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Short Paper

A Cost-Effective Forward Recovery Checkpointing Scheme in Multiprocessor Systems*

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This paper proposes a novel and cost-effective forward recovery checkpointing scheme for multiprocessor systems with duplex modular redundancy. In our scheme, one processing module is selected to retry the questionable checkpoint, and the other processing module executes toward the next checkpoint if a mismatched comparison between the two processing modules occurs at any checkpoint. Those schemes using a spare module to retry need much time to initiate the module, and the extra cost is high. Although the traditional rollback scheme retries the questionable checkpoint without any spare module, it has longer average completion time than our scheme for a job under any fault distribution. In our scheme, besides transient faults, permanent faults can be located as well. Experimental results based on our mathematical models demonstrate that, under burst errors, the average completion time of our scheme is reduced by 50% compared with that of the traditional rollback and is comparable with that of the scheme using a spare module to retry. In addition, our scheme has the least total execution time (the most cost-effectiveness) among the three schemes under any fault distribution.

Keywords: forward recovery, multiprocessor system, cost-effective, checkpointing scheme, transient fault, permanent fault

1. INTRODUCTION

A multiprocessor system with duplex modular redundancy (DMR) may employ two processing modules (PMs) to execute the same job concurrently. To reduce the number of restarts and to roll back to the last checkpointing state instead, the execution of a job is divided into *n* useful computation intervals, and each interval is followed by checkpointing [1-6]. The overhead for a checkpointing scheme is the time needed to handle these

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checkpointing states [7]. Some fault tolerant schemes are based on *fault masking*, which uses more redundant PMs to reduce the average completion time, such as in a triple module redundancy (TMR) system [8, 9] or voting [10]. Fault masking used by TMR is not suitable for a DMR system because it requires three PMs. Therefore, *fault retry* is used instead of fault masking. At each checkpoint, the states of two PMs are compared [7, 11, 12], and a retry is performed to locate a faulty checkpoint if the comparison mismatches. Two major types of retry are *retry on a spare PM* and *retry on an original PM*. The former can reduce the average completion time of a job, but the system cost is high because a spare module is required, and more time to initiate it before a retry can begin. In transaction systems, this is a serious problem because system availability is important [13]. To reduce the system cost, the latter may be preferred. However, it has problems related to *retry validation* and *longer average completion time*. To avoid an invalid retry on a faulty PM, the state of a PM needs to be made consistent with the last checkpoint at the beginning of the retry. In this paper, we propose a novel forward recovery checkpointing scheme (FCS) to provide a trade-off between the average completion time of a job and the number of PMs required. Two existing checkpointing schemes will be reviewed and compared: (1) the roll-forward checkpointing scheme (RFCS) and (2) the traditional rollback scheme (RB) [2, 7, 14]. RFCS needs a spare module to retry when a fault is detected, and RB uses both PMs simultaneously to retry the questionable checkpoint. FCS is different from the schemes in [7, 11] in terms of forward recovery without using a spare module and is unlike the traditional rollback scheme in terms of retry on a single PM, not on two PMs.

In digital computer systems, the occurrence probability of transient faults is larger than that of permanent faults [1, 15, 16]. Therefore, our analysis focuses on transient faults. We assume that there is no identical checkpointing state on two PMs when they have transient faults at the same checkpoint interval [7]. Nevertheless, our scheme can handle permanent faults, such as fail-stop faults and crash faults, as well.

The remainder of this paper is organized as follows. A DMR architecture is presented in section 2.1. Some notations are introduced in section 2.2. The basic concept of FCS is given in section 2.3. RB and RFCS schemes are reviewed in section 2.4. Section 3 illustrates in detail the FCS scheme in each fault case. Mathematical models and experimental results obtained using FCS, RB, and RFCS are presented in section 4. Section 5 depicts implementation issues. Some concluding remarks are given in section 6.

2. PRELIMINARIES

2.1 DMR Architecture for FCS

A DMR architecture for FCS is shown in Fig. 1. It consists of two PMs (P_1 and P_2), two reliable storages (RSs), and one reliable checkpoint processor (RCP). These components are connected by an interconnection network (e.g., a bus) in a multiprocessor system. To recover transient faults, no spare module is needed. Both P_1 and P_2 concurrently execute each replica of a job. At each checkpoint, both checkpointing states of P_1 and P_2 are sent to the RCP to be compared for fault detection. The fault history of each PM is recorded in the RCP. To improve the reliability of RCP, it may be configured as a TMR structure

[17]. Each PM has an RS used to save checkpointing states during fault recovery. The contents of an RS can be accessed by other processors (PMs and RCP). For brevity, we use checkpoint instead of checkpointing state in the following discussion.

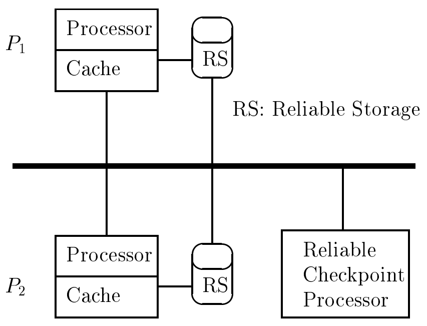
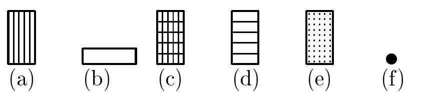


Fig. 1. DMR architecture for FCS.

2.2 Notations

We use six box notations for FCS, as shown in Fig. 2: (a) The time required for checkpointing is denoted by t_{ch}, which also includes the time needed for comparing the checkpoints of P_1 and P_2. (b) The time required for synchronization is denoted by t_w, where the faster PM waits for the slower PM. (c) The time required to compare the retried checkpoint ϕ_{j3} with two previously saved checkpoints ϕ_{j1} and ϕ_{j2} is denoted t_{cc}. (d) The time required to make the state of a faulty PM consistent with the state of the other PM is denoted t_{cp}. (e) The time required to make the states of both PMs consistent with a previous checkpoint is denoted t_r. (f) A fault is represented by a black dot. A box shaded with the same pattern represents the same operation that takes the same amount of time.



(a) Duplex checkpointing – t_{ch}
(b) Idle – t_w
(c) Comparing the retried checkpoint with two saved checkpoints – t_{cc}
(d) Restoring checkpoint – t_{cp}
(e) Rollback – t_r
(f) A fault

Fig. 2. Box notations for FCS.

2.3 Basic Concept

This section introduces the basic concept of fault detection, location, and recovery in FCS. Fault detection is achieved through comparison of a pair of checkpoints [7, 11, 12]. In Fig. 3, one PM is selected to retry the questionable checkpoint, and the other PM ex-

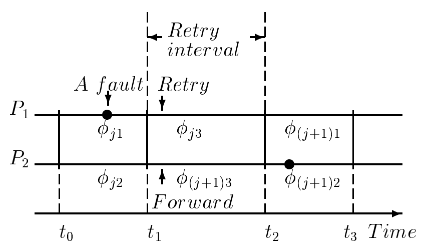


Fig. 3. FCS for a DMR system.

ecutes toward the next checkpoint continuously. Assume that the last checkpoint ϕ_{j-1} is completed at time t_0. The current checkpoint is denoted ϕ_j, and its following interval is denoted ϕ_{j+1}. In the j_{th} interval (I_j), both checkpoints of P_1 and P_2 are denoted ϕ_{j1} and ϕ_{j2}, respectively. Suppose that there is a mismatch between ϕ_{j1} and ϕ_{j2}, that one PM has a transient fault, and that the other PM is fault-free during interval I_j. Assume P_1 has a fault in interval I_j. At the beginning of the retry (t_1), P_1 is selected to retry the questionable checkpoint ϕ_j, and P_2 executes the next checkpoint ϕ_{j+1} forward. At the end of the retry interval (t_2), the checkpoint produced by P_1 is denoted ϕ_{j3}, the other checkpoint completed by P_2 is denoted a $\phi_{(j+1)3}$, ϕ_{j3} is compared with the two previously saved checkpoints ϕ_{j1} and ϕ_{j2}, and $\phi_{(j+1)3}$ is retained on the corresponding RS. The result of this comparison indicates that ϕ_{j3} is identical to ϕ_{j2} and different from ϕ_{j1}. The correct checkpoint ϕ_j for interval I_j can be derived from either ϕ_{j2} or ϕ_{j3}. ϕ_j is then retained on the RS, and the previous checkpoint ϕ_{j-1} is replaced to reduce the size of the RS. Thus, the fault in P_1 is recovered successfully. Then P_1 and P_2 execute at the next interval I_{j+1} continuously. At time t_3, the checkpoint $\phi_{(j+1)1}$ of P_1 and the checkpoint $\phi_{(j+1)2}$ of P_2 have been completed. If another fault happens in interval I_{j+1}, either on P_1 or P_2, the comparison between $\phi_{(j+1)1}$ and $\phi_{(j+1)2}$ will show them not to be identical. Then $\phi_{(j+1)3}$ is compared with $\phi_{(j+1)1}$ and $\phi_{(j+1)2}$ immediately. Note that $\phi_{(j+1)3}$ has completed at time t_2. The faulty $\phi_{(j+1)2}$ can be correctly located without the need for an additional retry interval.

2.4 RB and RFCS Schemes

For comparison with the other two schemes, the RFCS and RB schemes are illustrated in Fig. 4 and Fig. 5, respectively [7, 11]. Note that the retry for RFCS is performed on a spare module. As to RB, both PMs are rolled back to the previous checkpoint upon failure at the same time.

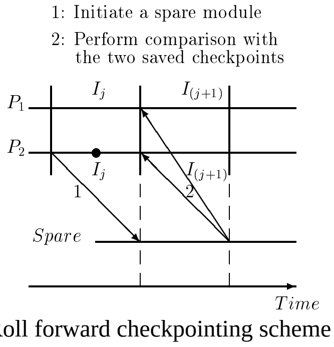

1: Initiate a spare module
2: Perform comparison with the two saved checkpoints
P_1
P_2
I_j
$I_{(j+1)}$
I_j
$I_{(j+1)}$
1
2
Spare
Time

Fig. 4. Roll forward checkpointing scheme (RFCS).

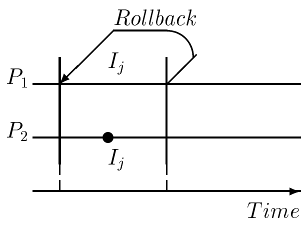


Fig. 5. Rollback scheme (RB).

3. FORWARD RECOVERY CHECKPOINTING SCHEME

3.1 Transient Faults

Now we will begin to describe our FCS under transient faults in detail. If there is a mismatched comparison at an interval, the second interval is needed to obtain a retried checkpoint and a forward checkpoint on two PMs, respectively. A forward checkpoint is valid if its previous checkpointing state is fault-free. Here the correctness of the forward checkpoint has not yet been determined. An additional interval is required to make sure whether the forward checkpoint is correct. In the following analysis, the states of at most three consecutive intervals are considered. That is, the scheme has three possible situations — (1) no retry interval, (2) one retry interval and the forward checkpoint is invalid, and (3) one retry interval and the forward checkpoint is valid. Depending on how faults occur, there are six possible fault cases. In the following, we will discuss these fault cases denoted (A) through (F).

(A) *No failure*: Both P_1 and P_2 are fault-free in interval I_j (see Fig. 6). At the end of the current interval I_j, ϕ_{j1} and ϕ_{j2} are found to be identical. The correct checkpoint ϕ_j is derived from either ϕ_{j1} or ϕ_{j2}. Then ϕ_j is saved in each RS, and the previous checkpoint ϕ_{j-1} is replaced. At time t_1, the current interval I_j has completed, and P_1 and P_2 are ready to execute at the next interval I_{j+1}. The time required for this case is $t_A = T = t_u + t_{ch}$, where t_u is the time required for useful computation in a checkpoint interval. The average completion times in this case are all the same for the three schemes.

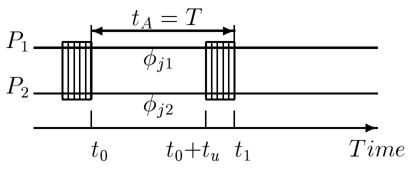


Fig. 6. Case (A) — No failure.

(B) *Recovering a failure, followed by a state restoration*: In this case, the current interval I_j has a fault, the retry checkpoint ϕ_{j3} is fault-free, and the forward checkpoint $\phi_{(j+1)3}$ will be invalid. Suppose that ϕ_{j1} is fault-free, and that ϕ_{j2} has a fault (see Fig. 7). A retry interval is inserted between intervals I_j and I_{j+1}. Suppose that P_1 is selected to retry due to a

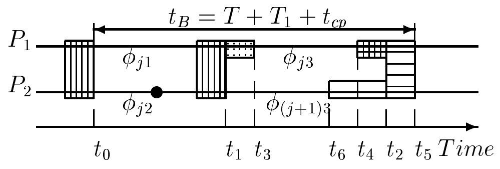


Fig. 7. Case (B) — Recovering a failure, followed by a state restoration.

faulty module selection miss. The forward checkpoint $\phi_{(j+1)3}$ is invalid because it is infected by the fault in ϕ_{j2}. We will discuss the situation of a selection hit in cases (D) through (F). At time t_1, ϕ_{j1} and ϕ_{j2} are found to be different. A retry interval is initiated. First, both ϕ_{j1} and ϕ_{j2} are saved in the respective RS. Then, the state of P_1 is made consistent with the state of the previous checkpoint $\phi_{j\text{-}1}$. The time required for this retry is $t_r = t_3 - t_1$. At time t_3, P_1 is ready to retry. Concurrently, P_2 continues to execute toward the forward checkpoint. At time t_6, this forward checkpoint is completed first. Then P_2 waits until P_1 completes the retry. The idle time is $t_w = t_2 - t_6$. At time t_4, P_1 completes the retry, and ϕ_{j3} is compared with the two saved checkpoints ϕ_{j1} and ϕ_{j2}. At time t_2, it is found that ϕ_{j3} is identical to ϕ_{j1} and different from ϕ_{j2}. The time required for this comparison is $t_{cc} = t_2 - t_4$. At the same time, the forward checkpoint $\phi_{(j+1)3}$ is determined to be invalid due to a fault in its previous interval. The state of P_2 is incorrect and must be made consistent with the correct state from P_1. The time required for this state restoration is $t_{cp} = t_5 - t_2$. At time t_5, P_1 and P_2 are ready to execute at the next interval I_{j+1}. FCS can recover a transient failure that appears in P_2 without the need for a spare module, but RFCS, which needs a spare module to retry, will fail if no spare module is available. The time required for this case is $t_B = T + T_1 + t_{cp}$, where $T_1 = t_r + t_u + t_{cc}$.

(C) *Rollback due to retry interval failure*: The retry interval may not succeed if at least two faults occur. There are three possible scenarios as listed in Table 1. In this case, at least two faults happen in interval I_j and in the retry interval. The number of faulty checkpoints is at least two among ϕ_{j1}, ϕ_{j2}, and ϕ_{j3}. RCP can not find the correct checkpoint ϕ_j. Both PMs are rolled back to the previous checkpoint ϕ_{j-1}. For the sake of illustration, consider scenario C_1 illustrated in Fig. 8. Here two faults happen in interval I_{j2} and retry interval I_{j3}, respectively. First, ϕ_{j1} and ϕ_{j2} are found to be different at time t_1. A retry interval is initiated. The retry is the same as that in case (B) except that the comparison mismatches at time t_2. RCP can not find two identical checkpoints because ϕ_{j2} and ϕ_{j3} are incorrect. Then the states of P_1 and P_2 are made consistent with the previous checkpoint ϕ_{j-1}. The time required for this rollback is $t_r = t_5 - t_2$. After the rollback is completed at time t_5, P_1 and P_2 are in a state identical to their respective state at t_0. No checkpoint is completed in this case. The time required for this case is $t_C = T + T_1 + t_r$.

Table 1. Three possible fault scenarios in case (C).

Scenario	Status in interval I_j		Status in the retry interval	
	ϕ_{j1}	ϕ_{j2}	ϕ_{j3}	$\phi_{(j+1)3}$
C_1	fault-free	faulty	faulty	don't-care
C_2	faulty	fault-free	faulty	don't-care
C_3	faulty	faulty	don't-care	don't-care

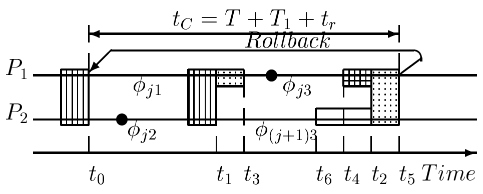


Fig. 8. Case (C_1) — Rollback due to retry interval failure.

(D) *Retry successful and no more failure in the next interval*: In this case and the following two cases, (E) and (F), the retry mechanism succeeds to recover a failure in the current interval I_j. The forward checkpoint $\phi_{(j+1)3}$ is also valid due to a selection hit. No more failures happen in the next interval I_{j+1} in this case. As shown in Fig. 9, there is a fault in P_1, and P_1 is selected to retry. At time t_1, ϕ_{j1} and ϕ_{j2} are found to be different. A retry interval is inserted between the current interval I_j and the next interval I_{j+1}. Up to time t_2, the case is similar to case (B). The forward checkpoint $\phi_{(j+1)3}$ is first saved in the corresponding RS. Both P_1 and P_2 continue to execute at the next interval I_{j+1}. After one interval of time, both checkpoints $\phi_{(j+1)1}$ and $\phi_{(j+1)2}$ are found to be identical at time t_5. The forward checkpoint $\phi_{(j+1)3}$ will not be used in this case. At time t_5, both P_1 and P_2 are ready to execute the following interval I_{j+2}. Thus, a single failure in P_1 is recovered successfully. The time required for this case is $t_D = 2T + T_1$.

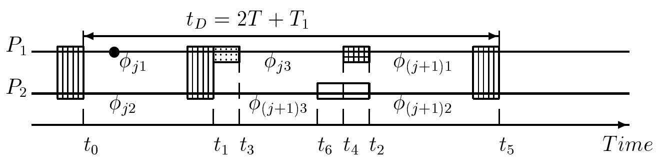


Fig. 9. Case (D) — Retry successful and no more failure in the next interval.

(E) *Retry successful and forward recovery of a failure in the next interval*: In this case, there are two faults in intervals I_j and I_{j+1}, respectively. The forward checkpoint $\phi_{(j+1)3}$ will be valid due to a selection hit. The infection of the fault is only one interval, but it is two intervals in RFCS. There are two possible fault scenarios as listed in Table 2. To illustrate, scenario E_1 is shown in Fig. 10. Since this case has been described in section 2. 3, we will only make some observations. Up to time t_5, this case is similar to case (D) except that a fault occurs in $I_{(j+1)2}$. Note that both failures in ϕ_{j1} and $\phi_{(j+1)1}$ are recovered successfully. In this case, FCS needs only one retry interval to recover two failures, but RB needs at least two rollback intervals. The time required for this case is $t_E = 2T + T_1 + t_{cc} + t_{cp}$.

Table 2. Two possible fault scenarios in case (E).

Scenario	Status in interval I_j		Status in the retry interval		Status in interval I_{j+1}	
	ϕ_{j1}	ϕ_{j2}	ϕ_{j3}	$\phi_{(j+1)3}$	$\phi_{(j+1)1}$	$\phi_{(j+1)2}$
E_1	faulty	fault-free	fault-free	fault-free	fault-free	faulty
E_2	faulty	fault-free	fault-free	fault-free	faulty	fault-free

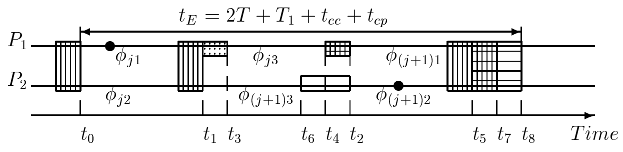


Fig. 10. Case (E_1) — Retry successful and forward recovery of a single failure in the next interval.

(F) *Retry successful and rollback in the next interval*: There are three possible fault scenarios in this case (see Table 3). To illustrate, consider scenario F_1 as shown in Fig. 11, where ϕ_{j1}, $\phi_{(j+1)3}$, and $\phi_{(j+1)2}$ are faulty. At time t_1, a fault is detected, and a retry interval is inserted between interval I_j and interval I_{j+1}. Up to time t_7, this case is similar to case (E) except that two faults occur in $\phi_{(j+1)3}$ and $\phi_{(j+1)2}$, respectively. RCP can not find the correct checkpoint ϕ_{j+1} in the next interval I_{j+1}. So both PMs are rolled back to the previous checkpoint ϕ_j. This rollback is similar to that of case (C). The time required for the rollback is $t_r = t_8 - t_7$. At time t_8, the states of P_1 and P_2 are the same as those at time t_2. In this case, a failure in I_j is recovered successfully. But the other two failures, in $\phi_{(j+1)2}$ and $\phi_{(j+1)3}$, can not be recovered. The time required for this case is $t_F = 2T + T_1 + t_{cc} + t_r$.

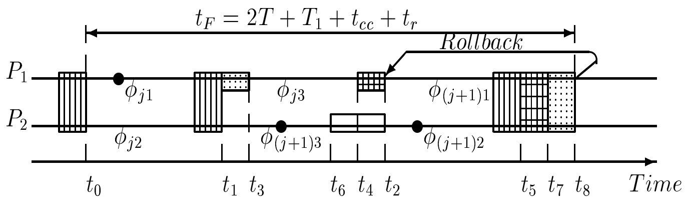


Fig. 11. Case (F_1) — Retry successful and rollback in the next interval.

Table 3. Three possible fault scenarios in case (F).

Scenario	Status in interval I_j		Status in the retry interval		Status in interval I_{j+1}	
	ϕ_{j1}	ϕ_{j2}	ϕ_{j3}	$\phi_{(j+1)3}$	$\phi_{(j+1)1}$	$\phi_{(j+1)2}$
F_1	faulty	fault-free	fault-free	faulty	fault-free	faulty
F_2	faulty	fault-free	fault-free	faulty	faulty	fault-free
F_3	faulty	fault-free	fault-free	don't-care	faulty	faulty

3.2 Permanent Faults

The scheme presented above can also detect and locate permanent faults in a PM. Remember that if a PM encounters failures in several consecutive intervals during a period of time, then the PM is assumed to have a permanent fault. We assume that P_1 and P_2 will not have permanent faults at the same time. Suppose P_1 has a permanent fault, and that P_2 is fault-free. As shown in Fig. 12, at time t_1, a fault is detected. P_1 is first selected to retry.

At time t_2, the retry fails, and P_1 and P_2 are rolled back to the previous checkpoint $\phi_{j\text{-}1}$ as in case (C). At time t_3, the execution still fails. Therefore, the retry is switched to P_2. At time t_4, this retry succeeds, and P_1 is identified as being faulty. The failure history is updated to indicate that P_1 is faulty. Then both P_1 and P_2 continuously execute in the next interval I_{j+1}. At time t_5, the comparison between the two checkpoints $\phi_{(j+1)1}$ and $\phi_{(j+1)2}$ mismatches. A retry is performed again. P_2 is selected to retry again. At time t_6, the correct checkpoint ϕ_{j+1} is found, and P_1 is found to have a fault once again. The results of the subsequent execution during t_4 to t_6 are similar to those during t_2 to t_4, which are similar to case (B) except that two consecutive faults occur in P_1. The results always indicate that P_1 is faulty. Since P_1 encounters several consecutive failures in a period of time, it is determined to have a permanent fault.

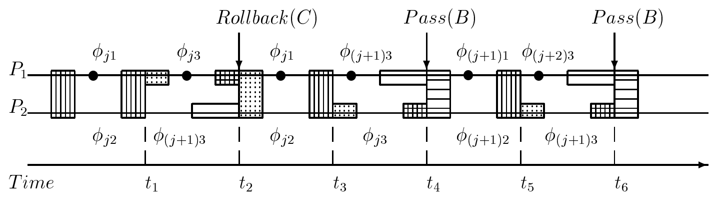


Fig. 12. Locating a module with a permanent fault: an example.

4. PERFORMANCE EVALUATION

In this section, we will evaluate the performance of FCS, RFCS, and RB quantitatively. Two performance measures will be examined: *average completion time* and *total execution time*. The total execution time can be used as a measure of cost-effectiveness for each scheme.

4.1 Poisson Distributed Faults

In section 3, we have described the six possible fault cases in detail. Now we will evaluate the occurrence probability for each case. The analysis presented here focuses on transient faults. Occurrence of transient faults in a module is first assumed to be a Poisson process with a constant failure rate λ [13, 18]. Therefore, the failure probability of a module is $1 - e^{-\lambda T}$ during time period T. Based on the probability and the time required for each case (see Table 4, where m is the selection miss rate), we can derive the average completion time and the total execution time for FCS. Now we will calculate the average completion time for a job with k checkpoints. If $k = 1$, then FCS is identical to RB. When a fault is detected, a DMR system will roll back instead of performing a retry. The occurrence probability for case (A), P_A, is derived under the no failure condition. P_{roll} is the probability of a rollback with $k = 1$. Thus $P_{roll} = 1 - P_A = 1 - e^{-2\lambda T}$. The average completion time $(\overline{\tau}_k)$ for $k = 1$ is derived as follows [7]:

$$\overline{\tau_1} = P_A t_A + P_{roll}(\overline{\tau_1} + t_{roll}), \tag{1}$$

Table 4. The likelihood of the six cases.

Case	Probability	required time
(A)	$P_A = e^{-2\lambda T}$	$t_A = T$
(B)	$P_B = 2me^{-\lambda(T+T1)}(1 - e^{-\lambda T})$	$t_B = T + T_1 + t_{cp}$
(C)	$P_C = 2e^{-\lambda T}(1 - e^{-\lambda T})(1 - e^{-\lambda T_1}) + (1 - e^{-\lambda T})^2$	$t_C = T + T_1 + t_r$
(D)	$P_D = 2(1 - m)e^{-\lambda(3T+T_1)}(1 - e^{-\lambda T})$	$t_D = 2T + T_1$
(E)	$P_E = 4(1 - m)e^{-2\lambda(T+T_1)}(1 - e^{-\lambda T})^2$	$t_E = 2T + T_1 + t_{cc} + t_{cp}$
(F)	$P_F = 2(1 - m)e^{-\lambda(T+T_1)}(1 - e^{-\lambda T})^2(1 + e^{-\lambda T} - 2e^{-\lambda(T+T_1)})$	$t_F = 2T + T_1 + t_{cc} + t_r$

where $t_{roll} = T + t_r$.
If $k = 2$, then FCS remains the same as RB. Therefore,

$$\overline{\tau_2} = 2\overline{\tau_1}. \tag{2}$$

For $k \geq 3$, a retry procedure is initiated if a fault is detected. Note that $P_A + P_B + P_C + P_D + P_E + P_F = 1$. The recursion for $\overline{\tau_k}$ is derived as follows:

$$\begin{aligned}\overline{\tau_k} = {} & P_A(\overline{\tau_{k-1}} + t_A) + P_B(\overline{\tau_{k-1}} + t_B) + P_C(\overline{\tau_k} + t_C) + P_D(\overline{\tau_{k-2}} + t_D) + \\ & P_E(\overline{\tau_{k-2}} + t_E) + P_F(\overline{\tau_{k-1}} + t_F).\end{aligned} \tag{3}$$

The DMR system is rolled back when a fault occurs in the last two intervals. The initial conditions are $\overline{\tau_1}$ and $\overline{\tau_2}$. Starting with the above recursive equation (3), the closed form of $\overline{\tau_n}$ is shown in equation (4):

$$\overline{\tau_n} = \frac{q_{DE}}{1+q_{DE}}\left(\begin{array}{l}(q_At_A + q_Bt_B + q_Ct_C + q_Dt_D + q_Et_E + q_Ft_F)((n-2)q_{DE}{}^{-1} \\ +\dfrac{1-(-q_{DE})^{n-2}}{1+q_{DE}}) + \overline{\tau}_1(q_{DE}{}^{-1} + (-q_{DE})^{n-1}) \\ +\overline{\tau}_1(2-(q_A+q_B+q_F))(q_{DE}{}^{-1} + (-q_{DE})^{n-2})\end{array}\right) \tag{4}$$

where $q_X = \dfrac{P_X}{1-P_C}$, for $X = A, B, C, D, E, F,$

$q_{DE} = q_D + q_E,$

$\overline{\tau_1} = \dfrac{T+t_r}{e^{-2\lambda T}} - t_r$, and

$\overline{\tau_2} = 2\overline{\tau_1}.$

The average completion time $\overline{\tau_n | f}$ of a job can then be derived. Note that $\overline{\tau_n | f}$ is equal to $\overline{\tau_n}$ except that at least one Poisson distributed fault (f) occurs during the execution of a job. If there are n checkpoints, the probability that there will be no fault in each execution interval is $P_A{}^n$. The value of $\overline{\tau_n | f}$ can be derived as follows [7]:

$$\overline{\tau_n | f} = \frac{\overline{\tau_n} - P_A{}^n nT}{1 - P_A{}^n}. \tag{5}$$

4.2 Burst Faults

The average completion time $\overline{\tau_n | f_b}$ when a DMR system encounters burst faults (f_b) can be derived as follows. Note that there are no faults in case (A), single faults in cases (B) and (D), multiple faults in cases (C), (E) and (F). Under burst faults, assume that cases (B) and (D) will not happen, and that their occerrence probabilities are distributed to cases (C), (E) and (F) based on their occurrence weights. Thus, we have

$$P_C' = P_C + W_C \times (P_B + P_D),$$
$$P_E' = P_E + W_E \times (P_B + P_D),$$
$$P_F' = P_F + W_F \times (P_B + P_D),$$

where $W_i = \dfrac{P_i}{P_C + P_E + P_F}$, $i = C, E,$ or F.

Therefore,

$$\overline{\tau_n | f_b} = \overline{\tau_n}$$

with P_A retained, $P_B = P_D = 0$, and P_C, P_E and P_F replaced by P_C', P_E' and P_F', respectively.

4.3 Analysis of RB and RFCS

We will derive the average completion time of RB and RFCS under Poisson distributed faults and burst faults. Since $\overline{\tau_n | f}$ can be derived from $\overline{\tau_n}$ by means of equation (5), only $\overline{\tau_n}$ for RB and RFCS are given here. Under burst faults, RB needs at least two intervals to roll back. Therefore, $\overline{\tau_n | f_b}$ of RB can be derived from $\overline{\tau_n}$ by multiplying the time of rollback interval by 2. In the case of RFCS under burst faults, situation B will not happen. Therefore, the occurrence probability of situation B is distributed to situations C and D based on their occurrence weights. Thus, we have

RB:

$$\overline{\tau_n} = n\left(\frac{T + t_r}{e^{-2\lambda T}} - t_r\right), \tag{7}$$

$$\overline{\tau_n | f_b} = n\left(\frac{2 \times (T + t_r)}{e^{-2\lambda T}} - t_r\right); \tag{8}$$

RFCS:

$$\overline{\tau_n} = \frac{q_B}{1 + q_B}\left(\begin{array}{l} (q_A t_A + q_B t_B + q_C t_C + q_D t_D)((n-2){q_B}^{-1} \\ + \dfrac{1 - (-q_B)^{n-2}}{1 + q_B}) + \overline{\tau_1}({q_B}^{-1} + (-q_B)^{n-1}) \\ +(\overline{\tau_2} - q_{AD}\overline{\tau_1})({q_B}^{-1} + (-q_B)^{n-2}) \end{array}\right), \tag{9}$$

$$\overline{\tau_n | f_b} = {q_{AD}}^{n-1}\overline{\tau_1} + (q_A t_A + q_C t_C + q_D t_D)(\frac{1 - {q_{AD}}^{n-1}}{1 - q_{AD}}). \tag{10}$$

4.4 Performance Comparison Among FCS, RB and RFCS

To conduct performance comparisons among these three schemes, we will use a hypothetical job called *job 1*. Parameters for job 1 are listed in Table 5, where T_u is defined as the *total useful execution time* of a job, t_s is the time needed to initialize a job, and t_{pr} is the time needed to initiate a spare module to retry [7]. The other notations have been defined in section 2. These parameters of checkpointing and fault recovery are very sensitive with the system environment [13], including the network delay and synchronization problem. However, they will not affect the performance comparisons.

Table 5. Parameters for job 1.

T_u	t_{ch}	t_r	t_s	t_{cc}	t_{cp}	t_{pr}
50	0.50	0.50	0.30	0.07	0.05	0.80

Comparison of the average completion time under Poisson distributed faults: In Fig. 13, $\overline{\tau_n | f}$ of each scheme is shown. The *x*-axis represents the failure rates from 10^{-2} to 10^{-8}, and the *y*-axis represents the average completion time with the optimal checkpoint number [19]. We use three miss rates to illustrate our FCS scheme. FCS_1, FCS_0.5, and FCS_0 represent the cases of $m = 1$, $m = 0.5$, and $m = 0$, respectively. RFCS has the shortest average completion time among the three schemes because it uses a spare module during a fault retry. Note that neither FCS nor RB employ any spare modules, and FCS performs better than RB. In addition, the miss rate has little impact on the average completion time under low failure rates, unless the failure rate is high. This means that the performance of our scheme is very stable.

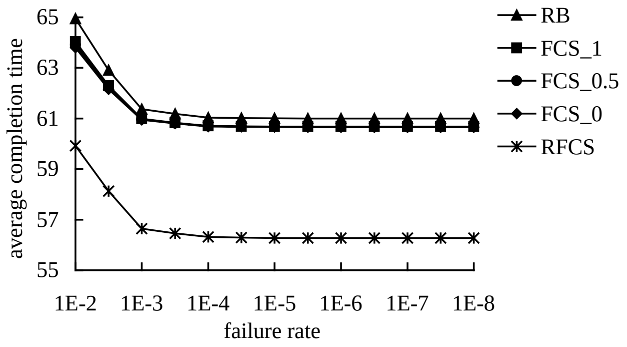


Fig. 13. $\overline{\tau_n | f}$ for job 1 with optimal checkpoint number.

Comparison of the average completion time under burst faults: In Fig. 14, $\overline{\tau_n | f_b}$ of each scheme is shown. The *x*-axis represents the failure rates from 10^{-1} to 10^{-7}, and the *y*-axis represents the average completion time with 10 checkpoints. The results show that FCS reduces by 50% the average completion time compared with RB and is comparable with RFCS in this respect. Therefore, our scheme is very suitable for situations with burst errors.

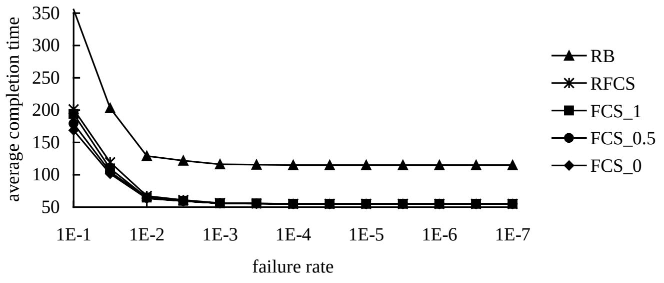


Fig. 14. $\overline{\tau_n | f_b}$ for job 1.

Comparison of the total execution time T_e (or cost-effectiveness): Without loss of generality, we only consider Poisson distributed faults. In a DMR system without a spare module for fault recovery, the total execution time is equal to twice the average completion time $\overline{\tau_n | f}$. FCS and RB are such cases because they use only two PMs. The execution time of the spare module must be considered in RFCS. That is, it uses two modules when no failures occur and three modules when a failure occurs. Four situations are considered [7]. In situation (A), there is no failure, and the number of PMs is two. In situations (B) through (D), RFCS uses a spare module to retry, and the number of PMs is three. The execution time of RFCS for each situation is listed in Table 6. For RFCS, derivation of the total execution time is similar to the derivation of $\overline{\tau_n | f}$ except that the required time for each situation is replaced by the corresponding execution time in Table 6. The total execution time for each of the three schemes (FCS, RB and RFCS) is equal to twice $\overline{\tau_n | f}$, summarized as follows:

Table 6. The execution time for each situation in RFCS.

Situation	Required execution time
(A)	$t'_A = 2T$
(B)	$t'_B = 4T + 2t_w + 2t_{cc} + 2t_u + 2t_{cp} + t_{pr}$
(C)	$t'_C = 4T + 2t_w + t_{cc} + t_u + 2t_r + t_{pr}$
(D)	$t'_D = 4T + 2t_w + 4t_{cc} + 4t_u + 2t_r + t_{pr}$

$$T_e(\text{FCS}) = 2 \times \overline{\tau_n | f}(\text{FCS}),$$
$$T_e(\text{RB}) = 2 \times \overline{\tau_n | f}(\text{RB}),$$
$$T_e(\text{RFCS}) = 2 \times \overline{\tau_n | f}(\text{RFCS}), \text{ with } \{t'_A,\ t'_B,\ t'_C,\ t'_D\} \text{ instead of } \{t_A,\ t_B,\ t_C,\ t_D\}.$$

In Fig.15, the *x*-axis represents the failure rates from 10^{-2} to 10^{-8}, and the *y*-axis represents the total execution time under Poisson distributed faults with the optimal checkpoint number [19]. In sum, the total execution time of our FCS is the smallest while that of RFCS is the largest among the three schemes under any fault distribution.

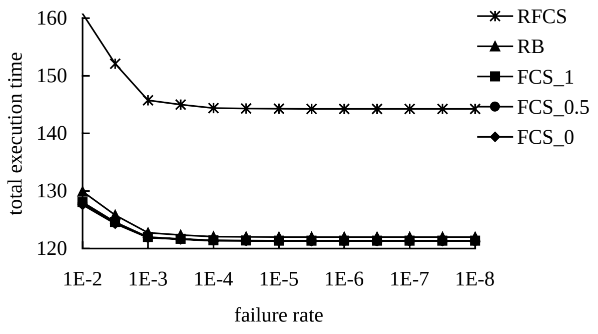


Fig. 15. T_e for job 1 with optimal checkpoint number.

5. DISCUSSION

Some implementation issues are considered here. All checkpointing schemes need reliable storages to save some checkpoints during execution of a job. Reliable storages are very important components of the proposed scheme. We assume that reliable storages are very dependable. The papers in [7, 14, 15, 17, 20] all assumed that reliable storages are available for fault recovery. The RFCS scheme requires that at most five checkpointing states be stored in the reliable storages when a failure occurs while FCS requires that at most three checkpointing states be stored for each case. In this respect, our scheme is better. Each checkpointing scheme needs a reliable checkpoint processor for comparison and synchronization. To avoid a single point of failure, the reliable checkpoint processor can be implemented by using a masking redundancy or by a distributed self-checking approach.

6. CONCLUSIONS

A very cost-effective forward recovery checkpointing scheme for multiprocessor systems with duplex modular redundancy has been described in this paper. We have compared our scheme (FCS) with the other two representative schemes: RB and RFCS. FCS only needs one retry interval to recover two transient faults at two consecutive checkpoints. RB needs at least two rollback intervals in the same fault situation. RFCS performs state restoration at the end of a retry, and execution on a faulty module is useless during the retry interval. Therefore, our scheme may be able to avoid wasting time on a faulty module during a retry interval. In addition, our scheme, without a spare module, has less system cost than those schemes which have a spare module in terms of hardware expense and the initialization time of the spare module. Mathematical models have been derived to evaluate the average completion time and the cost-effectiveness (total execution time) of the three schemes. Experimental results show that FCS can provide a better trade-off between average completion time and cost-effectiveness than the other two schemes under any fault distribution. In addition, under burst errors, the average completion time of FCS is decreased by 50% compared with that of RB and is comparable with that of RFCS. Furthermore, a processing module with a permanent fault can be located in FCS, and the size of the required reliable storage is smaller than that for RFCS.

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