Skyrmions as the ground states of quantum dots in strong magnetic fields

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We note that the properties of skyrmions capture many features of the ground states of quantum dots in strong magnetic fields. Therefore, in the present work we choose skyrmions with high winding numbers as trial ground states. In our results, we find that the occurrence of skyrmions fits an instability criterion of maximum density droplet very well. Interesting spin structures are found and such structures are recently supported by electron spin resonance experiments. The addition spectrum is calculated and the transition found in recent experiments is attributable to anti-skyrmion to skyrmion transition. The implication of pair tunneling from our trial wave function is discussed. [S0163-1829(99)10947-0]

The electronic structure of a quantum dot (QD) has been of great interest both theoretically and experimentally. The energy levels of a quantum dot can be inferred from the ''addition spectrum,'' i.e., the chemical potential $\mu_N=E_N^0-E_{N-1}^0$ [where E_N^0 is the ground-state energy of an N-electron-QD (Ref. 1)], obtained by transport measurements. In these experiments, electron-electron Coulomb interaction is proved to be of vital importance. For example, the phenomenological capacitance,[2] and Anderson model,[3] which consider the electron-electron interaction, are able to describe the correlation effect in QD to some extent. When the magnetic field B is applied, interesting phenomena can also be observed from the B-N phase diagram.[1,4–6] The phase diagram tells us how the ground state behaves as the magnetic field is changed. It was also shown that the ground states of QD in strong-magnetic field [or quantum Hall droplet (QHD)] may be the precursor of fractional quantum Hall (FQH) states.[7] Thus, there are interesting possibilities that one may observe some topological orders, similar to that of FQH, in a QHD.[8] This suggested that the electron-electron correlation in QHD would be very different from that of QD without magnetic field.

MacDonald *et al.*[9] have pointed out that for a N-electron QHD, the maximum density droplet (MDD) state ($|\mathrm{MDD}\rangle=\Pi_{m=0}^{N-1}a_{m\downarrow}^{+}|0\rangle$, where symmetric gauge is used, m is the orbital angular momentum and the magnetic field is applied in the $-z$ direction through out the paper) is the only eigenstate with the smallest orbital angular momentum (L_z) if all electrons are in the lowest Landau level (LLL). It would also be the exact ground state in the LLL approximation if the confining potential, $V(r)=\gamma r^2/2$, is steep enough (i.e., large γ). As the confining potential becomes smooth, the MDD state would expand to a larger droplet provided that the gain in Hartree energy exceeds the loss of confining energy. For a spin polarized system in the large N limit, MacDonald *et al.*[9] have shown an instability criterion for the MDD state. The instability is caused by the excitation of the $m=0$ electron to the edge and the MDD state becomes unstable when the following criterion is encountered

$$\overline{\gamma}_{crit}<0.514/\sqrt{N}, \tag{1}$$

where ε is the dielectric constant and $l=\sqrt{\hbar c/(eB)}$ is the magnetic length. (The dimensionless quantity $\overline{\gamma}$ is defined as $\overline{\gamma}\equiv\gamma l^2/(e^2/\varepsilon l)$. For GaAs, $\varepsilon\simeq 12.6$, $l=25.65/\sqrt{B[T]}$ (nm), and $e^2/(\varepsilon l)\simeq 4.4528\sqrt{B[T]}$ (meV).) We will show later on that the above criterion is also approximately valid when the MDD instability is caused by the excitation of $m\neq 0$ electron.[10]

While MacDonald *et al.* considered the excitation of *one*-electron out of the MDD state, Chamon and Wen[11] consider the occurrence strips in the edge. The origin of the strip state is the short range attractive exchange energy. In their Hartree-Fock approximation treatment, the MDD instability occurs if $\overline{\gamma}<0.083\ln(N/0.21)/\sqrt{N}$ for N lies between 20 and 70. Recently, calculations based on the density-functional theory were also performed on the same problem.[10,12–14]

On the other hand, for the $\nu=1$ quantum Hall system, it is known that the charge one excitation is a winding number 1 skyrmion with a large number of spin flipped.[15–18] Recently, Lilliehöök *et al.*[19] have also shown that the energy of a winding number 2 skyrmion is lower than the energy of two winding number one skyrmions in the limit of zero Zeeman energy. In the edge of a $\nu=1$ quantum Hall bar system, it is also shown that some edge spin texture (that is, the spin rotates in the x-y plane along the edge) developed when the confining potential is smooth enough.[20–22] If we consider the edge of a quantum Hall bar as the edge of a circular quantum Hall system with an infinite radius and smooth confining potential, then the edge spin texture will be similar to that of a skyrmion with a high winding number. In fact, if we replace the momentum k in the wave function of a quantum Hall bar by the angular momentum m of a circular quantum Hall system, the wave functions of the edge spin texture and the skyrmion will be similar within the framework of Hartree Fock approximation. The only exceptions are that the former has a higher winding number and its spins are all flipped with respect to the latter. In the center of a QHD, we expected the electrons behave as the skyrmion in a bulk quantum Hall system, but in the edge of a QHD one expects the electrons behave as the edge spin texture near the edge of a quantum Hall bar. This poses a question that whether the ground state of a QHD, which captures features of both bulk and edge quantum Hall system, is similar to that of a skyrmion or the edge spin texture.

In fact, in their nearly exact wave functions, Oaknin *et al.*[23] have shown that the lowest energy state in the (N,M) subspace (M is the total orbital angular momentum of a QHD) is of skyrmionic type. For these reasons, we choose the winding number Q skyrmions as the trial ground states

$$|Sky\rangle=\Pi_{m=0}^{N-1}(u_m a_{m\uparrow}^{+}+v_m a_{m+Q\downarrow}^{+})|0\rangle \tag{2}$$

for the quadratic confining potential $[V(r)=\gamma r^2/2]$. The Hamiltonian of the QHD is

$$H=H_p+H_Z+H_V, \tag{3}$$

$$H_p=\gamma l^2\sum_{m=0}(m+1)(a_{m\uparrow}^{+}a_{m\uparrow}+a_{m\downarrow}^{+}a_{m\downarrow}),$$

$$H_Z=-\frac{\Delta_Z}{2}\sum_{m=0}(a_{m\downarrow}^{+}a_{m\downarrow}-a_{m\uparrow}^{+}a_{m\uparrow}), \tag{4}$$

$$H_V=\frac{1}{2}\sum_{m_1,\dots,m_4}V_{m_1m_2m_3m_4}:(a_{m_1\uparrow}^{+}a_{m_2\uparrow}+a_{m_1\downarrow}^{+}a_{m_2\downarrow})\times(a_{m_3\uparrow}^{+}a_{m_4\uparrow}+a_{m_3\downarrow}^{+}a_{m_4\downarrow}):,$$

and

$$V_{m_1m_2m_3m_4}=\int\int d^2\mathbf{r}_1 d^2\mathbf{r}_2\,\phi_{m_1}^{*}(\mathbf{r}_1)\phi_{m_2}(\mathbf{r}_2)\frac{e^2}{\varepsilon|\mathbf{r}_1-\mathbf{r}_2|}\times\phi_{m_3}^{*}(\mathbf{r}_3)\phi_{m_4}(\mathbf{r}_4),$$

$$\phi_m(\mathbf{r})=\frac{(z/l)^m e^{-(r^2/4l^2)}}{\sqrt{\pi 2^{m+1}m!\,l^2}},$$

where $\Delta_Z=g\mu_B B$ is the Zeeman splitting energy. The form of ground state in Eq. (2) is exactly the winding number Q skyrmion.[17,19] If we flip all the spins in Eq. (2) then we have exactly the wave function of edge spin textures,[20–22] i.e., $|EST\rangle=\Pi_{m=0}^{N-1}(u_m a_{m\downarrow}^{+}+v_m a_{m+Q\uparrow}^{+})|0\rangle$. The two states have the same energy when the Zeeman energy is zero.

We compared the energies of MDD states and that of skyrmions of Eq. (2) from $Q=1$ to 18 when the Zeeman energy is zero. The result is shown in Fig. 1. Figure 1 presents the dependence of winding number Q of skyrmion (which has the smallest energy among these 19 states) on the value of $\overline{\gamma}$. Our results agree well with the criterion [Eq. (1)] of the instability of MDD states. For example, for $N=35$ (65), the MDD instability occurs when $\overline{\gamma}=0.08$ (0.067), which should be compared with 0.0856 (0.0636) obtained by Eq. (1). This result is better than that of spin polarized strip state[11] which yielded $\overline{\gamma}=0.0718$ (0.059). Although the instability criterion for spin polarized and spin unpolarized QHD would be different, one expects that the numerical values of these two criteria would not differ too much.

We may explain the overall features of Fig. 1 qualitatively by classical approximation.[7] Assuming that the electron density distribution ρ_c is uniform inside a disk of radius R and drops to zero abruptly outside that disk, i.e., $\rho_c(r)=N/(2\pi l^2 N_\phi)$ when $r<R$ and $\rho_c(r)=0$ when $r>R$, where

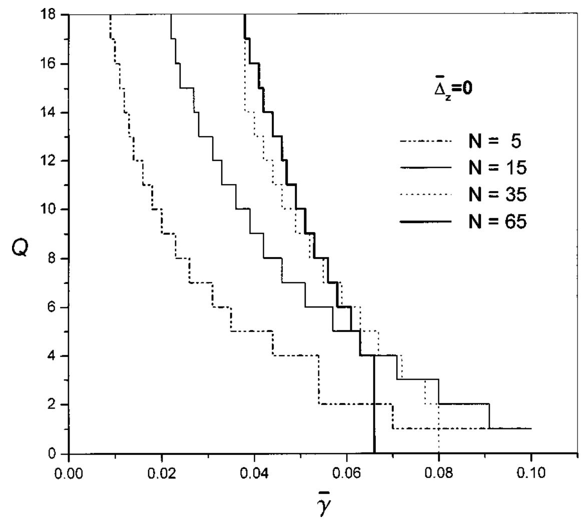

FIG. 1. The dependence of winding number Q of skyrmions on the confining potential $\overline{\gamma}=\gamma l^2/(e^2/\varepsilon l)$ for different numbers N of electrons. The Zeeman energy is zero. The state with $Q=0$ is the MDD state. The instability of MDD states for $N=65$ and $N=35$ occurs around the criterion of Eq. (1).

N_ϕ is the largest angular momentum m being occupied by electrons and $R=l\sqrt{2N_\phi}$. For our trial wave functions in Eq. (2), $N_\phi=N+Q$. The Coulomb potential energy (Hartree energy) E_H and the confinement energy E_C can be integrated to be

$$E_H=\frac{8\pi}{3}\frac{(\rho_c e)^2}{\varepsilon}R^3=\frac{4\sqrt{2}N^2}{3\pi\sqrt{N_\phi}}\left(\frac{e^2}{\varepsilon l}\right)\sim 0.6\frac{N^2}{\sqrt{N_\phi}}\left(\frac{e^2}{\varepsilon l}\right) \tag{5}$$

$$E_C=\frac{\pi}{4}\gamma\rho_c R^4=\frac{1}{2}\gamma l^2 N N_\phi. \tag{6}$$

One notes that the Zeeman energy varied as the first power of N while E_H and E_C varied roughly as $N^{3/2}$ and N^2, respectively, as can be noted in the above equations. Therefore, Zeeman energy can be safely ignored if only the distribution of charge is concerned. For a given value of γ and N, we can minimize the energy E_H+E_C with respect to the size (N_ϕ) of the droplet. This gives us the result

$$N_\phi=N+Q=\left(\frac{4\sqrt{2}N}{3\pi\overline{\gamma}}\right)^{2/3}\sim 0.711\left(\frac{N}{\overline{\gamma}}\right)^{2/3}. \tag{7}$$

We can prove the validity of the classical approximation by letting $Q=0$ in the above equation which corresponds to the MDD state instability. The calculated criterion $\overline{\gamma}_{crit}^{c}$ fits to that in Eq. (1) very well: $\overline{\gamma}_{crit}^{c}=0.6/\sqrt{N}\sim 1.17\overline{\gamma}_{crit}$. The overestimation of $\overline{\gamma}_{crit}$ is due to the absence of exchange energy in the classical approximation. If we substitute $\overline{\gamma}$ by $1.17\overline{\gamma}$ in the right hand side of Eq. (7), i.e., $N_\phi=0.711(N/(1.17\overline{\gamma}))^{2/3}=0.64(N/\overline{\gamma})^{2/3}$, then we find that the corrected N_ϕ (or Q) roughly resembles the curves in Fig. 1.

Figure 2 shows the spin polarization of the ground states of Fig. 1. The cusps occur when Q changes abruptly. We note that when the confining potential approaches zero, the

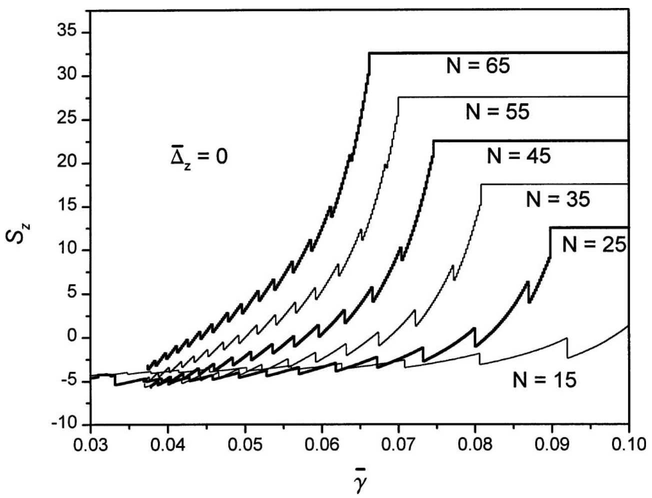

FIG. 2. The dependence of spin S_z on confining potential $\overline{\gamma} = \gamma l^2/(e^2/\varepsilon l)$ for different numbers N of electrons. The Zeeman energy is zero. The flat regions are the MDD states. The instability of MDD states for $N \geqslant 25$ occurs around the criterion of Eq. (1).

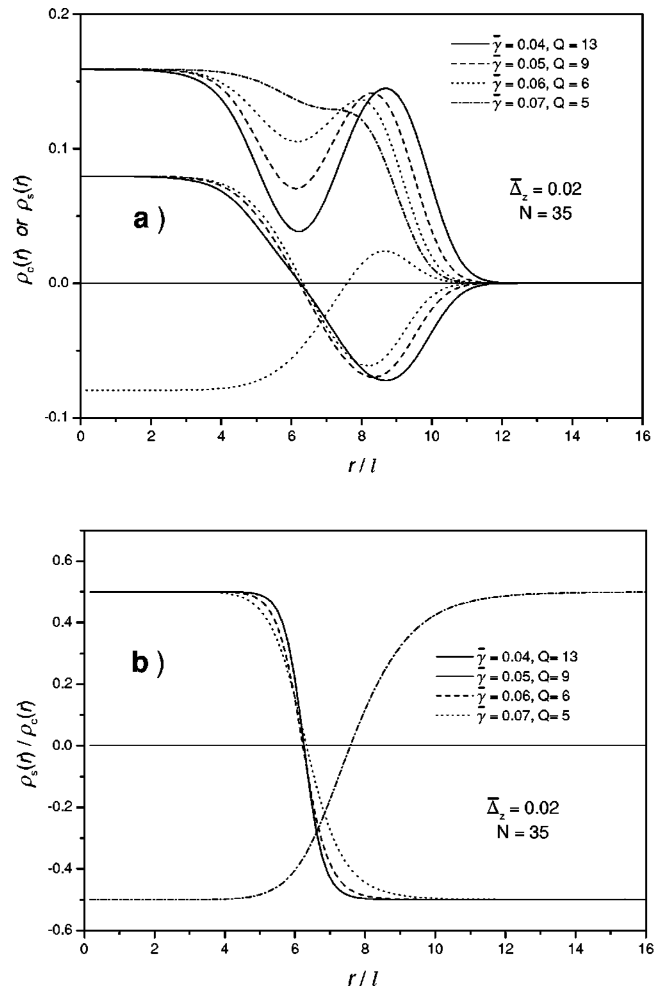

FIG. 4. (a) The charge density ρ_c and spin density ρ_s profiles of skyrmions for different confining potential $\overline{\gamma} = \gamma l^2/(e^2/\varepsilon l)$. The particle number is fixed to be $N=35$ and $\overline{\Delta}_z = \Delta_z/(e^2/\varepsilon l) = 0.02$. (b) The normalized spin ρ_s/ρ_c profile of (a).

spin polarization also becomes smaller. This is consistent with the exact results of few-particle systems[7] and the behavior of skyrmions in quantum Hall systems. Our results suggest that there are huge changes in the magnetic properties when the confining potential is changed. This can be verified by the measurement of spin susceptibility or circular polarized photoluminescence spectra.[24]

The dependence of the trial ground states (skyrmion or edge spin texture) on Zeeman energy is shown in Fig. 3 for different confining potential with fixed number of electrons, i.e., $N=35$. Note that when the Zeeman energy is zero, the energy of skyrmion is the same as that of the edge spin texture. But the energies are different for these two states when the Zeeman energy is not zero. We have calculated the energies of these two states and found that skyrmion [Eq. (2)] always have smaller energy except in the vicinity of MDD instability. That is, the trial ground state will change

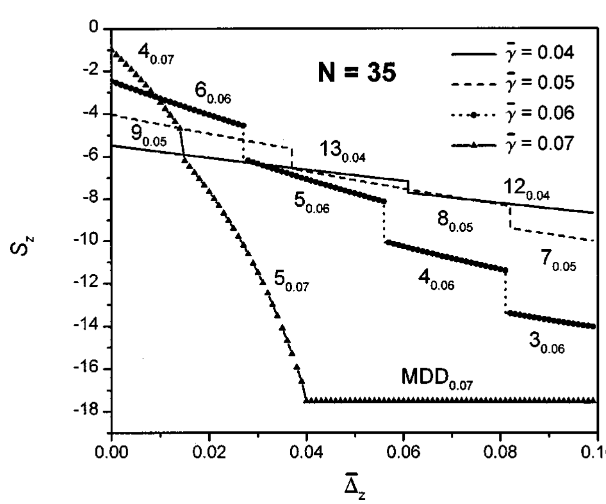

FIG. 3. The dependence of spin S_z on Zeeman energy and on different confining potential $\overline{\gamma} = \gamma l^2/(e^2/\varepsilon l)$ for a QHD of $N=35$ electrons. The integers adjacent to each line segments are the winding number Q of skyrmions and the subscript of Q is referred to $\overline{\gamma}$.

from MDD to edge spin texture and than to skyrmion as the confining potential is reduced. As expected, the spin polarization becomes larger when the Zeeman energy is increased. However, there are no reasons why the winding number Q or the size of QHD should be reduced (increased) for skyrmion (edge spin texture) when the Zeeman energy is increased. This may be due to our choice of trial ground states, which are not valid in the infinite Zeeman energy limit. But the invalidity does not pose serious problems when electron number is large. As one can note from Fig. 3 that the Q value does not change appreciably in the practical experimental range of Zeeman energies for GaAs, i.e., $\overline{\Delta}_z \equiv \Delta_z/(e^2/\varepsilon l) \lesssim 0.02$.

The charge density $\rho_c(r)$ and spin density $\rho_s(r)$ profiles are shown in Fig. 4(a). The oscillation of charge density near the edge is consistent with the exact results[11] of few-particle systems and resembles that of the Hartree-Fock approximation of edge spin texture of quantum Hall bar.[20–22] With classical approximation, we can also demonstrate, as follows, the instability of an uniform disk to the oscillation of charges. The Coulomb potential at position $\mathbf{r}$ produced by an uniform disk is

$$U_H(\mathbf{r})=\rho_c\int_0^{2\pi}d\theta\int_0^{r_\theta}dr'r'\frac{e^2}{\varepsilon r'}=\frac{2\sqrt{2}}{\pi}\frac{N}{\sqrt{N_\phi}}E\left(\frac{\pi}{2},\overline{r^2}\right)\left(\frac{e^2}{\varepsilon l}\right)$$

$$\cong 1.067\overline{\gamma}^{1/3}N^{2/3}E\left(\frac{\pi}{2},\overline{r^2}\right)\left(\frac{e^2}{\varepsilon l}\right), \tag{8}$$

where $r_\theta=r\cos\theta+\sqrt{r^2\cos^2\theta+(R^2-r^2)}$, $\overline{r}=r/R$, and $E(\alpha,\beta)$ is the elliptic function of the second kind. The last equality comes from Eq. (7). The confining potential at the same position is $U_C(\mathbf{r})=\gamma/2r^2=\gamma l^2N_\phi\overline{r^2}\cong 0.711\overline{\gamma}^{1/3}N^{2/3}\overline{r^2}(e^2/\varepsilon l)$. If one electron is removed from the uniform disk, then the removal of the electron locates at the position $r\cong 0.89R$ is favored because $U_H(\mathbf{r})+U_C(\mathbf{r})$ is maximum there. If the corrected N_ϕ $[=0.64(N/\overline{\gamma})^{2/3}]$ is used, then the depletion of electron would be occurred around $r\cong 0.81R$. This signifies the depletion of charges as shown in Fig. 4(a) and previous works.[10–12]

There are some differences between the usual edge spin textures of quantum Hall bar and our trial wave functions here. Our results show that both profiles oscillate more violently than the usual edge spin texture does. We have also observed in Fig. 4(b) that the profile of polarization density $[\rho_s(r)/\rho_c(r)]$ changes from 0.5 (-0.5) to -0.5 (0.5) quickly around the vicinity of the dip ($r/l\sim 6$) of density profile for skyrmion (edge spin texture) as the radius is increased. For larger winding number Q, a more flat line of $\rho_s(r)/\rho_c(r)=-0.5$ for $r/l\gtrsim 7$ is observed. Such spin configurations have recently been proposed[25] and confirmed from the electron spin resonance experiments. Our results suggested that there are rich spin structures in QHD and it may impose some novel transport properties, such as spin blockade,[26] involving the spin degree of freedom.

While the previous results are formulated in the dimensionless quantities $\overline{\gamma}$ and $\overline{\Delta}_z$, the experiments of B-N phase diagram[6] are performed at different magnetic fields B when the parameter γ of confining potential is fixed at some constant. The corresponding energy scales such as confining energy, Zeeman energy and Coulomb energy are then varied with magnetic field as $\gamma l^2=(\gamma\hbar c/e)B^{-1}$, $\Delta_z=(g\mu_B)B$, and $e^2/\varepsilon l=((e^2)/\varepsilon\sqrt{e/\hbar c})B^{1/2}$, respectively. Because the confining energy varies as B^{-1}, it will be the dominating energy in the low magnetic field limit. It is then easily understood that when the magnetic field is increased from zero, the filling fraction ν of QHD drops continuously until the MDD state ($\nu=1$) is reached and remained in that state for some range of magnetic field. For even larger magnetic field, the contribution from Coulomb energy (which is proportional to $B^{1/2}$) becomes important. The MDD state will thus becomes unstable and the QHD would expand to reduce the Coulomb energy. This makes the filling fraction being reduced further to be $\nu\lesssim 1$. If the magnetic field is increased further,[12] the spins of QHD will be polarized. The increment of magnetic field will trace a curve of $\overline{\gamma}\cdot\overline{\Delta}_z^3=$ constant, where the constant is dependent on the confining potential. Because our numerical results are obtained from LLL approximation, thus they will be valid between the magnetic field that makes the filling fraction of QHD just being smaller than 1 and the magnetic field that makes the onset of fully spin-polarized QHD.

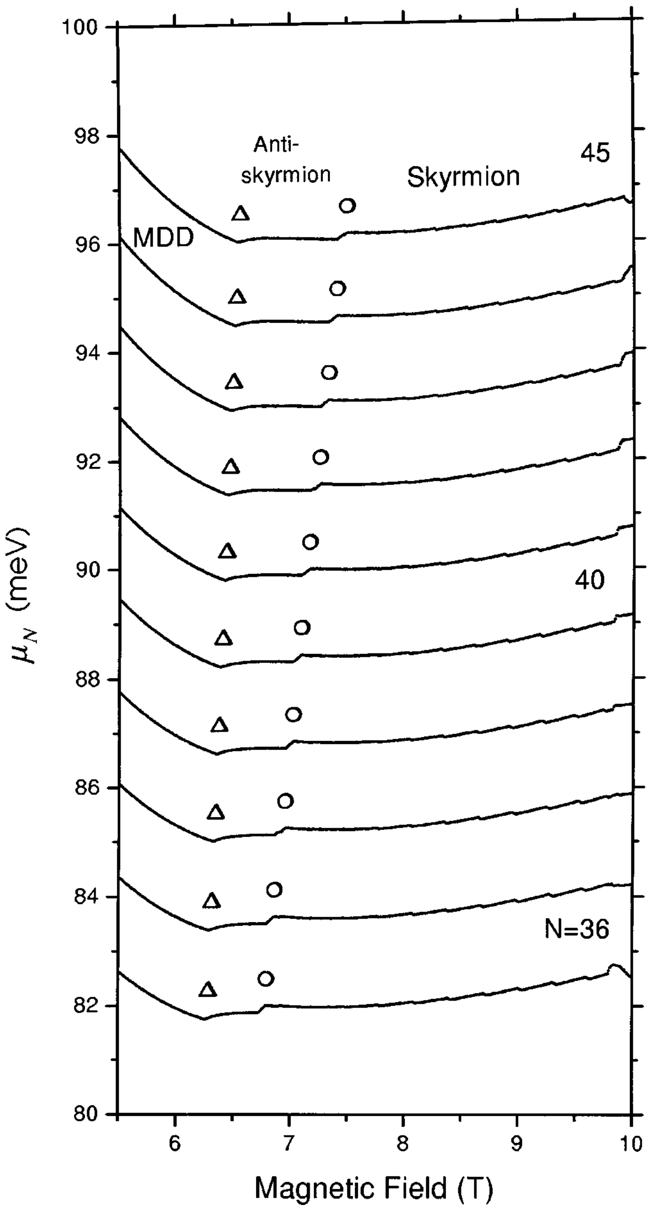

FIG. 5. The addition spectrum for a QHD where $\hbar\omega_0=3$ meV. The electron number ranges from 36 to 45. The region to the left of the open triangle is the MDD state, and the one to the right of open circle is the skyrmion. The region between them is the antiskyrmion. On increasing magnetic field, the winding number Q increases except in the vicinity of antiskyrmion to skyrmion transition where the winding number is deceased by 1.

Figure 5 shows the addition spectrum of a QHD for the electron number ranging from 36 to 45. The g factor in GaAs is chosen to be $g=0.44$ and the characteristic frequency ω_0 ($\gamma=m\omega_0^2$) is set as $\hbar\omega_0=3$ meV in the figure. On increasing magnetic field, the QHD evolves from MDD state to anti-skyrmion at the open triangle in Fig. 5. Antiskyrmion to skyrmion transition occurs when the magnetic field increases beyond the open circle in Fig. 5. There may be even possible a skyrmion to charge density wave transition for further larger magnetic field. In this work, we are not interested in that. The behavior of the calculated addition spectrum agrees with the recent experimental results of Ref. 6. It was pointed out in Ref. 6 that there are abrupt redistribution of charge when the two transition points are encountered. This is consistent with our calculation that the antiskyrmion adjacent to the MDD state owns a larger winding number ($Q=4$ or 5) rather than $Q=1$. The antiskyrmion to skyrmion transition is also accompanied with the decrement of winding number by 1. The features of the new transition discovered in recent experiment[6] are not fully understood. In the present paper, we consider the transition to be the antiskyrmion to skyrmion transition, but it remains to be justified from other experiments such as spin blockade. Since the skyrmion and the antiskyrmion have different spin orientations in the outer ring of QHD, it may result in abrupt change in tunneling phenomena if spin blockade is concerned.

It is interesting to note that, there are experiments[27] indicate the periodicity in addition spectrum. Among them, period 2 or pair tunneling at low magnetic field may be the most interesting one.[28] Some theories[29,30] have been proposed to explain it by relating the phenomena to the presence of impurities. Although there are no pairing observed from Fig. 5, we would like to point out another possible mechanism to explain the above experimental observation based on our trial wave function. The wave function in Eq. (2) looks similar to the BCS ground state. In fact if we define the vacuum as $|\mathrm{vac.}\rangle=\Pi_{m=0}^{N-1}a_{m\uparrow}^{+}|0\rangle$ and make the transformation $a_{m\uparrow}\rightarrow c_{-m\uparrow}^{+}$, for $m<N$, and $a_{m\sigma}\rightarrow c_{m\sigma}$ otherwise, then Eq. (2) can be rewritten as

$$|Sky\rangle=\Pi_{m=0}^{N-1}(u_m+v_m c_{-m\uparrow}^{+}c_{m+Q\downarrow}^{+})|\mathrm{vac.}\rangle, \qquad (9)$$

which is just the (one dimensional) BCS ground state except that $Q\neq 0$. If the coherence is preserved even when the magnetic field approaches zero, it can explain the pair tunneling by relating it to the one observed in superconducting island.[31,32] Measurement of Meisner effects of quantum dot at low-magnetic field will give some insight to the problem.

In summary, we consider the high winding number skyrmions as the trial ground states of QHD and find that they fit well the instability criterion of the MDD state. Our trial wave function would be better than the spin-polarized charge density wave in the low-Zeeman energy region. For the charge and spin structures, we found that there is a core where electrons lump together and an outer ring of electrons around the core. The spin of the core and the ring are almost totally polarized in the opposite directions. There are some experiments that support the results. We attribute the recently discovered transition to be anti-skyrmion to skyrmion transition and find that the addition spectrum agree with recent experimental results. Similarity of our trial wave function and BCS ground state is pointed out and the implication of pair tunneling is discussed.

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