ELSEVIER

An inventory model with deteriorating items under inflation when a delay in payment is permissible

Hung-Chang Liao[a], Chih-Hung Tsai[b], Chao-Ton Su[c],*

[a]*Department of Industrial Engineering and Management, Oriental Institute of Technology, Panchiao, Taipei, Taiwan, Republic of China*
[b]*Department of Industrial Engineering and Management, Ta Hwa Institute of Technology, Hsinchu, Taiwan, Republic of China*
[c]*Department of Industrial Engineering and Management, National Chiao Tung University, 1001 Ta Hsach Road, Hsinchu, Taiwan, Republic of China*

Abstract

This study develops an inventory model for initial-stock-dependent consumption rate when a delay in payment is permissible. In the inventory model, shortages are not allowed. The effect of the inflation rate, deterioration rate, initial-stock-dependent consumption rate and delay in payment are discussed. In the study, mathematical models are also derived under two different circumstances, i.e., Case I: The credit period is less than or equal to the cycle time for settling the account; and Case II: The credit period is greater than the cycle time for settling the account. Besides, expressions for an inventory system's total cost are derived for these two cases. Moreover, a computational procedure and GINO (Lasdon et al., ACM Transactions Mathematical Software 4 (1978) 34–50) are proposed to obtain the optimal order size and cycle time. The results can help managers determine the optimal total cost. Finally, a numerical example demonstrates the applicability of the proposed model. © 2000 Elsevier Science B.V. All rights reserved.

1. Introduction

In the conventional EOQ model, it is implicit that the customer must pay for the items as soon as the items are received. In practice, however, suppliers offer their customers a certain credit period without interest during the permissible delay time period. Allowing a delay in payment to the supplier is a form of price discount. Such a convenience is likely to motivate customers to order more quantities because paying later indirectly reduces the purchase cost. On the other hand, a decaying item such as photographic film, electronic item and fruit gradually loses its potential. When a price increase is anticipated, companies may purchase large amounts of items without considering related costs. However, ordering large quantities would not be economical if the items in the inventory system deteriorate and the demand depends on the stock level. Therefore, in this study, we develop an inventory model under inflation for stock-dependent

*Corresponding author. Tel.: + 886-3-573-1857; fax: + 886-3-572-2392.

E-mail address: ctsu@cc.nctu.edu.tw (C.-T. Su)

consumption rate and deterioration of items when delay in payment is permissible.

The area of permissible delay in payments has received some attention. Davis and Gaither [1] developed EOQ models for firms offering a one time opportunity to delay payments by their suppliers for the order of a commodity. Goyal [2] developed mathematical models for determining the EOQ under conditions of a permissible delay in payments. Shah et al. [3] studied the same model when a delay in payments of order and shortages are permitted. Mandal and Phaujdar [4] studied the same situation by considering the interest earned from the sales revenue. Shah [5] extended an EOQ model in which delays in payment are permissible and items in inventory deteriorate at a constant rate over time. Shah [6] also developed a probabilistic time-scheduling model for an exponentially decaying inventory when payment delays are permissible. Aggarwal and Jaggi [7] developed ordering policies of deteriorating items under permissible delay in payments. Shah and Sreehari [8] developed an EOQ model when the delay in payment is permitted and the capacity of own warehouse is limited. Jammal et al. [9] extended Aggarwal and Jaggi's model with allowable shortage.

Several studies have examined the inflationary effect on an inventory policy. Buzacott [10] developed an approach of modeling inflation by assuming a constant inflation rate. Misra [11] proposed an inflation model for the EOQ, in which the time value of money and different inflation rates were considered. Mangianeli et al. [12] not only reviewed and classified models appearing in previous literature, but also presented some examples with relaxed assumptions. Brahmbhatt [13] also developed an EOQ model under a variable inflation rate and marked-up prices. Later, Hwang and Sohn [14] developed a deterministic inventory model for items that deteriorate continuously and follow an exponential distribution when a price increase is anticipated. Gupta and Vrat [15] developed a multi-item inventory model for a resource constraint system under a variable inflation rate.

Other investigators have described inventory policies for decaying items. Ghare and Schrader [16] first analyzed the decaying inventory problem,

and also developed a relatively simple economic order quantity model with a constant decay rate. Covert and Philip [17] derived a revised form of the EOQ model under the assumption of Weibull distribution for deterioration. Also, Cohen [18] formulated and solved an inventory model by simultaneously considering pricing and ordering policies for exponentially decaying inventory. Dave [19] proposed a deterministic inventory model in continuous units and discrete time for deteriorating items.

Many studies have modified inventory policies by considering the "stock dependent consumption rate". Gupta and Vrat [20] considered this phenomenon by using the following relation:

$$\lambda = \alpha + \beta Q^\tau, \ \lambda = \alpha - \beta Q^\tau, \ \lambda = \alpha + \beta e^Q, \ \lambda = \alpha - \beta e^Q,$$

where α, β, τ are positive constants (τ is initial-stock-dependent consumption rate parameter; and Q is order size). Their calculation of the average system cost was based on order size rather than inventory level. Mandal and Phaujdar [21] suggested that the demand rate depends on the current stock level. Mandal and Phaujdar [22] also developed a model for deteriorating items with a stock-dependent consumption rate. In 1988, Baker and Urban [23] developed an inventory model with an inventory level-dependent demand rate. In addition, Vrat and Padmanabhan [24] developed an inventory model under a constant inflation rate for initial-stock-dependent consumption rate. Padmanabhan and Vrat [25] proposed an EOQ model for items with initial-stock-dependent consumption rate and exponential decay. Moreover, Datta and Pal [26] studied the inventory problems for deteriorating items with inventory level-dependent demand rate and shortages. Pal et al. [27] developed a deterministic inventory model for deteriorating items with stock-dependent demand rate. Urban [28] developed inventory models with the demand rate dependent on stock and shortage levels. The model includes the effect of a stock-dependent demand rate, considering both initial-stock-dependent demand and instantaneous-stock-dependent demand rate. Karabi et al. [29] developed an inventory model with two-component demand rate and shortages. Su et al. [30] developed an inventory model under inflation

for initial-stock dependent consumption rate and exponential decay.

This study develops an inventory model for initial-stock-dependent consumption rate when a delay in payment is permissible. Shortages are not allowed and the effect of the inflation rate, deterioration rate, initial-stock-dependent consumption rate and delay in payment are discussed. Mathematical models are also derived under two different circumstances, i.e.,

Case I: The credit period is less than or equal to the cycle time for settling the account and

Case II: The credit period is greater than the cycle time for settling the account.

Also, expressions for an inventory system's total cost are derived for the above two cases. Moreover, a computational procedure and GINO [31] are proposed to obtain the optimal ordering size and cycle time. Those results help the decision-makers in accurately determining the optimal total cost. Finally, a numerical example demonstrates the applicability of the proposed model.

2. Notations and assumptions

The notations adopted in this paper are as follows:

H length of planning horizon
T cycle time
I inventory level
I_t inventory level at time t
Q order size
k constant rate of inflation (\$/\$/unit time)
$C(t)$ unit purchase cost for an item bought at time t. That is, $C(t) = C_0 e^{kt}$ where C_0 is the unit price at time zero
$A(t)$ ordering cost for an order placed at time t. That is, $A(t) = A_0 e^{kt}$, where A_0 is the ordering cost at time zero
i_i inventory holding cost per unit per year excluding interest charges
θ a constant fraction of the on-hand inventory which deteriorates per unit time
i_e Annual interest that can be earned per unit
i_c Annual interest charges payable per unit (Note: We generally have $i_c > i_e$)

M permissible delay period for settling accounts
Q_1^* the optimal order size in Case I
Q_2^* the optimal order size in Case II
T_1^* the optimal cycle time in Case I
T_2^* the optimal cycle time in Case II

The following assumptions are made:

(1) The unit price is subject to the same inflation rate as other inventory related costs, thereby implying that the ordering size can be determined by minimizing the total cost over a planning period.
(2) The inflation rate is constant.
(3) The replenishment rate is infinite, i.e., the replenishment is instantaneous.
(4) Backlogging is not allowed.
(5) Lead time is zero.
(6) The demand rate is known and constant.
(7) Initial-stock-dependent consumption rate is assumed, in which the demand rate depends on the order size and follows the function $\lambda = \alpha + \beta Q^\tau$, where α, β, τ are positive constants and Q is order size ($Q \neq 0$).
(8) The inventory carrying charge is a constant.
(9) A constant fraction, θ, of the on-hand inventory which deteriorates per unit time and there is no repair or replenishment of the deteriorated inventory during a cycle time T.
(10) During the fixed credit period M, a deposit is made of the unit cost of generated sales revenue into an interest bearing account. The daily expenses of the system can be met by retaining the difference between retail price and unit cost. At the end of the credit period, the account is settled and interest charges are payable on the account in stock.

The following considers the period in which accounts of the purchased quantities are not settled. The generated sales revenue is deposited in an interest bearing account which earns an annual interest at a rate of i_e per unit. After the account is settled, the system starts remitting interest charges on the outstanding amount in inventory at a rate of i_c per unit. Only the unit cost from the generated revenue is assumed herein to be deposited in an interest bearing account. Consequently, the system

can retain the difference between the retail price and unit cost to meet daily expenses.

3. Model development

Assume that $H = mT$, where m is an integer for the number of replenishments to be made during period H, and T is a constant interval of time between replenishments. The change in the inventory level during an infinitesimal time is a function of the deterioration rate, demand rate and inventory level. Thus, we have

$$- \mathrm{d}I = I_t\theta \, \mathrm{d}t + \lambda \, \mathrm{d}t. \tag{1}$$

After adjusting the constant of integration, the solution of Eq. (1) is

$$I_t = \frac{\lambda}{\theta} (\mathrm{e}^{\theta(T - t)} - 1), \quad 0 \leqslant t \leqslant T. \tag{2}$$

Consequently, initial inventory after replenishment becomes

$$I_0 = Q = \frac{\lambda}{\theta} (\mathrm{e}^{\theta T} - 1). \tag{3}$$

Since the inventory model considers delay in payment effect, there are two distinct types of cases in inventory system.

Case I: $M \leqslant T$

Let $A(t)$ and $C(t)$ denote the replenishment cost and unit purchasing cost at time t, respectively.

Then replenishment cost in $(0, H)$ is

$$C_r = A(0) + A(T) + A(2T) + \cdots + A((m - 1)T)$$

$$= A_0\left(\frac{\mathrm{e}^{kH} - 1}{\mathrm{e}^{kT} - 1}\right), \tag{4}$$

and purchasing cost in $(0, H)$ is

$$C_p = Q[C(0) + C(T) + C(2T) + \cdots + C((m - 1)T)]$$

$$= QC_0\left(\frac{\mathrm{e}^{kH} - 1}{\mathrm{e}^{kT} - 1}\right). \tag{5}$$

For inventory carrying cost, let $I(t)$ be the inventory level at time t. Since $Q = (\lambda/\theta)(\mathrm{e}^{\theta T} - 1)$, we have

$$I(nT + t) = \frac{\lambda}{\theta} (\mathrm{e}^{\theta(T - t)} - 1), \quad 0 \leqslant t \leqslant T \tag{6}$$

and holding cost in $(0, H)$ is

$$C_c = i_i \sum_{n=0}^{m-1} C(nT) \int_0^T I(nT + t) \, \mathrm{d}t$$

$$= i_i C_0 \sum_{n=0}^{m-1} \mathrm{e}^{knT} \int_0^T \frac{\lambda}{\theta} (\mathrm{e}^{(\theta T - t)} - 1) \, \mathrm{d}t$$

$$= \frac{\lambda i_i C_0}{\theta^2} (\mathrm{e}^{\theta T} - \theta T - 1)\left(\frac{\mathrm{e}^{kH} - 1}{\mathrm{e}^{kT} - 1}\right). \tag{7}$$

Interest charged for the inventory not being sold after the due date M in $(0, H)$ is

$$C_i = i_c \sum_{n=0}^{m-1} C(nT) \int_M^T I(nT + t) \, \mathrm{d}t$$

$$= \frac{\lambda i_c C_0}{\theta}\left[(M - T) + \frac{1}{\theta} (\mathrm{e}^{\theta(T - M)} - 1)\left(\frac{\mathrm{e}^{kH} - 1}{\mathrm{e}^{kT} - 1}\right)\right], \tag{8}$$

and interest earned in $(0, H)$ is

$$C_{e1} = i_e \sum_{n=0}^{m-1} C(nT) \int_0^M I(nT + t) \, \mathrm{d}t$$

$$= \frac{\lambda i_e C_0 M^2}{\theta^2}\left(\frac{\mathrm{e}^{kH} - 1}{\mathrm{e}^{kT} - 1}\right). \tag{9}$$

From Eqs. (4), (5), (7)–(9), the total system cost over $(0, H)$ is

$$TC_1(H, T) = C_r + C_p + C_c + C_i - C_{e1}$$

$$= \left\{A_0 + QC_0 + \frac{\lambda i_i C_0}{\theta^2} (\mathrm{e}^{\theta T} - \theta T - 1)\right.$$

$$+ \frac{\lambda i_c C_0}{\theta}\left[(M - T) + \frac{1}{\theta} (\mathrm{e}^{\theta(T - M)} - 1)\right]$$

$$\left. - \frac{\lambda i_e C_0 M^2}{2}\right\}\left(\frac{\mathrm{e}^{kH} - 1}{\mathrm{e}^{kT} - 1}\right). \tag{10}$$

Case II: $M > T$

No interest charged in $(0, H)$ because the supplier can be paid in full at the permissible delay and interest earned in $(0, H)$ is

$$C_{e2} = i_e \sum_{n=0}^{m-1} C(nT)\left[\int_0^T \lambda \, \mathrm{d}t + (M - T) \int_0^T \lambda \, \mathrm{d}t\right]$$

$$= i_e C_0 \lambda (TM - \tfrac{1}{2}T^2)\left(\frac{\mathrm{e}^{kH} - 1}{\mathrm{e}^{kT} - 1}\right). \tag{11}$$

From Eqs. (4), (5), (7) and (11), the total cost over $(0, H)$ is

$$TC_2(H, T) = C_r + C_p + C_c - C_{e2}$$
$$= \left\{ A_0 + QC_0 + \frac{\lambda i_i C_0}{\theta^2} (e^{\theta T} - \theta T - 1) \right.$$
$$\left. - i_e C_0 \lambda (TM - \tfrac{1}{2}T^2) \right\} \left(\frac{e^{kH} - 1}{e^{kT} - 1} \right).$$
(12)

Substituting the quadratic approximation of

$$e^{\theta T} = 1 + \theta T + (\theta T)^2 / 2,$$

$$e^{\theta(T-M)} = 1 + \theta(T - M) + \frac{[\theta(T - M)]^2}{2},$$

$$e^{kT} = 1 + kT + \frac{(kT)^2}{2}, \quad \text{and}$$

$$Q = \frac{\lambda}{\theta} (e^{\theta T} - 1)$$

in Eqs. (10) and (12) yield

$$TC_1(H, Q)$$
$$= \left[A_0 + QC_0 + \frac{i_i C_0 QT}{2 + \theta T} + \frac{QC_0 i_c (T - M)^2}{T(2 + \theta T)} \right.$$
$$\left. - \frac{C_0 Q i_e M^2}{T(2 + \theta T)} \right] \left(\frac{e^{kH} - 1}{kT + (k^2 T^2/2)} \right),$$
(13)

$$TC_2(H, Q)$$
$$= \left[A_0 + QC_0 + \frac{i_i C_0 QT}{2 + \theta T} - \frac{QC_0 i_e (2M - T)}{2 + \theta T} \right.$$
$$\left. \times \left(\frac{e^{kH} - 1}{kT + (k^2 T^2/2)} \right) \right].$$
(14)

In the proposed model, consumption rate is assumed to follow the relationship $\lambda = \alpha + \beta Q^\tau$. Incorporating this into Eqs. (13) and (14) yields

$$TC_1(H, Q)$$
$$= \left[A_0 + QC_0 + \frac{i_i C_0 QN(Q)}{2 + \theta N(Q)} + \frac{QC_0 i_c (N(Q) - M)^2}{N(Q)(2 + \theta N(Q))} \right.$$
$$\left. - \frac{C_0 Q i_e M^2}{N(Q)(2 + \theta N(Q))} \right] \left(\frac{e^{kH} - 1}{kN(Q) + (k^2 N(Q)^2/2)} \right)$$
(15)

and

$$TC_2(H, Q)$$
$$= \left[A_0 + QC_0 + \frac{i_i C_0 QN(Q)}{2 + \theta N(Q)} - \frac{QC_0 i_e (2M - N(Q))}{2 + \theta N(Q)} \right.$$
$$\left. \times \left(\frac{e^{kH} - 1}{kN(Q) + (k^2 N(Q)^2/2)} \right) \right],$$
(16)

where

$$T = N(Q) = \frac{-1 + \sqrt{1 + (2Q\theta/(\alpha + \beta Q^\tau))}}{\theta}.$$
(17)

4. Special cases

Case (i): When $M = 0$, $i_e = 0$ and $i = i_i + i_c$, Eqs. (15) and (16) reduce to Su et al. [30]. Setting $(dTC_1(H, Q))/dQ = 0$ and $(dTC_2(H, Q))/dQ = 0$, the optimal value of Q becomes

$$C_0 + C_0 i * \frac{QM(Q)W(Q) + M(Q)N(Q) - Q\theta N(Q)W(Q)}{\{M(Q)\}^2}$$
$$- \frac{(A_0 + QC_0 + QiC_0 N(Q)/M(Q))(W(Q) + kN(Q)W(Q))}{N(Q) + k/2\{N(Q)\}^2}$$
$$= 0,$$
(18)

$$M(Q) = 1 + \sqrt{1 + \frac{2Q\theta}{\alpha + \beta Q^\tau}},$$
(19)

$$W(Q) = \frac{(\alpha + \beta Q^\tau) - \tau \beta Q^\tau}{(\alpha + \beta Q^\tau)^2 \sqrt{1 + (2Q\theta/(\alpha + \beta Q^\tau))}}.$$
(20)

Case (ii): When $M = 0$, $i_e = 0$, $i = i_i + i_c$, and $\theta = 0$, Eqs. (15) and (16) reduce to Vrat and Padmanabhan [24]. Setting $(dTC_1(H, Q))/dQ = 0$ and $(dTC_2(H, Q))/dQ = 0$, the optimal value of Q becomes

$$\frac{1}{2(\alpha + \beta Q^\tau) + kQ}$$
$$* \left[\frac{k\beta \tau Q^\tau}{\alpha + \beta Q^\tau} \left(\frac{Q^2 C_0 i + 4(\alpha + \beta Q^\tau)(A_0 + QC_0)}{2(\alpha + \beta Q^\tau)Q} \right) \right.$$
$$+ QC_0 i - \frac{2(A_0 + QC_0)(\alpha + \beta Q^\tau(1 - \tau) + kQ)}{Q} \right]$$
$$+ C_0 = 0.$$
(21)

Case (iii): When $M = 0$, $i_e = 0$, $i = i_i + i_c$, $k = 0$ and $\theta = 0$, Eqs. (15) and (16) reduce to Gupta and Vrat [20]. Setting $(\mathrm{d}TC_1(H, Q))/\mathrm{d}Q = 0$ and $(\mathrm{d}TC_2(H, Q))/\mathrm{d}Q = 0$, the optimal value of Q becomes

$$Q^2 C_0 i - 2A_0 \alpha + 2A_0 \beta Q^\tau(\tau - 1) + 2\beta\tau C_0 Q^{\tau+1} = 0. \tag{22}$$

Case (iv): When $M = 0$, $i_e = 0$, $i = i_i + i_c$, $\theta = 0$ and $\beta = 0$, i.e., $\lambda = \alpha$, Eqs. (15) and (16) reduce to Buzacott [10]. Setting $(\mathrm{d}TC_1(H, Q))/\mathrm{d}Q = 0$ and $(\mathrm{d}TC_2(H, Q))/\mathrm{d}Q = 0$, the optimal value of Q becomes

$$Q^2 C_0 i - 2\alpha A_0 - k(2Q A_0 + Q^2 C_0) = 0. \tag{23}$$

Case (v): When $M = 0$, $i_e = 0$, $i = i_i + i_c$, $k = 0$, $\theta = 0$ and $\beta = 0$, Eqs. (15) and (16) reduce to Classical EOQ. Setting $(\mathrm{d}TC_1(H, Q))/\mathrm{d}Q = 0$ and $(\mathrm{d}TC_2(H, Q)/\mathrm{d}Q = 0$, the optimal value of Q becomes

$$Q = \sqrt{\frac{A\alpha}{C_0 i}}. \tag{24}$$

5. Computational procedure

Eqs. (15) and (16) are the total system cost for $M \leqslant T$ and $M > T$, respectively. Since these equations are nonlinear, the optimal value of Q can be found by using a commercial software GINO which utilizes a powerful CRG2 [31] optimizer. By doing so the total cost can be minimized. The optimal order size, cycle time and total cost can be obtained as follows:

Step 1: Determine Q_1^* from Eq. (15) and T_1^* from Eq. (17), if $T_1^* \geqslant M$, obtain $TC_1(H, Q_1^*)$ from Eq. (15).

Step 2: Determine Q_2^* from Eq. (16) and T_2^* from Eq. (17), if $T_2^* < M$, obtain $TC_2(H, Q_2^*)$ from Eq. (16).

Step 3: By comparing $TC_1(H, Q_1^*)$ and $TC_2(H, Q_2^*)$, select the order size and cycle time with the least total cost evaluated in Step 1, Step 2.

6. Numerical example

An IC packaging factory needs a chemical compound i.e., *molding compound*, to produce a 64-Mega DRAM. The molding compound consumption rate depends on initial-stock and is deteriorating due to inappropriate storage, temperature, or expiration date. There is a constant ordering interval; permissible delay time period in payments for the factory is 0.1, 0.4 and 0.7 year. It is assumed that the deterioration rate is constant. The following data are used:
$\alpha = 500$ units
$\beta = 1.00$
$\tau = 0.6$
$i_i = 0.18$ per year
$i_e = 0.09$ per year
$i_c = 0.11$ per year
$H = 1$ year
$C_0 = \$2.50$ per unit
$A_0 = \$7.50$ per unit
$k = \$0.1$ per unit per $\$$
$\theta = 0.1$

According to the proposed computational procedure, the results listed in Table 1 are obtained for $M = 0.1$, 0.4 and 0.7 year. This table reveals that an increase in the permissible delay causes both the order size and cycle time to increase. Moreover, the total cost is markedly reduced.

The effect of θ on the optimal order size, cycle time and total cost is not significant [7,9]. To illustrate the intersection effect M, k and τ, they are assumed 0.05, 0.1 or 0.15 (change in the same rate 50%). Meanwhile, the other parameter values follow those data mentioned above. By using the computational procedure, Table 2 displays the

Table 1
Results obtained from the example

	$M = 0.1$ year	$M = 0.4$ year	$M = 0.7$ year
Case	I	II	II
Ordering size	79.45	80.77	81.06
Cycle time	0.153	0.155	0.156
Total cost	1417.90	1381.44	1345.24

Table 2
Effects of M, τ and k on the optimal ordering size, cycle time and total cost

k	M	0.05			0.1			0.15		
	τ	0.05	0.1	0.15	0.05	0.1	0.15	0.05	0.1	0.15
0.05	Ordering size	95.35	95.08	94.69	95.91	95.65	95.25	96.87	96.61	96.22
	Cycle time	0.188	0.188	0.187	0.190	0.189	0.188	0.191	0.191	0.190
	Total cost	1359.72	1360.57	1361.63	1353.20	1354.04	1355.10	1347.00	1347.85	1348.90
0.1	Ordering size	103.82	103.49	103.01	104.38	104.06	103.58	105.37	105.05	104.57
	Cycle time	0.205	0.204	0.203	0.206	0.205	0.204	0.208	0.207	0.206
	Total cost	1387.80	1388.69	1389.81	1381.10	1381.99	1383.10	1374.72	1375.60	1376.71
0.15	Ordering size	114.88	114.47	113.86	115.42	115.01	114.40	116.43	116.03	115.43
	Cycle time	0.227	0.226	0.224	0.228	0.227	0.225	0.230	0.229	0.227
	Total cost	1416.14	1417.07	1418.25	1409.26	1410.19	1411.37	1402.68	1403.61	1404.79

optimal order size, cycle time and total cost. From above results, we can infer that:

1. When the parameter k increases and parameters M and τ remain unchanged, the optimal order size, cycle time and total cost increase;
2. When the parameter τ increases and parameters M and k remain unchanged, the optimal order size and cycle time decrease only slightly and the optimal total cost increases only slightly;
3. When the parameters τ and k increase and parameter M remains unchanged, the optimal order size, cycle time and total cost increase;
4. When the parameters M and k increase and parameter τ remains unchanged, the optimal order size, cycle time and total cost increase; and
5. When the parameters M and τ increase and parameter k remains unchanged; the optimal order size, cycle time and total cost decrease only slightly.

From Tables 1 and 2, we can know that the parameters M, k and τ affect the optimal order size, cycle time and total cost. In terms of the intersection effect, the parameter k is stronger than the parameters τ and M about the optimal order size, cycle time and total cost. The parameter τ is stronger than the parameter M about the optimal order size and cycle time. However, the parameter M is stronger than the parameter τ about the optimal total cost.

7. Concluding remarks

This study presents deterministic inventory models for initial-stock-dependent consumption rate under a situation in which the supplier offers a credit period to settle the account of a purchased quantity. Shortages are not allowed and the effect of the inflation rate, deterioration rate, initial-stock-dependent consumption rate and delay in payment are discussed as well. Five special cases are also presented. The proposed models can assist the manager in concisely determining the order size, cycle time and total cost. Numerical results indicate that an increase in the permissible delay causes both the order size and cycle time to increase. Moreover, the total cost is markedly reduced. The intuitive reason is that, when the permissible payment period increases, the purchaser earns more by investing the cash from the sales of inventory resulting in lower costs. Also, the purchaser tends to hold more inventory which extends the cycle time. The results of sensitivity analysis indicate that the parameters M and k are more sensitive toward the optimal total cost. That is, for the parameters M, k, θ and τ, the effects of delay in payment and inflation are strong. The proposed model can be used in inventory control of certain decaying items such as photographic film, electronic components, and radio active materials. A future study should further incorporate the

proposed model into more realistic assumptions, such as probabilistic demand, the demand which depends on the current stock and a finite rate of replenishment.

References

[1] R.A. Davis, N. Gaither, Optimal ordering policies under conditions of extended payment privileges, Management Science 31 (1985) 499–509.

[2] S.K. Goyal, EOQ under conditions of permissible delay in payments, Journal of Operation Research Society 36 (1985) 335–338.

[3] V.R. Shah, N.C. Patel, D.K. Shah, Economic ordering quantity when delay in payments of order and shortages are permitted, Gujarat Statistical Review 15 (2) (1988) 52–56.

[4] B.N. Mandal, S. Phaujdar, Some EOQ models under permissible delay in payments, International Journal of Management Science 5 (2) (1989) 99–108.

[5] N.H. Shah, Probabilistic time-scheduling model for an exponentially decaying inventory when delays in payments are permissible, International Journal of Production Economics 32 (1993) 77–82.

[6] N.H. Shah, A lot-size model for exponentially decaying inventory when delay in payments is permissible, Cahiers du CERO 35 (1993) 115–123.

[7] S.P. Aggarwal, C.K. Jaggi, Ordering policies of deteriorating items under permissible delay in payments, Journal of Operation Research Society 46 (1995) 658–662.

[8] V.R. Shah, M. Sreehari, An inventory model for a system with multiple storage facilities and conditional credit facility, Opsearch 33 (2) (1996) 96–106.

[9] A. Jamal, B. Sarker, S. Wang, An ordering policy for deteriorating items with allowable shortage and permissible delay in payment, Journal of Operation Research Society 48 (1997) 826–833.

[10] J.A. Buzacott, Economic order quantities with inflation, Operation Research Quarterly 26 (1975) 553–558.

[11] R.B. Misra, A note on optimal inventory management under inflation, Naval Research, Logistics Quarterly 26 (1979) 161–165.

[12] P.M. Mangiameli, J. Banks, H. Schwarzbach, Static inventory models and inflationary cost increases, The Engineering Economist 26 (1981) 91–112.

[13] A.C. Brahmbhatt, Economic order quantity under variable rate of inflation and mark-up prices, Productivity 23 (1982) 127–130.

[14] H. Hwang, K.I. Sohn, Management of deteriorating inventory under inflation, The Engineering Economist 28 (1983) 191–206.

[15] R. Gupta, P. Vrat, Inventory model with multiitems under constraint systems for stock dependent consumption rate, Proceedings of XIX Annual Convention of Operation Research Society of India 2 (1986) 579–609.

[16] P.M. Ghare, G.F. Schrader, A model for exponentially decaying inventories, Journal of Industrial Engineering 14 (1963) 234–238.

[17] R.P. Covert, G.C. Philip, A EOQ model for items with Weibull distribution deterioration, AIIE Transactions 5 (1973) 323–326.

[18] M.A. Cohen, Joint pricing and ordering policy for exponentially decaying inventory with known demand, Naval Research Logistics Quarterly 24 (1977) 257–268.

[19] U. Dave, On a discrete-in-time order-level inventory model for deteriorating items, Journal of Operation Research Society 30 (1979) 349–354.

[20] R. Gupta, P. Vrat, Inventory models for stock dependent consumption rate, Opsearch 23 (1986) 19–24.

[21] B.N. Mandal, S. Phaujdar, A note on an inventory model with stock-dependent consumption rate, Opsearch 26 (1989) 43–46.

[22] B.N. Mandal, S. Phaujdar, An inventory model for deteriorating items and stock-dependent consumption rate, Journal of Operation Research Society 40 (1989) 483–488.

[23] R.C. Baker, T.L. Urban, A deterministic inventory system with an inventory-level-dependent demand rate, Journal of Operation Research Society 39 (1988) 823–831.

[24] P. Vrat, G. Padmanabhan, An inventory model under inflation for stock dependent consumption rate items, Engineering Costs and Production Economics 19 (1990) 379–383.

[25] P. Vrat, G. Padmanabhan, An EOQ model for items with stock dependent consumption rate and exponential decay, Engineering Costs and Production Economics 18 (1990) 241–246.

[26] T.K. Datta, A.K. Pal, Deterministic inventory system for deteriorating items with inventory level-dependent demand rate and shortages, Opsearch 27 (1990) 213–224.

[27] S. Pal, A. Goswami, K.S. Chaudhuri, A deterministic inventory model for deteriorating items with stock-dependent demand rate, International Journal of Production Economics 32 (1993) 291–299.

[28] T.L. Urban, Inventory models with the demand rate dependent on stock and shortage levels, International Journal of Production Economics 40 (1995) 21–28.

[29] P. Karabi, T.K. Datta, K.S. Chaudhuri, A.K. Pal, An inventory model with two-component demand rate and shortages, Journal of Operation Research Society 47 (1996) 1029–1036.

[30] C.-T. Su, L.-T. Tong, H.-C. Liao, An inventory model under inflation for stock dependent consumption rate and exponential decay, Opsearch 33 (2) (1996) 71–82.

[31] L.S. Lasdon, A.D. Waren, A. Jain, M. Ratner, Design and testing of a generalized reduced gradient code for nonlinearprogramming, ACM Transactions Mathematical Software 4 (1978) 34–50.