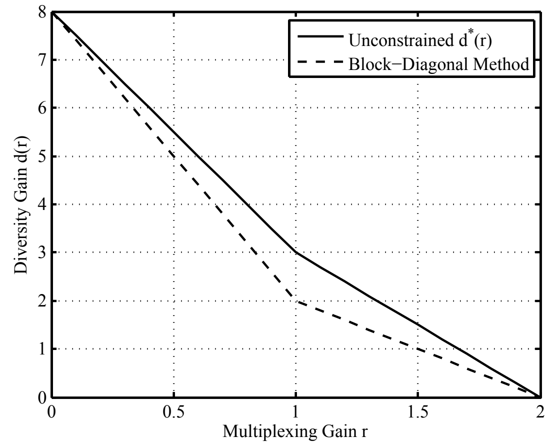

Fig. 1. The DMT performances of unconstrained coding schemes and codes derived from block-diagonal constructions [8].

II. Proposed DMT Optimal Transmission Scheme for Constrained Asymmetric MIMO Systems

In the previous section, we have shown that in order to employ ZF, sphere, or MMSE decoding techniques for decoding the transmitted signal matrix in an asymmetric MIMO channel, the number of transmit antennas used in each channel use cannot be larger than the number of receive antennas n_r. In this section, we will focus on the case when $n_r = 2$. For any $n_t > 2$ we will present a DMT optimal transmission scheme that can achieve the optimal DMT $d^*(r)$ in the $(n_t \times 2)$ constrained asymmetric MIMO channel. To describe the proposed transmission scheme, we first define the following.

Definition 1: In an $(n_t \times n_r)$ constrained asymmetric MIMO channel, let $\mathcal{T} = \{T_1, \cdots, T_{n_t}\}$ be the set of indices of n_t transmit antennas. We say

$$\mathcal{S} \;:=\; \{(\mathcal{T}_1, n_1), \cdots, (\mathcal{T}_s, n_s)\} \tag{8}$$

is an *antenna-selection transmission scheme* if the antenna selection patterns $\mathcal{T}_i$ are distinct proper subset of $\mathcal{T}$ and have size $1 \leq |\mathcal{T}_i| \leq n_r < n_t$ for each i. Moreover, each antenna selection pattern $\mathcal{T}_i$ will be used for n_i transmissions and it is assumed that the MIMO channel remains fixed for T channel uses with

$$T \;\geq\; \sum_{i=1}^{s} n_s. \tag{9}$$

■

For example, the block-diagonal coding method proposed in [8] for the (4×2) constrained asymmetric MIMO channel can be regarded as an antenna-selection transmission scheme of

$$\mathcal{S}_{BD} \;=\; \{(\{T_1, T_2\}, 2), (\{T_3, T_4\}, 2)\}\,. \tag{10}$$

However, it has already been seen in Section I that the above scheme $\mathcal{S}_{BD}$ is not DMT optimal in the (2×4) constrained asymmetric MIMO channel. On the other hand, for any antenna-selection transmission scheme $\mathcal{S} =$

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$\{(\mathcal{T}_1, n_1), \cdots, (\mathcal{T}_s, n_s)\}$ with $|\mathcal{T}_i| = n_r$, it is clear that the ergodic channel capacity achieved by $\mathcal{S}$ is the same as that achieved by the unconstrained schemes. To see this, let H_i denote the channel matrix associated with the selection pattern $\mathcal{T}_i$ and the set of all receive antennas, and let $\underline{x}_{i_j}$, $1 \le i \le s$ and $1 \le j \le n_i$ be i.i.d. zero-mean complex Gaussian random vectors having same covariance matrix $K = \frac{\text{SNR}}{n_r} I_{n_r}$. Then following the same approach as in [1] the ergodic channel capacity achieved by $\mathcal{S}$ using random $\underline{x}_{i_j}$ as transmitted signal vectors is

$$\begin{aligned} \mathcal{C}(\text{SNR}) &= \frac{1}{\sum_{i=1}^{s} n_s} \sum_{i=1}^{s} \mathbb{E} n_i \log_2 \det \left(I_{n_r} + H_i K H_i^\dagger \right) \\ &= \mathbb{E} \log_2 \det \left(I_{n_r} + \frac{\text{SNR}}{n_r} H_1 H_1^\dagger \right) \\ &\approx n_r \log_2 \text{SNR} \end{aligned} \quad (11)$$

at high SNR regime and is the same as that achieved by the unconstrained schemes.

To improve the DMT performance, for any $n_t > n_r = 2$ below we provide another transmission scheme and we will prove that it can achieve the optimal DMT $d^*(r)$ given in (6). Clearly, in this case, the maximal value of multiplexing gain r is upper bounded by $\min\{n_t, n_r\} = 2$, hence $0 \le r \le 2$. The proposed scheme is the following.

Theorem 1 (Main Result): In an $(n_t \times 2)$ constrained asymmetric MIMO system with $n_t > 2$, let $\mathcal{T} = \{T_1, \cdots, T_{n_t}\}$ be the set of indices of n_t transmit antennas. Given the desired multiplexing gain r,

1) if the multiplexing gain r falls within the range of $[1, 2]$, the following antenna-selection scheme

$$\begin{aligned} \mathcal{S}_1 = \{ &(\{T_1, T_2\}, 2), (\{T_2, T_3\}, 2), \cdots, \\ &(\{T_{n_t-1}, T_{n_t}\}, 2)\} \end{aligned} \quad (12)$$

achieves the optimal DMT $d^*(r)$ of (6), and

2) if $r \in [0, 1)$,

$$\begin{aligned} \mathcal{S}_2 = \{ &(\{T_1, T_2\}, 4), (\{T_2, T_3\}, 2), \cdots, \\ &(\{T_{n_t-2}, T_{n_t-1}\}, 2), (\{T_{n_t-1}, T_{n_t}\}, 4)\} \end{aligned} \quad (13)$$

is DMT optimal in terms of $d^*(r)$. ■

First of all, the only difference between the selection patterns $\mathcal{S}_1$ and $\mathcal{S}_2$ is that when $0 \le r < 1$, the sets $\{T_1, T_2\}$ and $\{T_{n_t-1}, T_{n_t}\}$ are used twice more than the others. Secondly, for the case of (4×2) constrained asymmetric MIMO channel, the scheme in Theorem 1 is given by

$$\mathcal{S}_1 = \{(\{T_1, T_2\}, 2), (\{T_2, T_3\}, 2), (\{T_3, T_4\}, 2)\}$$

for multiplexing gain $r \in [1, 2]$ and

$$\mathcal{S}_2 = \{(\{T_1, T_2\}, 4), (\{T_2, T_3\}, 2), (\{T_3, T_4\}, 4)\}$$

for $r \in [0, 1)$. Comparing to the block-diagonal method $\mathcal{S}_{BD}$, the proposed scheme requires two more transmissions for $r \ge 1$ and six more for $r < 1$. However, the price of using more transmissions is well paid off by having a much better error performance and achieving the same DMT performance as the unconstrained systems.

Below we provide a proof to Theorem 1

Proof: As the difference between the schemes $\mathcal{S}_1$ and $\mathcal{S}_2$ lies only in the number of times used for each antenna selection pattern $\mathcal{T}_i$, here we consider the following general form:

$$\mathcal{S} := \{(\{T_1, T_2\}, n_1), \cdots (\{T_{n_t-1}, T_{n_t}\}, n_{n_t-1})\}. \quad (14)$$

For the ith selection, let $\underline{x}_i$ be the length-n_r, zero-mean, complex Gaussian random vector with covariance $K_i = \frac{\text{SNR}}{2} I_{n_r}$ as the random code vector; then the corresponding received signal vector is given by

$$\underline{y}_i = H_i \underline{x}_i + \underline{w}_i \quad (15)$$

where $H_i := \left[\underline{h}_i \ \underline{h}_{i+1}\right]$ and $\underline{h}_i$ is length-2 vector consisting of the fading coefficients between the ith transmit antenna T_i and the receive antennas. $\underline{w}_i$ is the zero-mean complex Gaussian random vector of length 2 used to model the effect of additive white Gaussian noise. Thus, given the channel matrix H_i, the mutual information between the transmit and receive signal vectors is

$$I\left(\underline{x}_i; \underline{y}_i | H_i\right) \approx \log_2 \det\left(I_2 + \text{SNR} H_i H_i^\dagger\right), \quad (16)$$

where we have neglected the 2 appearing in the denominator of $\frac{\text{SNR}}{2}$ as here we are only interested in the high SNR regime for the sake of DMT performance analysis. Define

$$N := \sum_{j=1}^{n_t-1} n_j. \quad (17)$$

Given the desired multiplexing gain r, the channel outage probability of $\mathcal{S}$ is

$$\begin{aligned} P_{\text{out}}(r) := &\left\{ \sum_{i=1}^{n_t-1} n_i \log\det\left(I_2 + \text{SNR} H_i H_i^\dagger\right) \le \right. \\ &\left. N r \log \text{SNR} \right\} \doteq \text{SNR}^{-d(r)}. \end{aligned} \quad (18)$$

In particular, the mutual information associated with $\{T_1, T_2\}$ can be rewritten as

$$\begin{aligned} &\log\det\left(I_2 + \text{SNR} H_1 H_1^\dagger\right) \\ = &\log\det\left(I_2 + \text{SNR} \underline{h}_1 \underline{h}_1^\dagger + \text{SNR} \underline{h}_2 \underline{h}_2^\dagger\right) \\ = &\log\det\left(I_2 + \text{SNR} D_1\right) + \\ &\log\left(1 + \text{SNR} \underline{h}_2^\dagger U_1 \left(I_2 + \text{SNR} D_1\right)^{-1} U_1^\dagger \underline{h}_2\right) \\ = &\log\left(1 + \text{SNR} \left|\underline{h}_1\right|_F^2\right) + \\ &\log\left(1 + \text{SNR} \underline{g}_2^\dagger \left(I_2 + \text{SNR} D_1\right)^{-1} \underline{g}_2\right), \end{aligned}$$

where $U_1 D_1 U_1^\dagger$ is the eigen-decomposition of the rank-1 matrix $\underline{h}_1 \underline{h}_1^\dagger$ and where $\underline{g}_2 := U_1^\dagger \underline{h}_2$ has the same joint probability density function as that of $\underline{h}_1$. By $|\underline{h}_1|_F$ we mean the Frobenius norm of vector $\underline{h}_1$. Hence, without affect the

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calculation of (18), we can set the channel matrix associated with the second selection pattern $\{T_2, T_3\}$ as

$$H_2' = \begin{bmatrix} \underline{g}_2 & \underline{h}_3 \end{bmatrix} \tag{19}$$

and the corresponding mutual information changes to

$$I\left(\underline{x}_2; \underline{y}_2 | H_2'\right) = \log\left(1 + \mathrm{SNR}\left|\underline{g}_2\right|_F^2\right) + \log\left(1 + \mathrm{SNR}\underline{g}_3^\dagger \left(I_2 + \mathrm{SNR}D_2\right)^{-1} \underline{g}_3\right),$$

where $U_2 D_2 U_2^\dagger$ is the eigen-decomposition of the rank-1 matrix $\underline{g}_2 \underline{g}_2^\dagger$ and $\underline{g}_3 = U_2^\dagger \underline{h}_3$. Continuing in this fashion, we can rewrite the overall mutual information associated with scheme $\mathcal{S}$ as

$$\begin{aligned} & \sum_{i=1}^{n_t-1} n_i \log\det\left(I_2 + \mathrm{SNR} H_i' H_i'^\dagger\right) \\ = & \sum_{i=1}^{n_t-1} n_i \left[\log\left(1 + \mathrm{SNR}\left|\underline{g}_i\right|_F^2\right) + \right. \\ & \left. \log\left(1 + \frac{\mathrm{SNR}\left|g_{i+1,1}\right|^2}{1 + \mathrm{SNR}\left|\underline{g}_i\right|_F^2} + \mathrm{SNR}\left|g_{i+1,2}\right|^2\right)\right], \end{aligned} \tag{20}$$

where we have set $\underline{g}_1 = \underline{h}_1$, and for $i = 2, \cdots, n_t$, $\underline{g}_i = U_{i-1}^\dagger \underline{h}_i = [g_{i,1} \;\; g_{i,2}]^t$. $U_i D_i U_i^\dagger$ is the eigen-decomposition of $\underline{g}_i \underline{g}_i^\dagger$.

Now define

$$\left|g_{i,j}\right|^2 \doteq \mathrm{SNR}^{-\alpha_{i,j}} \tag{21}$$

and we can rewrite (20) as

$$\begin{aligned} & \frac{1}{\log \mathrm{SNR}} \sum_{i=1}^{n_t-1} n_i \log\det\left(I_2 + \mathrm{SNR} H_i' H_i'^\dagger\right) \\ \approx & \sum_{i=1}^{n_t-1} n_i \left[\left(\underbrace{\max_j \left\{(1-\alpha_{i,j})^+\right\}}_{:=(1-\beta_i)^+}\right) + \right. \\ & \left. \max\left\{\left(1 - \alpha_{i+1,1} - (1-\beta_i)^+\right)^+, (1-\alpha_{i+1,2})^+\right\}\right], \end{aligned}$$

where $(x)^+ := \max\{0, x\}$. Thus, the diversity gain achieved by the general scheme $\mathcal{S}$ is

$$d(r) = \inf_{\mathcal{A}(r)} \sum_{i=1}^{n_t} \sum_{j=1}^{2} \alpha_{i,j}, \tag{22}$$

where

$$\begin{aligned} \mathcal{A}(r) = \Bigg\{ & (\alpha_{1,1}, \cdots, \alpha_{n_t,2}) \; : \; \sum_{i=1}^{n_t-1} n_i \left[(1-\beta_i)^+ + \right. \\ & \left. \max\left\{\left(1 - \alpha_{i+1,1} - (1-\beta_i)^+\right)^+, (1-\alpha_{i+1,2})^+\right\}\right] \\ & \leq Nr, \text{ and } \alpha_{i,j} \geq 0 \Bigg\}. \end{aligned} \tag{23}$$

While the optimization of $d(r)$ subject to the constraint (23) appears to be a non-linear optimization problem, below we will convert it to a problem of linear programming. First note that for each $\alpha_{i,j}$, the probability of $\alpha_{i,j} < 0$ is zero. Secondly, to minimize the diversity gain $d(r)$, we do not need $\alpha_{i,j}$ to be larger than 1 as $(1-\alpha_{i,j})^+ = 0$ for $\alpha_{i,j} \geq 1$ and setting $\alpha_{i,j} = 1$ minimizes the cost of $d(r)$. Thus, we have the following sets of linear constraints:

$$0 \leq \alpha_{i,j} \leq 1 \quad \text{for all } i = 1, \cdots, n_t - 1, \; j = 1, 2. \tag{24}$$

Next, for $i = 1, 2, \cdots, n_t - 1$, setting

$$(1-\beta_i)^+ = \max\{(1-\alpha_{i,1})^+, (1-\alpha_{i,2})^+\} := r_{i,1} \tag{25}$$

yields the following linear constraints:

$$\alpha_{i,1} \geq 1 - r_{i,1} \tag{26}$$

$$\alpha_{i,2} \geq 1 - r_{i,1} \tag{27}$$

$$1 \geq r_{i,1} \geq 0. \tag{28}$$

Again, for $i = 1, 2, \cdots, n_t - 1$, setting

$$\max\left\{\left(1 - \alpha_{i+1,1} - (1-\beta_i)^+\right)^+, (1-\alpha_{i+1,2})^+\right\} = r_{i,2} \tag{29}$$

gives the following linear constraints:

$$\alpha_{i+1,1} \geq 1 - r_{i,1} - r_{i,2} \tag{30}$$

$$\alpha_{i+1,2} \geq 1 - r_{i,2} \tag{31}$$

$$1 \geq r_{i+1,2} \geq 0. \tag{32}$$

To achieve the desired multiplexing gain r, the linear constraint on the $r_{i,j}$ is given by

$$\sum_{i=1}^{n_t-1} n_i \left(r_{i,1} + r_{i,2}\right) \leq Nr.. \tag{33}$$

Using standard linear programming techniques to minimize $d(r)$ of (22) subject to the constraints of (24), (26), (27), (28), (30), (31), (32), and (33), it can be shown that

1) for the scheme $\mathcal{S}_1$, i.e. $n_i = 2$ for all i, we have $N = 2(n_t - 1)$ and

$$d(r) \geq (n_t - 1)(2 - r) \text{ and } d(r) \geq 2n_t - 2(n_t - 1)r. \tag{34}$$

Hence for the region of $1 \leq r \leq 2$, the DMT achieved by $\mathcal{S}_1$ is given by

$$\begin{aligned} d(r) & \geq \max\{(n_t-1)(2-r), 2n_t - 2(n_t-1)r\} \\ & = (n_t - 1)(2 - r), \quad \text{for } 1 \leq r \leq 2. \end{aligned} \tag{35}$$

2) for the scheme $\mathcal{S}_2$, i.e. the case when $n_1 = 4$, $n_{n_t-1} = 4$ and the remaining $n_i = 2$, we have $N = 2n_t + 2$, and

$$d(r) \geq 2n_t - (n_t + 1)r \text{ and } 2d(r) \geq (n_t + 1)(2 - r). \tag{36}$$

Thus for the region of $0 \leq r \leq 1$, the DMT achieved by scheme $\mathcal{S}_2$ is given by

$$d(r) \geq \max\left\{2n_t - (n_t + 1)r, \frac{n_t + 1}{2}(2 - r)\right\}$$

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$$= \quad 2n_t - (n_t+1)r, \quad \text{for } 0 \le r \le 1. \qquad (37)$$

The proof is now complete after noting that the DMTs (36) and (37) achieved respectively by schemes $\mathcal{S}_1$ and $\mathcal{S}_2$ in the region of $r \in [1,2]$ and $r \in [0,1)$ match exactly the optimal DMT $d^*(r)$ given in (6). ■

In Fig. 2 we have provided the exact DMT performances of the transmission schemes $\mathcal{S}_1$ and $\mathcal{S}_2$ proposed in Theorem 1 for the (4×2) constrained asymmetric MIMO system. It can be easily seen that the schemes are DMT optimal and achieves the optimal DMT $d^*(r)$ of (6) within the designated regions.

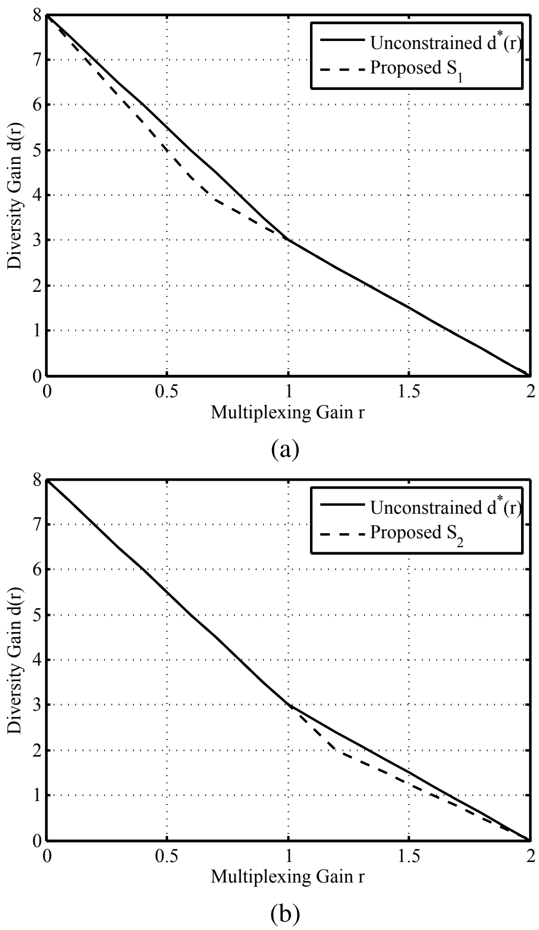

Fig. 2. DMT performances of (a) the proposed scheme $\mathcal{S}_1$ and (b) the proposed scheme $\mathcal{S}_2$ for the (4×2) constrained asymmetric MIMO system.

III. DMT Optimal Codes for Constrained Asymmetric MIMO Systems

In Section II, we have identified the DMT optimal transmission schemes for the constrained asymmetric MIMO systems with $n_t > n_r = 2$. To achieve this optimal DMT performance, it turns out that the multi-block space-time codes [9] that are originally designed to achieve the optimal DMT in multi-block fading channel can be modified to cater to the constrained channel. Due to limited space, below we provide without proof the construction of DMT optimal constrained codes for the case when $n_t > n_r = 2$.

Theorem 2: Given $n_t > n_r = 2$ and the desired multiplexing gain r, let $\mathcal{S}$ be the DMT optimal transmission scheme specified in Theorem 1. Let $\mathcal{C}$ be a $(2 \times 2L)$ multi-block space-time code given in [9] with number of coded fading blocks $L = n_t - 1$ if $r \in [1,2]$ and $L = n_t + 1$ if $r \in [0,1)$. For any code matrix $C \in \mathcal{C}$, transmit C according to the transmission scheme $\mathcal{S}$. Then the codeword error probability achieved by $\mathcal{C}$ is

$$P_{\text{cwe}}(\text{SNR}) \doteq \text{SNR}^{-d^*(r)}, \qquad (38)$$

meaning that the constrained code $\mathcal{C}$ is DMT optimal. ■

IV. Conclusion

When the number of transmit antennas n_t is strictly larger than the number of receive n_r, all the currently available DMT optimal codes require that all the transmit antennas are active during transmission, hence forbid the possibility of having a ZF, sphere, or MMSE decoder. To remedy this, the number of active transmit antennas must be constrained to be less than or equal to n_r. When $n_r = 2$ and for all $n_t > 2$, an optimal transmission scheme satisfying the above constraint was presented in this paper and was shown to achieve the same DMT performance as the unconstrained. A systematic construction of DMT optimal constrained codes was also provided.

Acknowledgment

This research was supported by Taiwan National Science Council under Grants NSC 97-2219-E-009-014 and NSC 96-2628-E-009-172-MY3.

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This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE "GLOBECOM" 2008 proceedings.