Mechanics of Structures and Machines
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/lmbd19

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Published online: 26 Apr 2007.

To cite this article: S. D. WU , J. C. Chiou & Y. C. Lin (2000) Modified Adams-Moulton Predictor-Corrector Method in Solving Multibody Dynamical Systems, Mechanics of Structures and Machines, 28:2-3, 201-218

To link to this article: http://dx.doi.org/10.1081/SME-100100618

MECH. STRUCT. & MACH., 28(2&3), 201–218 (2000)

Modified Adams-Moulton Predictor-Corrector Method in Solving Multibody Dynamical Systems

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ABSTRACT

A modified Adams-Moulton predictor-corrector method is proposed to solve multibody dynamical systems. The proposed method is obtained by combining an Adams-Bashforth predictor method and an Adams-Moulton corrector method with derived weighting coefficients. With the modification, the accuracy of the proposed method is almost one order of magnitude better than the Adams-Moulton predictor-corrector method with the same step size. Stability limits of the proposed method are also studied. Because the present method has greater stability limits than Adams-Moulton predictor-corrector methods, the proposed method has good robustness during the process of time integration. A crank-slider mechanism is used as an example to investigate the capability of the proposed method in solving multibody dynamic systems.

*Communicated by E. J. Haug.

I. INTRODUCTION

Computer simulation of multibody dynamic (MBD) systems has been motivated by growing interest in the design and analysis of such engineering problems as robot manipulators, ground vehicles, and space vehicles. The dynamic equations of MBD systems can be derived and expressed in a set of differential algebraic equations (DAE). However, solution procedures for DAE suffer such drawbacks as constraint violation and numerical stiffness in computer implementation. These difficulties have motivated researchers to seek alternative solution procedures.

In solving MBD systems written in terms of DAE, a constraint violation stabilization technique in conjunction with a numerical integration method is often used. In the past two decades, several constraint violation stabilization techniques have been proposed to solve DAE. These include the stabilization technique of Baumgarte [1,2], the adaptive constraint violation stabilization method of Chang and Nikravesh [3], the penalty staggered stabilized procedure of Park and Chiou [4], the gradient feedback technique of Yoon et al. [5], and the input/output feedback linearization technique of Chiou and Wu [6]. Many numerical integration methods have also been proposed to solve DAEs. These include the midpoint rule [4], Runge-Kutta integration methods [7], symplectic integration methods [8], Adams-Moulton predictor-corrector methods [5,6], and the back difference formula [9]. Of the numerical integration methods cited, Adams-Moulton predictor-corrector (AMPC) methods are generally thought to be a good candidate for solving multibody dynamic systems, for the following reasons:

1. AMPC methods are "strongly stable" integration methods [10]. They can be used to solve most differential equations, in both ordinary differential equation (ODE) and DAE forms.
2. AMPC methods are multistep integration methods and therefore less computational time is expected during the process of time integration.

In this article, a new integration method based on AMPC methods is developed to solve MBD systems. The integration method, called the modified Adams-Moulton predictor-corrector (MAMPC) method, is a linear combination of the Adams-Bashforth predictor (ABP) and the Adams-Moulton corrector (AMC) methods, with newly derived weighting coefficients.

This article is organized as follows. Section II presents a review of the Adams-Bashforth and Adams-Moulton methods. Section III presents the proposed modified Adams-Moulton predictor-corrector method. Section IV gives an outline for applying the MAMPC method with the Baumgarte stabilization technique to solve MBD systems. In Section V, a numerical example is used to demonstrate the efficiency of the proposed method. The main contribution of this article is summarized in Section VI.

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II. ADAMS-BASHFORTH AND ADAMS-MOULTON METHODS

To begin the derivation of multistep methods, if the initial-value problem is integrated over the interval $[t_i, t_{i+1}]$, then

$$y(t_{i+1}) = y(t_i). + \int_{t_i}^{t_{i+1}} f(t,\ y(t))dt \tag{1}$$

where $f(t,\ y(t))$ is the first derivative of $y(t)$. To derive an Adams-Bashforth explicit m-step (ABP_m) method, Newton's backward difference formula with a set of equally spaced points, $t_{i+1-m}, \ldots, t_{i-1}, t_i$, is used to approximate the integral $\int_{t_i}^{t_{i+1}} f(t,\ y(t))dt$ as

$$\int_{t_i}^{t_{i+1}} f(t,\ y(t))dt = h\cdot \sum_{k=0}^{m-1} \left[\nabla^k\ f(t_i,\ y(t_i))\cdot(-1)^k \int_0^1 C_k^{-s}\ ds\right] + h^{m+1} f^{(m)}(\mu_i,\ y(\mu_i))(-1)^m \int_0^1 C_m^{-s} ds \tag{2}$$

where

$$C_k^{-s} = \frac{-s(-s-1)\cdots(-s-k+1)}{k!} \tag{3}$$

and $\nabla f(x_i)$ represents the backward difference operator that is defined by

$$\nabla f(x_i) = f(x_i) - f(x_{i-1}). \tag{4}$$

Higher orders of backward difference are defined recursively by

$$\nabla^k f(x_i) = \nabla(\nabla^{k-1} f(x_i)). \tag{5}$$

In contrast, the Adams-Moulton implicit $(m-1)$-step (AMC_$(m-1)$) method is derived by using the set of equally spaced points, $t_{i+2-m}, \ldots, t_i, t_{i+1}$, and the integral $\int_{t_i}^{t_{i+1}} f(t, y(t))dt$ is approximated by

$$\int_{t_i}^{t_{i+1}} f(t,\ y(t))dt = h\cdot \sum_{k=0}^{m-1} \left[\nabla^k f(t_{i+1},\ y(t_{i+1}))\cdot(-1)^k \int_0^1 C_k^{-s+1} ds\right] + h^{m+1} f^{(m)}(\xi_i,\ y(\xi_i))(-1)^m \int_0^1 C_m^{-s+1} ds \tag{6}$$

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where

$$C_k^{-s+1} = \frac{(-s+1)(-s)(-s-1)\cdots(-s-k+2)}{k!} \tag{7}$$

To simplify notation, $f(t_i, y(t_i))$ is defined to be $f(t_i)$. By using Eqs. (2) and (6), Adams-Bashforth m-step and Adams-Moulton $(m-1)$-step methods can be expressed as follows:

Adams-Bashforth *m*-step (ABP_*m*) method

$$\begin{aligned} y^{AB\_m}(t_{i+1}) &= y(t_i) + \int_i^{i-1} f(t, y(t))dt \\ &= y(t_i) + h \cdot \sum_{k=0}^{m-1} \left[\nabla^k f(t_i) \cdot (-1)^k \int_0^1 C_k^{-s} ds\right] \end{aligned} \tag{8}$$

Adams-Moulton (*m* – 1)-step (AMC_(*m* – 1)) method

$$\begin{aligned} y^{AM\_(m-1)}(t_{i+1}) &= y(t_i) + \int_i^{i+1} f(t, y(t))dt \\ &= y(t_i) + h \cdot \sum_{k=0}^{m-1} \left[\nabla^k f(t_{i+1}) \cdot (-1)^k \int_0^1 C_k^{-s+1} ds\right] \end{aligned} \tag{9}$$

Both integrals $(-1)^k \int_0^1 C_k^{-s}\, ds$ and $(-1)^k \int_0^1 C_k^{-s+1} ds$, for various values of k, are easily evaluated and are listed in Table 1 [10]. Note that the local truncation error for both ABP_m and AMC_$(m-1)$ methods are of mth order of the integration step size.

The typical types of ABP and AMC methods are listed in the appendix. By investigating the local truncation error of the ABP and AMC methods, it is found that ABP methods have positive errors and AMC methods have negative ones. The mth order local truncation error can be cancelled by combining the

TABLE 1

The values of $(-1)^k \int_0^1 C_k^{-s}\, ds$ and $(-1)^k \int_0^1 C_k^{-s+1}\, ds$ for different k.

k	0	1	2	3	4	5
$(-1)^k \int_0^1 C_k^{-s}\, ds$	1	$\frac{1}{2}$	$\frac{5}{12}$	$\frac{3}{8}$	$\frac{251}{720}$	$\frac{95}{288}$
$(-1)^k \int_0^1 C_k^{-s+1}\, ds$	1	$-\frac{1}{2}$	$-\frac{1}{12}$	$-\frac{1}{24}$	$-\frac{19}{720}$	$-\frac{3}{160}$

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ABP_m and AMC_$(m-1)$ methods with appropriate weighting. For example, third-order local truncation error can be cancelled by using the weighted operators, $1/10 \cdot y^{AB\_3} + (1 - 1/10) \cdot y^{AM\_2}$. One may notice that $1/10 \cdot y^{AB\_3} + (1 - 1/10) \cdot y^{AM\_2}$ is equal to y^{AM_3} by a simple calculation. Thus, the accuracy level of the AMPC method can be improved by using a set of derived parameters by combining the ABP and AMC methods. In the next section, the above mentioned concept will be proven.

III. MODIFIED ADAMS-MOULTON PREDICTOR-CORRECTOR METHOD

In order to show that higher-order AMPC methods can be generated by combining lower order ABP and AMC methods with derived weighting coefficients, the following propositions are given:

Proposition 1

$$\nabla^k f(t_{i+1}) = f(t_{i+1}) - \sum_{r=0}^{k-1} \nabla^r f(t_i) \tag{10}$$

Proof

By induction,

1. For $k = 1$, Eq. (10) holds.
2. Suppose for $k = n$, Eq. (10) holds; i.e.,

$$\nabla^n f(t_{i+1}) = f(t_{i+1}) - \sum_{r=0}^{n-1} \nabla^r f(t_i) \tag{11}$$

When $k = n + 1$,

$$\begin{aligned}
\nabla^{n+1} f(t_{i+1}) &= \nabla(\nabla^n f(t_{i+1})) \\
&= \nabla(f(t_{i+1}) - \sum_{r=0}^{n-1} \nabla^r f(t_i)) \\
&= f(t_{i+1}) - f(t_i) - \sum_{r=1}^{n} \nabla^r f(t_i) \\
&= f(t_{i+1}) - \sum_{r=0}^{n} \nabla^r f(t_i)
\end{aligned}$$

Q.E.D.

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Proposition 2

$$(-1)^k \int_0^1 C_k^{-s} ds - (-1)^k \int_0^1 C_k^{-s+1} ds = (-1)^{k-1} \int_0^1 C_{k-1}^{-s} ds \tag{12}$$

Proof

$$(-1)^k \int_0^1 C_k^{-s} ds = (-1)^k \int_0^1 \frac{-s(-s-1)(-s-2)\cdots(-s-k+1)}{k!}\, ds$$

$$= \int_0^1 \frac{s(s+1)(s+2)\cdots(s+k-1)}{k!}\, ds \tag{13}$$

$$(-1)^k \int_0^1 C_k^{-s+1} ds = (-1)^k \int_0^1 \frac{(-s+1)(-s)(-s-1)\cdots(-s-k+2)}{k!}\, ds$$

$$= \int_0^1 \frac{(s-1)s(s+1)\cdots(s+k-2)}{k!}\, ds$$

$$(-1)^k \int_0^1 C_k^{-s} ds - (-1)^k \int_0^1 C_k^{-s+1} ds$$

$$= \int_0^1 \frac{s(s+1)(s+2)\cdots(s+k-2)(s+k-1)}{k!}\, ds$$

$$- \int_0^1 \frac{(s-1)s(s+1)(s+2)\cdots(s+k-2)}{k!}\, ds$$

$$= \int_0^1 \frac{s(s+1)(s+2)\cdots(s+k-2)}{(k-1)!}\, ds$$

$$= (-1)^{k-1} \int_0^1 C_{k-1}^{-s}\, ds \tag{14}$$

Q.E.D.

Proposition 3

$$\sum_{r=0}^{k} (-1)^r \int_0^1 C_r^{-s+1} ds = (-1)^k \int_0^1 C_k^{-s}\, ds \tag{15}$$

Proof

By induction,

1. $k = 0$ arbitrary.
2. When $k = 1$,

$$1 + (-1)\int_0^1 C_1^{-s+1} ds = \frac{1}{2} = (-1)\int_0^1 C_1^{-s} ds \tag{16}$$

3. Suppose for $k = n$, Eq. (15) holds; i.e.,

$$\sum_{r=0}^{n} (-1)^r \int_0^1 C_r^{-s+1} ds = (-1)^n \int_0^1 C_n^{-s} ds$$

When $k = n + 1$,

$$\begin{aligned}\sum_{r=0}^{n+1} (-1)^r \int_0^1 C_l^{-s+1} ds &= \sum_{r=0}^{n} (-1)^r \int_0^1 C_r^{-s+1} ds + (-1)^{n+1} \int_0^1 C_{n+1}^{-s+1} ds \\ &= (-1)^n \int_0^1 C_n^{-s} ds + (-1)^{n+1} \int_0^1 C_{n+1}^{-s+1} ds \\ &= (-1)^{n+1} \int_0^1 C_{n+1}^{-s} ds\end{aligned}$$

Q.E.D.

Note that the ABP_m step method is given by

$$\begin{aligned}y^{AB\_m}(t_{i+1}) &= y(t_i) + \int_i^{i+1} f(t,\ y(t))dt \\ &= y(t_i) + h \cdot \sum_{k=0}^{m-1} \left[\nabla^k f(t_i) \cdot (-1)^k \int_0^1 C_k^{-s} ds\right]\end{aligned} \tag{17}$$

whereas the AMC_$(m-1)$ step method is given as

$$\begin{aligned}y^{AM\_(m-1)}(t_{i+1}) &= y(t_i) + \int_i^{i+1} f(t,\ y(t))dt \\ &= y(t_i) + h \cdot \sum_{k=0}^{m-1} \left[\nabla^k f(t_{i+1}) \cdot (-1)^k \int_0^1 C_k^{-s+1} ds\right]\end{aligned} \tag{18}$$

$$\begin{aligned}y^{AM\_(m-1)}(t_{i+1}) = y(t_i) &+ hf(t_{i+1})(-1)^{m-1} \int_0^1 C_{m-1}^{-s} ds \\ &+ h \cdot \sum_{k=0}^{m-1} \left\{\nabla^k f(t_i) \cdot \left[-(-1)^{m-1} \int_0^1 C_{m-1}^{-s} ds + (-1)^k \int_0^1 C_k^{-s} ds\right]\right\}\end{aligned} \tag{19}$$

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To establish a relationship between the ABP_m and AMC_$(m-1)$ step methods, the following theorem is proposed and proved.

Theorem 1

$$\frac{W_1}{W_1+W_2}\,y^{AB\_m}(t_{i+1})+\frac{W_2}{W_1+W_2}\,y^{AM\_(m-1)}(t_{i+1})=y^{AM\_m}(t_{i+1}) \tag{20}$$

where

$$W_1=-(-1)^m\int_0^1 C_m^{-s+1}ds$$

$$W_2=(-1)^m\int_0^1 C_m^{-s}ds$$

$$W_1+W_2=(-1)^{m-1}\int_0^1 C_{m-1}^{-s}ds \tag{21}$$

Proof

Define

$$\theta(t_{i+1})\equiv\frac{W_1}{W_1+W_2}\,y^{AB\_m}(t_{i+1})+\frac{W_2}{W_1+W_2}\,y^{AM\_(m-1)}(t_{i+1}) \tag{22}$$

Evaluating the coefficient of $y(t_i)$, $h\cdot f(t_{i+1})$, and $h\cdot\nabla^k f(t_i)$ in $\theta(t_{i+1})$

$$y(t_i):\quad \frac{W_1}{W_1+W_2}\cdot 1+\frac{W_2}{W_1+W_2}\cdot 1=1 \tag{23}$$

$$h\cdot f(t_{i+1}):\quad \frac{W_2}{W_1+W_2}\cdot(-1)^{m-1}\int_0^1 C_{m-1}^{-s}ds=(-1)^m\int_0^1 C_m^{-s}ds \tag{24}$$

$$\begin{aligned}h\cdot\nabla^k f(t_i):\quad &\frac{W_1}{W_1+W_2}\cdot(-1)^k\int_0^1 C_k^{-s}ds+\frac{W_2}{W_1+W_2}\cdot\left[-(-1)^{m-1}\int_0^1 C_{m-1}^{-s}ds\right.\\ &\left.+(-1)^k\int_0^1 C_k^{-s}ds\right]=-(-1)^m\int_0^1 C_m^{-s}ds+(-1)^k\int_0^1 C_k^{-s}ds\end{aligned} \tag{25}$$

$$\begin{aligned}\theta(t_{i+1})&=y(t_i)+hf(t_{i+1})(-1)^m\int_0^1 C_m^{-s}ds+h\\ &\quad\cdot\sum_{k=0}^{m-1}\left\{\nabla^k f(t_i)\cdot\left[-(-1)^m\int_0^1 C_m^{-s}ds+(-1)^k\int_0^1 C_k^{-s}ds\right]\right\}\\ &=y(t_i)+hf(t_{i+1})(-1)^m\int_0^1 C_m^{-s}ds+h\\ &\quad\cdot\sum_{k=0}^{m}\left\{\nabla^k f(t_i)\cdot\left[-(-1)^m\int_0^1 C_m^{-s}ds+(-1)^k\int_0^1 C_k^{-s}ds\right]\right\}\\ &\quad\left(\because\nabla^m f(t_i)\cdot\left[-(-1)^m\int_0^1 C_m^{-s}ds+(-1)^m\int_0^1 C_m^{-s}ds\right]=0\right)\end{aligned} \tag{26}$$

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$\theta(t_{i+1})$ is the formulation of the AMC_m step method.
Q.E.D.

By using Theorem 1, the MAMPC method is as follows:

$$y^{AB\_m}(t_{i+1}) = y(t_i) + h\cdot \sum_{k=0}^{m-1}\left[\nabla^k f(t_i)\cdot(-1)^k \int_0^1 C_k^{-s} ds\right] \tag{27a}$$

$$y^{AM\_(m-1)}(t_{i+1}) = y(t_i) + hf(t_{i+1}, y^p(t_{i+1}))(-1)^{m-1}\int_0^1 C_{m-1}^{-s} ds + h\cdot \sum_{k=0}^{m-1}\left\{\nabla^k f(t_i)\cdot\left[-(-1)^{m-1}\int_0^1 C_{m-1}^{-s} ds + (-1)^k \int_0^1 C_k^{-s} ds\right]\right\} \tag{27b}$$

$$y^{MAMPC\_m}(t_{i+1}) = \frac{W_1}{W_1 + W_2}\cdot y^{AB\_m}(t_{i+1}) + \frac{W_2}{W_1 + W_2}\cdot y^{AM\_m-1}(t_{i+1}) \tag{27c}$$

To examine the robustness and efficiency of the proposed method, stability and accuracy analyses are given as follows.

Accuracy analysis: y^{mpc} and y^{AM_m} can be approximated by

$$y^{mpc}(t_{i+1}) = y(t_i) + hf(t_{i+1}, y^p(t_{i+1}))(-1)^m \int_0^1 C_m^{-s} ds + h \cdot \sum_{k=0}^{m}\left\{\nabla^k f(t_i)\cdot\left[-(-1)^m \int_0^1 C_m^{-s} ds + (-1)^k \int_0^1 C_k^{-s} ds\right]\right\} \tag{28}$$

$$y^{AM\_m}(t_{i+1}) = y(t_i) + hf(t_{i+1}, y(t_{i+1}))(-1)^m \int_0^1 C_m^{-s} ds + h \cdot \sum_{k=0}^{m}\left\{\nabla^k f(t_i)\cdot\left[-(-1)^m \int_0^1 C_m^{-s} ds + (-1)^k \int_0^1 C_k^{-s} ds\right]\right\} \tag{29}$$

Furthermore, the assumption is made that $y(t_i)$ and $\nabla^k f(t_i)$ in Eqs. (28) and (29) are equal. With this assumption, one has the following:

$$\begin{aligned} y^{AM\_m}(t_{i+1}) - y^{MAMPC\_m}(t_{i+1}) &= h(-1)^m \int_0^1 C_m^{-s} ds\cdot[f(t_{i+1}, y(t_{i+1})) - f(t_{i+1}, y^p(t_{i+1}))] \\ &= h(-1)^m \int_0^1 C_m^{-s} ds\cdot f'(\vartheta_i)\cdot(y(t_{i+1}) - y^p(t_{i+1})) \\ &\approx h^{m+2}\left[(-1)^m \int_0^1 C_m^{-s} ds\right]^2 f'(\vartheta_i) f^{(m)}(\mu_i, y(\mu_i)) \end{aligned} \tag{30}$$

Hence, the dominant truncation errors of y^{MAMPC_m} and y^{AM_m} are of the order of h^{m+1}. In Eq. (30), it is shown that y^{MAMPC_m} and y^{AM_m} have the same order of accuracy. Thus, it is concluded that the MAMPC methods are about one order of magnitude more accurate than the ABMPC methods (with the same step size).

Stability analysis: For the linear test problem

$$\dot{z} = \lambda z \tag{31}$$

Figures 1, 2, and 3 show that the domain of stability of the characteristic equation of Eq. (31) connecting the points $h\lambda$ for which the roots of these equations have a modulus less than unity. From Figs. 1, 2, and 3, it is concluded that the stability domains of the MAMPC methods are larger than those of the ABMPC methods.

IV. BAUMGARTE STABILIZATION TECHNIQUE TO SOLVE MBD SYSTEMS

The governing equations of multibody dynamics can be written as the following differential algebraic equations (DAEs) [8]:

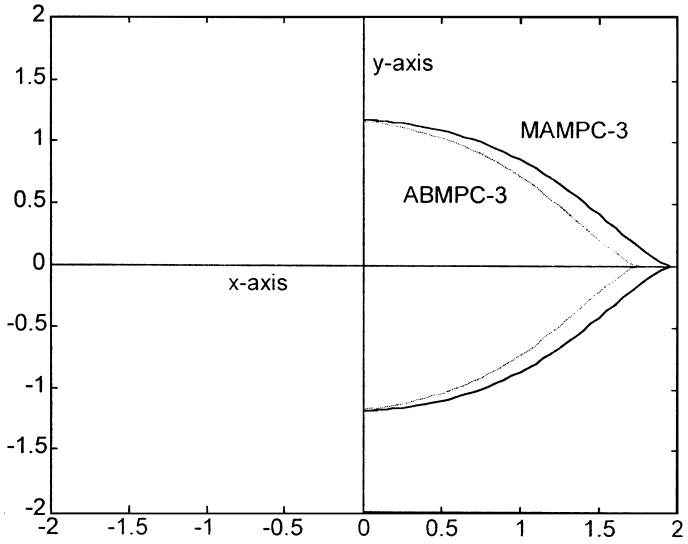

Fig. 1. The stability region of ABMPC_3 and MAMPC_3.

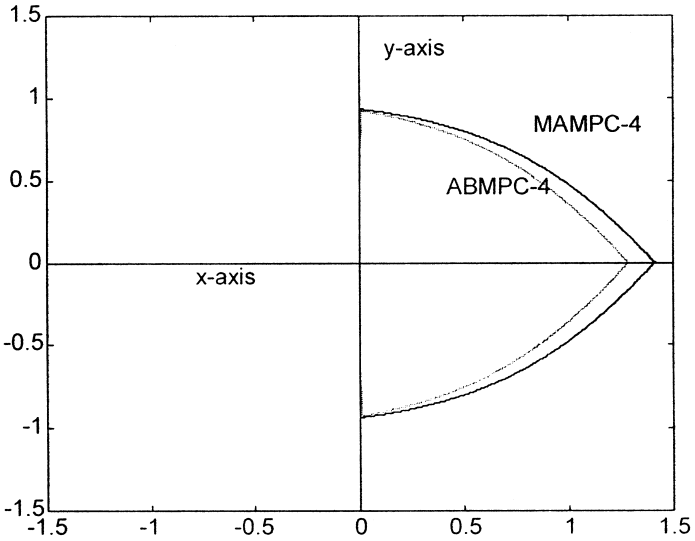

Fig. 2. The stability region of ABMPC_4 and MAMPC_4.

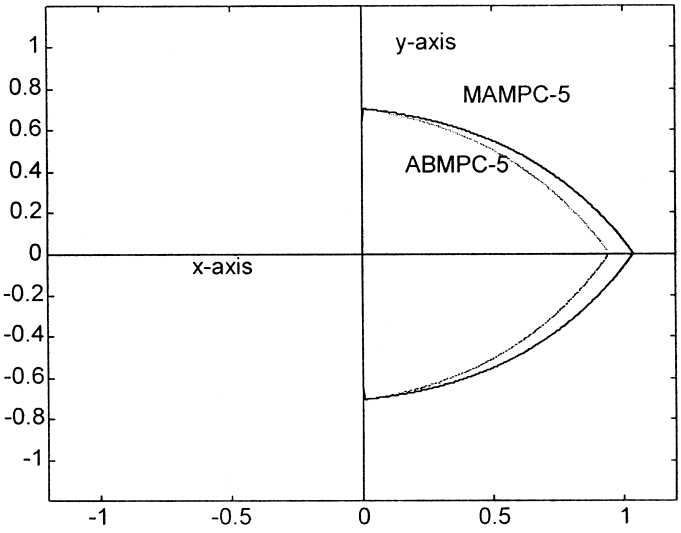

Fig. 3. The stability region of ABMPC_5 and MAMPC_5.

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$$\mathbf{M}\ddot{q} + \mathbf{B}^T\lambda = F(q, \dot{q}, t) \tag{32a}$$

$$\Phi(q) = 0 \tag{32b}$$

where $\mathbf{M} \in \mathrm{R}^n$ denotes the mass matrix of the system, $q \in \mathrm{R}^n$ is the vector of generalized coordinates, $\Phi \in \mathrm{R}^m$ are constraint functions, $\mathbf{B} \in \mathrm{R}^{m\times n}$ is the Jacobian matrix of the constraint equations, $\lambda \in \mathrm{R}^m$ is the Lagrangian multiplier, and $F(q, \dot{q}, t)$ consists of applied force, centrifugal and Coriolis force, and internal spring force.

Although there exist many good constraint stabilization techniques, in the present development, Baumgarte's technique is employed because

1. It is easy to implement in a computer program.
2. It requires no special integration method.
3. It is relatively easy to obtain the stabilization parameters, as long as the parameters satisfy the stability condition of the numerical integration method employed.

In the Baumgarte technique [8], one replaces Eq. (32b) by

$$\ddot{\Phi} + 2\alpha\dot{\Phi} + \beta^2\Phi = 0 \tag{33}$$

where α and β are positive real numbers. Chang and Nikravesh [3] suggested that this constraint violation stabilization technique was at its best by choosing α and β at the critical damping condition, i.e., $\alpha = \beta$. Indeed, for most MBD analysis problems previously conducted, the most accurate solution is achieved when the critical damping condition is used. Furthermore, it is observed that the magnitude of α greatly affects position and velocity constraint errors during the process of time integration of Eq. (33). In other words, a large α will stabilize constraint errors faster than a small α. However, the simulated MBD system will become numerically unstable if a randomly large α is chosen. By imposing these conditions, the stabilization parameter is decided by the inequality

$$\alpha \cdot h \le SL \tag{34}$$

where h is the integration step size and SL is the stability limit of the integration method. Table 2 presents stability limits of ABP, AMPC, and MAMPC methods, with different orders. Note that the stability limits of MAMPC methods are the largest. In other words, the range of α in MAMPC methods is greater than ABP methods and AMPC methods.

TABLE 2
Stability limits of ABP, AMPC, and MAMPC methods of order k.

Order k	3	4	5
ABP	0.545	0.300	0.164
AMPC	1.729	1.285	0.947
MAMPC	1.935	1.412	1.040

V. NUMERICAL EXAMPLE

The crank mechanism is used as an example to demonstrate the accuracy and robustness of the proposed method, in comparison with Adams-Moulton predictor-corrector methods. Shown in Fig. 4 is the classical crank mechanism, whose governing equations of motion are characterized by the following matrices and constraints:

$$M = \begin{bmatrix} J_1 & & & \\ & J_2 & & \\ & & m & \\ & & & m \end{bmatrix}, \qquad \Phi = \begin{Bmatrix} r\cos\theta - (x - l_1\cos\phi) \\ r\sin\theta - (y - l_1\sin\phi) \\ (l - l_1)\sin\phi + y \end{Bmatrix} = 0 \tag{35}$$

$$q = [\dot{\theta}\ \phi\ x\ y]^T, \qquad F = \{0\ 0\ 0\ -mg\}^T \tag{36}$$

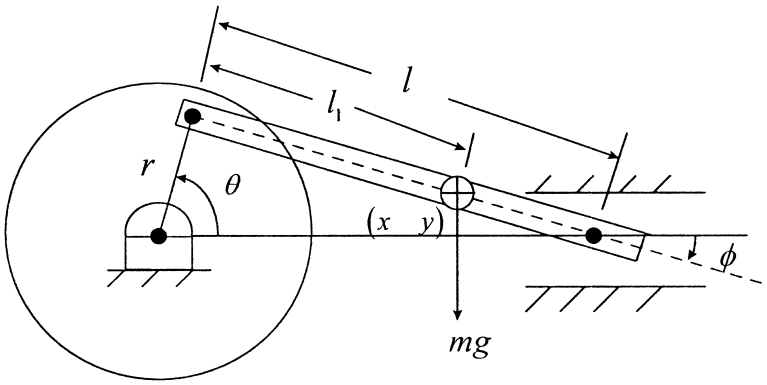

Fig. 4. Crank-slider mechanism.

where $J_1 = 0.045$, $J_2 = 33/4800$, $m = 1$, $r = 0.3$, $l = 0.5$, and $l_1 = 0.3$. The initial conditions are

$$\theta(0) = 0.9851, \quad \phi(0) = -0.5236, \quad x(0) = 0.4256, \quad y(0) = 0.1$$
$$\dot{\theta}(0) = 0, \quad \dot{\phi}(0) = 0, \quad \dot{x}(0) = 0, \quad \dot{y}(0) = 0 \tag{37}$$

In the present study, Baumgarte's technique with stabilization parameters $\alpha = \beta = 5$ is used to stabilize constraint violations. The performance of the Baumgarte technique for the present example is presented in Figs. 5 and 6, using the same integration methods as given in this example. Two integration step sizes, $h = 0.01$ s and $h = 0.001$ s, are used in this simulation. The starting procedure for these methods is a Runge-Kutta-Fehlberg method with truncation error equal to 10^{-15}. Table 3 shows the average 2-norm of constraint errors of AMPC_3 and MAMPC_3.

From Table 3, the accuracy of MAMPC_3 is observed to be one order of magnitude higher than AMPC_3, by reducing by one-tenth of the integration step size. The time histories of constraint errors are shown in Figs. 5 and 6. In Fig. 7, it is observed that AMPC_3 becomes unstable when $\alpha \geq 168$, with $h = 0.01$ s. However, MAMPC_3 is still stable when $168 \leq \alpha \leq 188$.

Although not addressed here, other MAMPC methods (MAMPC_4 and

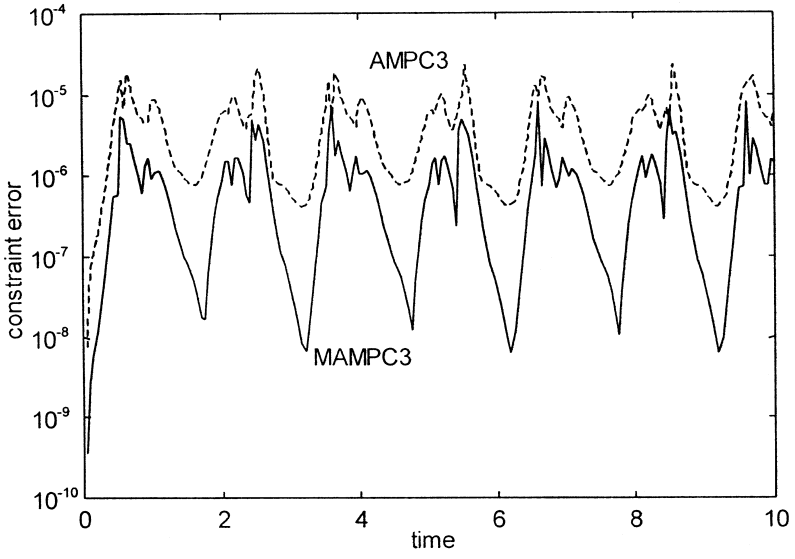

Fig. 5. 2-norm of constraint errors ($\alpha = \beta = 5$ and $h = 0.01$ s).

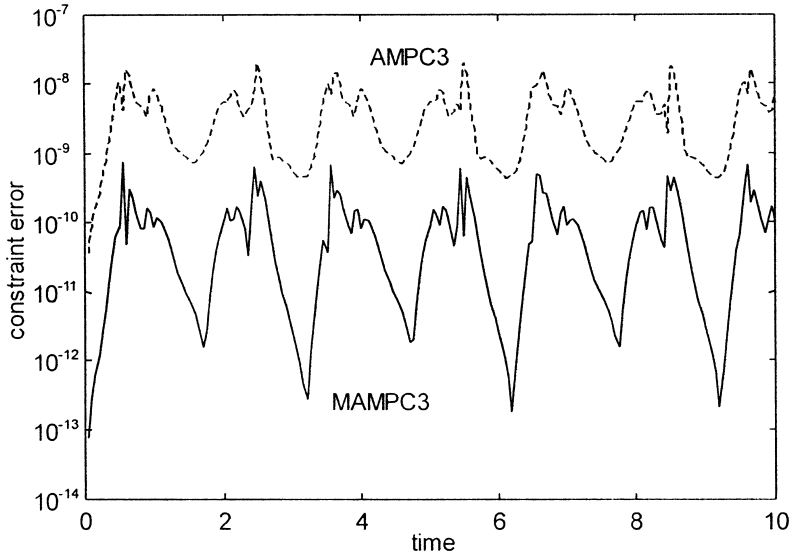

Fig. 6. 2-norm of constraint errors ($\alpha = \beta = 5$ and $h = 0.001$ s).

MAMPC_5) have also been studied. Similar results were obtained. Thus, it is concluded that the MAMPC methods are more accurate and robust than the AMPC method.

VI. CONCLUSION

A modified predictor-corrector method is presented. By making a small modification, the proposed method has increased the accuracy by about one order of magnitude, in comparison with AMPC methods. By employing Baumgarte's

TABLE 3
Average absolute constraint errors of AMPC_3 and MAMPC_3.

Method	$h = 0.01$	$h = 0.001$
AMPC_3	4.65×10^{-6}	4.14×10^{-9}
MAMPC_3	9.8×10^{-7}	9.6×10^{-11}

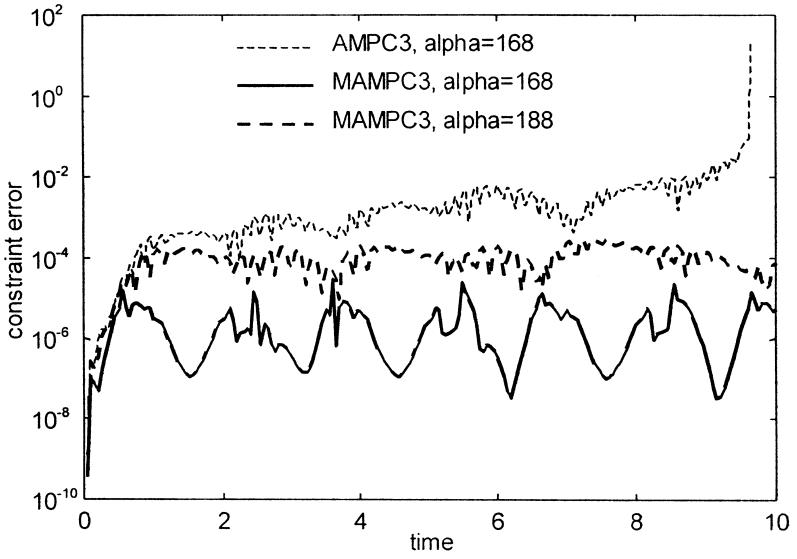

Fig. 7. 2-norm of constraint errors ($h = 0.01$ s).

technique to correct the constraint violation during computer simulation of MBD systems, the proposed method has demonstrated its robustness in handling stabilization parameter α. A numerical example has been used to show the superiority of the proposed integration method.

ACKNOWLEDGMENT

This work was supported by the NSC/PIDC and National Science Council under Grant NSC89-NSPO(A)-PC-FA06-01.

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APPENDIX

Adams-Bashforth Integrators

1. Two-step (first-order) method:

$$y_{i+1} = y_i + \frac{h}{2}(3f_i - f_{i-1}) \tag{A1}$$

The local truncation error is $O_{i+1}(h) = \frac{5}{12} y^m(\mu_i)h^2$, for some $u_i \in (t_{i-1}, t_{i+1})$.

2. Three-step (second-order) method:

$$y_{i+1} = y_i + \frac{h}{12}(23f_i - 16f_{i-1} + 5f_{i-2}) \tag{A2}$$

The local truncation error is $O_{i+1}(h) = \frac{3}{8} y^{(4)}(\mu_i)h^3$, for some $u_i \in (t_{i-2}, t_{i+1})$.

3. Four-step (third-order) method:

$$y_{i+1} = y_i + \frac{h}{24}(55f_i - 59f_{i-1} + 37f_{i-2} - 9f_{i-3}) \tag{A3}$$

The local truncation error is $O_{i+1}(h) = \frac{251}{720} y^{(5)}(\mu_i)h^4$, for some $u_i \in (t_{i-3}, t_{i+1})$.

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4. Five-step (fourth-order) method:

$$y_{i+1} = y_i + \frac{h}{720}(1901 f_i - 2774 f_{i-1} + 2616 f_{i-2} - 1274 f_{i-3} + 251 f_{i-4}) \quad \text{(A4)}$$

The local truncation error is $O_{i+1}(h) = \frac{95}{288} y^{(6)}(\mu_i) h^5$, for some $u_i \in (t_{i-4}, t_{i+1})$.

Adams-Moulton Integrators

1. Two-step (second-order) method:

$$y_{i+1} = y_i + \frac{h}{12}(5 f_{i+1} + 8 f_i - f_{i-1}) \quad \text{(A5)}$$

The local truncation error is $O_{i+1}(h) = -\frac{1}{24} y^{(4)}(\hat{\mu}_i) h^3$, for some $\hat{u}_i \in (t_{i-1}, t_{i+1})$.

2. Three-step (third-order) method:

$$y_{i+1} = y_i + \frac{h}{24}(9 f_{i+1} + 19 f_i - 5 f_{i-1} + f_{i-2}) \quad \text{(A6)}$$

The local truncation error is $O_{i+1}(h) = -\frac{19}{720} y^{(5)}(\hat{\mu}_i) h^4$, for some $\hat{u}_i \in (t_{i-2}, t_{i+1})$.

3. Four-step (fourth-order) method:

$$y_{i+1} = y_i + \frac{h}{720}(251 f_{i+1} + 646 f_i - 264 f_{i-1} + 106 f_{i-2} - 19 f_{i-3}) \quad \text{(A7)}$$

The local truncation error is $O_{i+1}(h) = -\frac{3}{160} y^{(6)}(\hat{\mu}_i) h^5$, for some $\hat{u}_i \in (t_{i-3}, t_{i+1})$.

Received May 1998
Revised November 1999

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