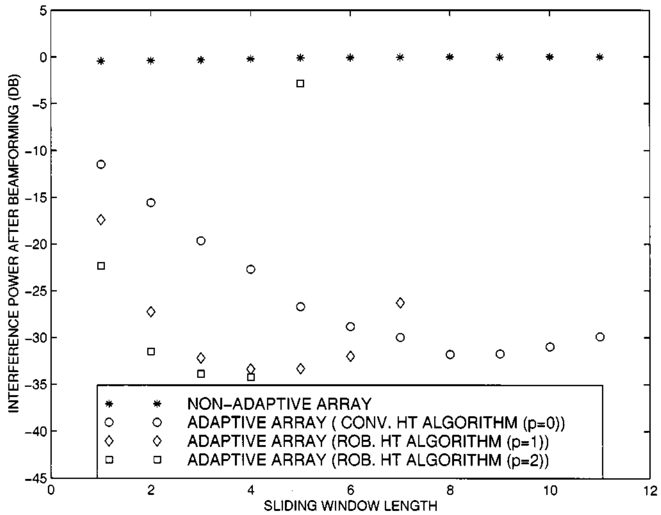


Fig. 6. Array outputs (20) versus $N$. Conventional beamformer is compared with the conventional and robust ($p = 1$) HT algorithms. The outputs (20) were additionally averaged over the snapshot index $t$.

## VI. Conclusion

Several basic adaptive beamforming algorithms have been compared using experimental shallow sea sonar data with a horizontal ULA of 15 hydrophones and moving interfering sources originated from shipping noise. Our results show the relationship between the practical performances of adaptive and nonadaptive techniques in terms of the output SINR or the related measure given by the noncompensated postbeamforming interference power. They demonstrate performance improvements that can be achieved using several robust adaptive algorithms relative to traditional adaptive beamforming techniques operating in a real sonar environment.


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# Applications of a Variable Step Size Algorithm to QCEE Adaptive IIR Filters


Pau-Lo Hsu, Tsung-Yu Tsai, and Fu-Ching Lee



***Abstract*—The quadratic constraint equation error (QCEE) method was proposed by Ho and Chan to achieve bias removal for the equation error adaptive IIR filters. However, this approach is limited to slow convergence in estimation when a small step size is used; large step sizes lead to significant misadjustment. This correspondence presents a variable step size (VSS) technique to greatly improve the QCEE under its quadratic constraint to achieve both fast convergence and reduced misadjustment for adaptive IIR filters.**



***Index Terms*—Adaptive IIR filter, estimation, quadratic constraint, step size.**


## I. Introduction

In adaptive IIR filters, the output error method is updated directly by minimizing its mean square error function. However, its error surface is not quadratic, and its gradient search algorithm may fail to reach the global minimum if initial points are not suitably chosen [2]. On the other hand, the equation error (EE) method separates an IIR filter into two FIR filters, and thus, global convergence can always be achieved because of the quadratic error surface. However, in the presence of


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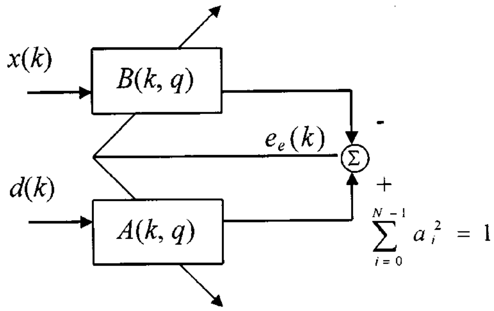


Fig. 1. Structure of QCEE adaptive IIR filters with white noise.

measurement noise, the EE approach tends to obtain biased estimation results, as reported in [1] and [3]–[5]. With a quadratic constraint $A^T A = 1$ on the feedback coefficients in the QCEE, the noise contributes a constant term to the mean square error (MSE) only. Thus, the QCEE achieves desirable estimation results without bias for the IIR adaptive filter. However, when the step size is larger in the QCEE method, the convergence speed increases, but the estimation misadjustment increases as well. Conversely, when the step size is smaller, the misadjustment decreases, but the convergence slows as well. Therefore, using a variable step size (VSS) algorithm to enhance the QCEE adaptive filter to achieve both fast convergence and suppressed misadjustment is desirable.

Several VSS methods have been employed in different applications. Harris *et al.* used two consecutive sign changes of the gradient to change the step size [6]. Park added an exponential term in the LMS adaptive algorithm [7]. Kwong and Johnston's step size depended on the variation in the mean-square error [8]. However, because the $A^T A = 1$ constraint is required in the QCEE, those reported VSS methods cannot be directly employed. In this correspondence, the projection length of $\partial \text{MSE}/\partial A$ on the span$(\perp A)$ space, which corresponds to the closeness between the estimated parameter and its optimal solution, is derived to determine the varied step size of the VSS-QCEE. Consequently, when the estimated result is far from its optimal solution, the VSS automatically increases its step size to accelerate the convergence speed. Moreover, when the estimation result gets close to its optimal solution, the step size is automatically reduced to suppress the misadjustment. Simulation results have proven the effectiveness and accuracy of the proposed VSS-QCEE.

## II. The QCEE Algorithm

Without loss of generality, the relationship between the input and output signals for an IIR filter can be written as

$$y(k) = \frac{1}{a_0^o}\left(-\sum_{i=1}^{N-1} a_i^o y(k-i) + \sum_{j=0}^{M-1} b_j^o x(k-j)\right) \tag{1}$$

where $x(k)$ is the input signal, $y(k)$ is the desired output signal, $\sum_{i=1}^{N-1} a_i^{o^2} = 1$ and $a_0^o \neq 0$ [1]. Suppose that the output noise $n(k)$ is a white noise signal with power $\sigma_n^2$ and $d(k) = y(k) + n(k)$; then, the QCEE adaptive IIR filter shown in Fig. 1 is

$$A(k, q^{-1}) = \sum_{i=0}^{N-1} a_i(k) q^{-i} \tag{2}$$

$$B(k, q^{-1}) = \sum_{j=0}^{M-1} b_j(k) q^{-j} \tag{3}$$

where $a_i$ and $b_j$ are adaptive parameters, and $q^{-i}$ is the delay $i$th sampling time. Moreover

$$R_{dd} = E[D(k)D^T(k)] \tag{4}$$

$$R_{xx} = E[X(k)X^T(k)] \tag{5}$$

$$R_{dx} = E[D(k)X^T(k)] \tag{6}$$

where $X$ and $D$ are input and output signal vectors, respectively. The covariance matrix $R$ is defined as

$$R = \begin{bmatrix} R_{dd} & -R_{dx} \\ -R_{dx} & R_{xx} \end{bmatrix}. \tag{7}$$

The equation error $e_e$ and MSE can be represented as

$$e_e(k) = \sum_{i=0}^{N-1} a_i d(k-i) - \sum_{j=0}^{M-1} b_j x(k-j) \tag{8}$$

and

$$\begin{aligned} \text{MSE} &= E[e_e^2(k)] \\ &= E\left[\left(\sum_{i=0}^{N-1} a_i y(k-i) - \sum_{j=0}^{M-1} b_j x(k-j)\right)^2\right] \\ &\quad + \sigma_n^2 \sum_{i=0}^{N-1} a_i^2. \end{aligned} \tag{9}$$

From (9), if the quadratic constraint $\sum_{i=0}^{N-1} a_i^2 = 1$ on the feedback coefficients is specified, the noise contributes to the MSE with a constant term only. As a result, the noise component in the MSE only shifts up the error surface but does not affect the global minimal location. An unbiased solution can thus be obtained by minimizing the MSE.

In the QCEE adaptation, the updating rules for the LMS adaptive method with the constraint $A^T A = 1$ at each adaptive step is as follows [1]:

$$\hat{A}(k+1) = A(k) - 2\mu_a (I_N - A(k)A^T(k)) e_e(k) D(k) \tag{10}$$

$$A(k+1) = \frac{\hat{A}(k+1)}{\|\hat{A}(k+1)\|} \tag{11}$$

and

$$B(k+1) = B(k) + 2\mu_b e_e(k) X(k) \tag{12}$$

where $\mu_a$ and $\mu_b$ are the step size for the FIR filter $A(k, q^{-1}$ and $B(k, q^{-1})$, respectively. Moreover, the condition to ensure convergence is [1], [11]

$$0 < \mu_a, \mu_b < \frac{1}{\lambda_i}, \qquad i = 1, 2, \cdots, N+M, \qquad \lambda \neq 0 \tag{13}$$

where $\lambda_i$ are the eigenvalues of the matrix $R$ in (7).

## III. Performance of the Optimal Solution on the $A^T A = 1$ Constraint

The QCEE method searches the solution $\theta = [A^T(k) \quad B^T(k)]^T$ for the minimum point $\theta^*$ on the MSE surface with the $A^T A = 1$ constraint [9]. Suppose $A^*$ and $B^*$ are optimal solutions and that they must satisfy the necessary conditions as given below.

*Lemma—Necessary Conditions for First-Order Equality Constraints [10]:* Let $\theta^*$ be a regular point of the constraint $h(\theta) = A^T A - 1 = 0$ and a local extreme point (either minimum $z \in E^n$ or maximum) of MSE is subject to this constraint. Then, all that satisfy

$$\nabla_\theta g(\theta^*) z = 0 \tag{14}$$

must also satisfy

$$\nabla_\theta \text{MSE}(\theta^*) z = 0. \tag{15}$$

In (14) and (15), $z$ can be any vector on the tangent plane of $\theta^*$, and $\nabla_\theta \mathrm{MSE}(\theta^*)$ must be orthogonal to the tangent plane of $\theta^*$. Let $A \in E^N = \Re_A$, $B \in E^M = \Re_B$, $\theta \in E^{N+M} = \Re_A \oplus \Re_B = \Re_{A+B}$ and $z \in \Re_{A+B}$, where $z = [\, z_1^T \quad z_2^T \,]^T$, $z_1 \in \Re_A$, and $z_2 \in \Re_B$ is one vector of the tangent plane on $\theta^* = [\, A^{*^T} \quad B^{*^T} \,]^T$, and

$$\nabla_\theta h(\theta^*) = [\, A^{*^T} \quad 0 \,]^T. \tag{16}$$

From (14), the vector $z = [\, z_1^T \quad z_2^T \,]^T$ on the tangent plane must satisfy

$$\nabla_\theta h(\theta^*) z = A^{*^T} \cdot z_1 + 0 \cdot Z_2 = A^{*^T} \cdot z_1 = 0. \tag{17}$$

Thus, the vector of tangent plane $z_1 \in$ span$(\perp A^*)$ and $z_2 \in \Re_B$, where span$(\perp A^*)$ refers to the space spanned by all vectors that are orthogonal to the optimal $A^*$. Moreover, (15) indicates that

$$\nabla_\theta \mathrm{MSE}(\theta^*) \cdot z = \left[ \frac{\partial \mathrm{MSE}^T}{\partial A} \, \frac{\partial \mathrm{MSE}^T}{\partial B} \right]_{A=A^*,\, B=B^*} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = 0. \tag{18}$$

Equations (17) and (18) indicate that the optimal $B^*$ at the minimum point on the $\Re_B$ space is a general FIR adaptive filter as

$$\left. \frac{\partial \mathrm{MSE}}{\partial B} \right|_{B=B^*}^T = \vec{0}. \tag{19}$$

Therefore, $\partial \mathrm{MSE}/\partial A^{*^T} \cdot z_1 = 0$ and (17)–(19) indicate that under the constraint $A^T A = 1$, $\partial$MSE$/\partial A \cdot$span$(\perp A) = 0$ at the optimal $A^*$. In the present VSS-QCEE algorithm, the projection length of $\partial$MSE$/\partial A$ on the span$(\perp A)$ can be used as the estimation of the closeness between the parameter $A$ and its optimal solution $A^*$. However, the projection length in the estimation procedure may vanish if $A$ is near an eigenvector corresponding to a saddle point of the MSE.

## IV. The VSS-QCEE Adaptive Algorithm

The step size in the QCEE adaptive algorithm is fixed [1]. In the present VSS-QCEE algorithm, we will apply the proposed VSS technique to the $A(k,\, q^{-1})$ FIR filter under a quadratic constraint. On the other hand, a fixed step size method is still applicable for the $B(k,\, q^{-1})$ FIR filter. By employing the LMS adaptive algorithm, the gradient of MSE $\nabla_\theta$MSE$(\theta) = [\, g_{A(k)}^T \quad g_{B(k)}^T \,]^T$ with $\theta = [\, A^T(k) \quad B^T(k) \,]^T$ are estimated as $[\, \hat{g}_{a(k)}^T \quad \hat{g}_{B(k)}^T \,]^T$ [11] such that

$$\hat{g}_{A(k)} = 2e_e(k)D(k) \tag{20}$$

$$\hat{g}_{B(k)} = -2e_e(k)X(k). \tag{21}$$

For this unbiased estimation method, we obtain

$$E[\hat{g}_{A(k)}] = g_{A(k)} \tag{22}$$

$$E[\hat{g}_{B(k)}] = g_{B(k)}. \tag{23}$$

In the present VSS-QCEE algorithm, the fixed step size $\mu_a$ in (10) is changed to the variable step size $\mu_a(k)$ and is varied according to the projection length of $E[\hat{g}_{A(k)}]$ on the space span$(\perp A(k))$. Moreover, $\overline{g}_{A(k)}$ can be determined to approximate $E[\hat{g}_{A(k)}]$ as

$$\overline{g}_{A(k+1)} = \alpha \overline{g}_{A(k-1)} + (1-\alpha)\hat{g}_{A(k)} \tag{24}$$

where $\alpha$ is the forgetting factor, and $\alpha < 1$. When the adaptive parameters $\theta \rightarrow \theta^*$, then $\hat{g}_{A(k+1)} \approx \hat{g}_{A(k)}$ and $\hat{g}_{B(k+1)} \approx \hat{g}_{B(k)}$. $\alpha$ is chosen as 0.99, and $\overline{g}_{A(k)}$ is thus close to $E[\hat{g}_{A(k)}]$. The projection $P_{A(k)}$ of $\overline{g}_{A(k)}$ on the space of span$(\perp A(k))$ can be calculated as

$$P_{A(k)} = \sqrt{\|\overline{g}_{A(k)}\|^2 - (\overline{g}_{A(k)}^T \cdot A(k))^2}. \tag{25}$$

Note that since the projection length will also vanish near an eigenvector corresponding to a saddle point of the MSE and a large VSS will cause unstable estimation, determination of the upper bound and the lower bound for the VSS is thus required in the present VSS-QCEE algorithm. Practically, $\mu_a(k)$ can be determined within the range $\mu_{\min} \leq \mu_a(k) \leq \mu_{\max}$ based on $P_{A(k)}$. Note that $\mu_a(k)$ must be less than the inverse of the maximal eigenvalue of the covariance matrix as in (13) [1], [11]. Thus, the step size of the VSS-QCEE algorithm can be simply set as

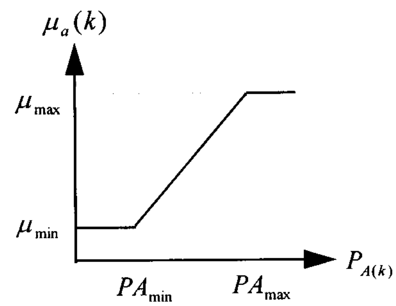


Fig. 2 Determination of the VSS $\mu_a(k)$ by the calculated projection $P_{A(k)}$.

$$\mu_a(k) = \begin{cases} \mu_{\min}, & \text{if } P_{A(k)} < PA_{\min} \\ \mu_{\max}, & \text{if } P_{A(k)} > PA_{\max} \\ \mu_{\min} + \dfrac{\mu_{\max} - \mu_{\min}}{PA_{\max} - PA_{\min}} & \\ \quad \cdot (P_{A(k)} - PA_{\min}), & \text{otherwise} \end{cases} \tag{26}$$

where $PA_{\max} > PA_{\min} > 0$. As shown in Fig. 2, a linear method was used to define $\mu_a(k)$. Generally speaking, the closer $\alpha$ is to 1, the more precise the $E[\hat{g}_{A(k)}]$ can be estimated, and a smaller $PA_{\min}$ can be set to achieve smaller misadjustment. Alternatively, when the $\alpha$ is far less than 1 in real applications, a larger $PA_{\max}$ would be more appropriate. The proposed algorithm is summarized in the Appendix.

## V. Simulation Results

To verify the present VSS-QCEE adaptive filtering, examples are provided with simulation results. Suppose we have an unknown system

$$H(z) = \frac{0.5 - 0.1z^{-1}}{1 - 1.4z^{-1} + 0.49z^{-2}} = \frac{0.2795 - 0.0599z^{-1}}{0.5590 - 0.7826z^{-1} + 0.2739z^{-2}} = \frac{B^0(z)}{A^0(z)}$$

where the second term of $H(z)$ is the normalization from the first term to satisfy the constraint $A^T A = 1$. The corresponding coefficients are $a_1 = -1.4$, $a_2 = 0.49$, $b_0 = 0.5$, $b_1 = -0.1$, $a_0^o = 0.5590$, $a_1^o = -0.7826$, $a_2^o = 0.2739$, $b_0^o = 0.2795$, and $b_1^o = -0.0599$. The initial parameters are set as $A(0) = [1 \quad 0 \quad 0]^T$, $B(0) = [0 \quad 0]^T$ in the simulation. Moreover, we define the index MSD to indicate the state of convergence as

$$\begin{aligned} \mathrm{MSD} &= \mathrm{MSD}(A) + \mathrm{MSD}(B) \\ &= 20\log(\tilde{A}^T(k)\tilde{A}(k) + \tilde{B}^T(k)\tilde{B}(k)) \end{aligned} \tag{27}$$

where $\tilde{A}(k) = A(k) - A^0$, $\tilde{B}(k) = B(k) - B^0$.

*Example 1—The VSS-QCEE Algorithm:* In this example, its input signal was white noise with power 1, and the output signal was contaminated by uncorrelated white noise with power 0.01. We set the parameters in the VSS-QCEE algorithm as $\mu_{\min} = 0.001$, $\mu_{\max} = 0.02$, $\mu_b = 0.001$, $\alpha = 0.99$, $PA_{\max} = 0.2$, and $PA_{\min} = 0.03$. Simulation results in Fig. 3(a) indicate that the misadjustment of the present VSS-QCEE was similar to the results of the QCEE with a very small step size $\mu = 0.001$. Moreover, Fig. 3(b) indicates that the present VSS-QCEE leads to a fast convergence speed, and the result is similar to that of a very large step size $\mu = 0.02$ in the QCEE. In addition, the present method achieves greatly attenuated MSD results, as shown in Fig. 3(b). In addition, because $\mu_a(k)$ is variable in the

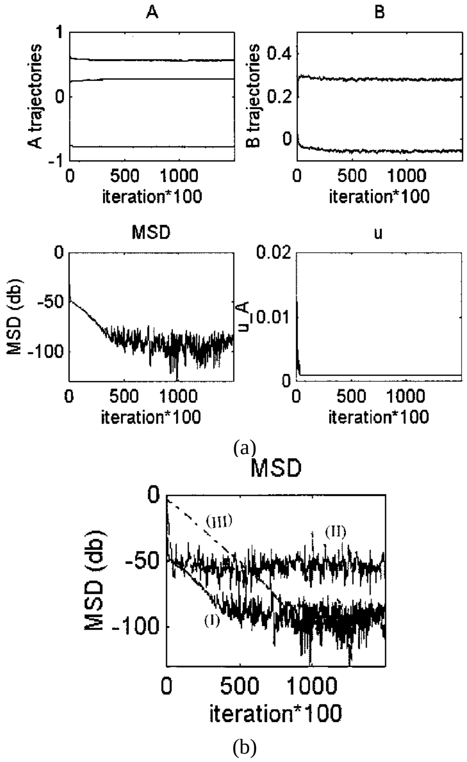


Fig. 3. (a) Estimation results of the VSS-QCEE and (b) MSD with the VSS-QCEE algorithm (solid, I), QCEE algorithm with the fixed step size $\mu = 0.02$ (dashed, II) and $\mu = 0.001$ (dash-dot, III).

present QCEE method, (10) can be processed as the same format in (12) to render similar results in real applications.

*Example 2—Applications of VSS-QCEE to Estimate Parameter Variation:* System parameters may vary, or faults may occur in the system, sensors, or actuators in real applications. In this simulation, when parameter $a_1$ was suddenly changed from −1.4 to −0.3, Fig. 4(a) shows the trajectories of the QCEE algorithm with a fixed step size $\mu = 0.001$, and Fig. 4(b) shows the results of the present VSS-QCEE algorithm. Apparently, the tracking performance of the present method is much better than the QCEE method, and a prompt indication of the parameter variation can thus be successfully set.

## VI. Conclusions

Although the QCEE algorithm achieves the bias removal, its fixed step size in the estimation brings disadvantages as either a slower converging speed or the misadjustment. This correspondence calculated the projection length of $\partial\text{MSE}/\partial A$ on the span$(\perp A)$ space to determine the varied step size of the present VSS-QCEE under the constraint $A^T A = 1$. Therefore, the step size of the present approach is independent of input or output variables and is directly determined by the estimated projection length $P_{A(k)}$ as the closeness to the optimal solution.

The proposed VSS-QCEE algorithm appropriately provides automatic adjustment for the step size in the adaptive estimation. Compared twith the QCEE, results indicate that the proposed VSS-QCEE method satisfactorily achieves both a fast convergence speed and reduced misadjustment. Moreover, in monitoring the variation of system parameters, results from the present VSS-QCEE indicate that this new method provides better tracking performance. In the present modeling, the order of the model is known, and the noise is white. For undermodeled cases, the unit-norm method converges to a model having nice impulse response and autocorrelation matching properties [9], [12]. For color noise contamination in real applications, further studies of the present VSS-QCEE are promising in future research.

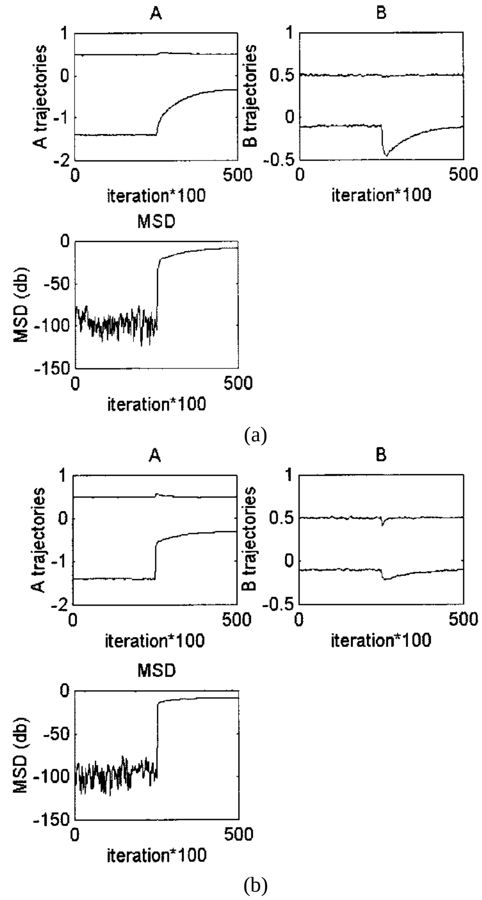


Fig. 4. Estimation results of $a_1$, $a_2$, $b_1$, and $b_2$ as $a_1$ changes from −1.4 to −0.3. (a) QCEE algorithm. (b) VSS-QCEE algorithm.

## Appendix
## VSS-QCEE Algorithm

Provided with

a) the input vector $X$ and the noisy output vector $D$;
b) the given values of $\mu_{\min}$, $\mu_{\max}$, $PA_{\min}$, $PA_{\max}$;
c) the initial values of $A = [1 \quad 0 \quad 0 \quad \cdots]$, $B = [0 \quad 0 \quad \cdots]$, $\mu_a(0) = \mu_{\min}$, $\overline{g}_A(0) = 0$, $k = 0$.

*Iteration Procedures:*

Step 1) Obtain the equation error $e_e(k)$ from (8).
Step 2) Obtain the directly calculated gradient $\hat{g}_{A(k)}$ from (20).
Step 3) Obtain the estimated gradient $\overline{g}_{A(k)}$ from (24).
Step 4) Obtain the estimated projection length $P_{A(k)}$ from (25).
Step 5) Obtain the VSS $\mu_a(k)$ from (26).
Step 6) Obtain the estimated parameters $A(k+1)$ from (10) and (11).
Step 7) Obtain the estimated parameters $B(k+1)$ from (12).
Step 8) Go to Step 1).

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# Spectral Analysis of Subband Adaptive Digital Filters

Shuichi Ohno and Hideaki Sakai

***Abstract*—Subband adaptive digital filters at stationary points are analyzed in the frequency domain. Approximate expressions for the optimal subband filters are presented, and then, a spectral representation of the error variance is derived. These are expressed in the frequency domain and, hence, enable us to see the aliasing effects in subband adaptive filtering.**

***Index Terms*—Aliasing effect, error variance, subband adaptive filtering.**

## I. Introduction

Adaptive filtering is now widely used for various applications such as adaptive system identification, adaptive equalization, and acoustic echo cancellation [1]. However, some applications that need adaptive filters with a large number of taps suffer from its heavy computational burden and slow convergence [2]. To overcome these problems, subband adaptive filtering has been proposed [3], [4].

Fig. 1 shows a schematic diagram of $M$-band subband adaptive filtering, where $x(n)$ is the input of the unknown system $F(z)$, $d(n)$ is the output of the system corrupted by a noise $w(n)$, and $\downarrow L$ and $\uparrow L$, respectively, denote the $L$-fold decimation and the $L$-fold interpolation. The input signal $x(n)$ and the desired signal $d(n)$ are split into subband input signals $v_i(n)$ and subband desired signals $d_i(n)$. Since adaptive digital filters (ADF's) are adapted in each subband, the computational complexity per input sample is reduced by a factor of the decimation rate [3]–[5]. It is also said that the subband ADF has a better convergence property since the condition numbers of correlation matrices of subband input signals are smaller than that for the fullband case [5], [6].

One of disadvantages of subband ADF is the error due to the aliasing of nonideal filters of filter banks. Gilloire and Vetterli [5] gave theoretical results on the aliasing effects for deterministic input signals and presented the subband ADF with cross filters to compensate the aliasing effects. However, the convergence speed of the subband ADF with cross filters is slow [5]. Other subband techniques avoiding the aliasing effects have been developed [7]–[10]. However, there are no simple expressions for the optimal subband filters and the error variance for stochastic input signals in the frequency domain.

In this correspondence, we give approximate expressions of the optimal subband adaptive filters by using the subband ADF with cross filters. Then, we present a spectral representation of the error variance when paraunitary filter banks are used. These representations enable us to see the aliasing effects in the conventional subband ADF. A numerical example is included to confirm our approximate expressions.

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## II. Subband Adaptive Filtering with Cross Filters

We consider $M$-band ($M \geq L$) adaptive filtering, as depicted in Fig. 1, with least mean-squared (LMS) algorithm for adaptation. We assume that all of the signals are real and that the input $x(n)$ and the additive noise $w(n)$ are (wide-sense) stationary processes with zero mean. The additive noise is assumed to be uncorrelated with the input signal. We express the $z$-transform of a sequence denoted by a small letter by the corresponding capital letter.

To analyze subband adaptive filtering, we use polyphase matrices [11]. The polyphase matrix of analysis filters $\{H_0(z),\, H_1(z),\, \cdots,\, H_{M-1}(z)\}$ is defined as

$$[H_p(z)]_{il} = \sum_n h_i(Ln+l)z^{-n} \quad \text{for } 0 \leq i \leq M-1 \text{ and } 0 \leq l \leq L-1 \tag{1}$$

where $[\cdot]_{il}$ denotes the $(i,\, l)$th element of a matrix. Similarly, the polyphase matrix of synthesis filters $\{G_0(z),\, G_1(z),\, \cdots,\, G_{M-1}(z)\}$ is defined as

$$[G_p(z)]_{L-1-l,\, i} = \sum_n g_i(Ln+l)z^{-n}. \tag{2}$$

By using the polyphase matrices and block version of signals [11]

$$x_i(n) = x(Ln-i), \quad w_i(n) = w(Ln-i) \quad \text{for } i = 0,\, \cdots,\, L-1 \tag{3}$$

the system in Fig. 1 can be express as that in Fig. 2, where $F_p(z)$ is the pseudo-circulant matrix constructed from the polyphase components of the unknown system $F(z) = \sum_{l=0}^{L-1} F_l(z^L)z^{-l}$ as

$$F_p(z) = \begin{bmatrix} F_0(z) & F_1(z) & \cdots & F_{L-1}(z) \\ z^{-1}F_{L-1}(z) & F_0(z) & \cdots & F_{L-2}(z) \\ \vdots & \vdots & \ddots & \vdots \\ z^{-1}F_1(z) & z^{-1}F_2(z) & \cdots & F_0(z) \end{bmatrix}. \tag{4}$$

Let us suppose the subband adaptive filtering with cross filters among subbands [5], in which the ADF $C(z)$ is a matrix filter. We remark that the following results for deterministic signals for $L = M$