Efficient Algorithm for Reliability of a Circular Consecutive-k-out-of-n:F System

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Key Words — **Circular consecutive-*k*-out-of-*n*:F system, System reliability, Algorithm.**

Reader Aids —
Purpose: Report a new algorithm
Special math needed for explanations: Probability theory
Special math needed to use results: Same
Results useful to: Reliability analysts and theoreticians

Summary & Conclusions — **The time complexities of previously published algorithms for circular consecutive-*k*-out-of-*n*:F system are $O(n \cdot k^2)$ and $O(n \cdot k)$. This paper proposes a method to improve upon the original $O(n \cdot k^2)$ algorithm and hence derives an $O(n \cdot k)$ algorithm.**

1. INTRODUCTION

A consecutive-k-out-of-n:F system consists of a sequence of n ordered components where each component either functions or fails. The system fails if and only if at least k consecutive components fail. There are two topologies for this system: a line and a circle. The reliability analysis of such systems was first studied by Chiang & Niu [2], and later by Derman, Lieberman, Ross [3], Hwang [4], Shanthikumar [5], Antonopoulou & Papastavridis [1], and Wu & Chen [6]. Hwang [4] proposed a recursive $O(n)$ algorithm for linear consecutive-k-out-of-n:F system and an $O(n \cdot k^2)$ algorithm for a circular consecutive-k-out-of-n:F system. Antonopoulou & Papastavridis [1] announced that they had found an $O(n \cdot k)$ recursive algorithm for computing the reliability of a circular system. Wu & Chen [6] also found a new $O(n \cdot k)$ algorithm for such system. This paper proposes an improvement upon the Hwang [4] $O(n \cdot k^2)$ algorithm and introduces a new $O(n \cdot k)$ algorithm.

2. MODEL

Assumptions

1. Each component, subsystem, and system either functions or fails.
2. All n component-states are mutually s-independent.
3. Components 1, 2, ..., n are arranged as to form a circle in that order.
4. The system or subsystem fails if and only if at least k consecutive components fail.

Notation

- n — number of components in a system
- k — minimum number of consecutive failed components that causes system failure
- p_i, q_i — probability that component i [functions, fails]; $q_i + p_i \equiv 1$
- $R_L(i,j)$, $R_C(i,j)$ — reliability of [linear, circular] system consisting of components i, $i+1$, $i+2$, ..., j

Other, standard notation is given in "Information for Readers & Authors" at the rear of each issue.

3. IMPROVEMENT

Hwang [4] proposed a recursive $O(n)$ algorithm for linear consecutive-k-out-of-n:F systems and an $O(n \cdot k^2)$ algorithm for circular consecutive-k-out-of-n:F system. The $O(n \cdot k^2)$ algorithm is:

$$R_C(1,n) = \sum_{s-1+n-l<k} \left\{ \prod_{i=1}^{s-1} q_i \cdot p_s \cdot R_L(s+1,l-1) \cdot p_l \cdot \prod_{i=l+1}^{n} q_i \right\} \tag{1}$$

Now, we derive the $O(n \cdot k)$ algorithm for circular consecutive-k-out-of-n:F systems by (1):

$$R_C(1,n) = \sum_{s=1}^{k} \sum_{l=n-k+s}^{n} \left\{ \prod_{i=1}^{s-1} q_i \cdot p_s \cdot R_L(s+1,l-1) \cdot p_l \cdot \prod_{i=l+1}^{n} q_i \right\}$$

$$= \sum_{i=2}^{k+1} \sum_{j=n-k-2+i}^{n-1} \left\{ \prod_{m=1}^{i-2} q_m \cdot p_{i-1} \cdot R_L(i,j) \cdot p_{j+1} \cdot \prod_{m=j+2}^{n} q_m \right\}. \tag{2}$$

In (2), we need to generate:

$R_L(2,n-k)$, ..., $R_L(2,n-1)$;

$R_L(3,n-k+1)$, ..., $R_L(3,n-1)$; ...; $R_L(k+1,n-1)$

$$\prod_{i=1}^{1} q_i, \ldots, \prod_{i=1}^{k-1} q_i;$$

$$\prod_{i=n-k+2}^{n} q_i, \ldots, \prod_{i=n}^{n} q_i.$$

The Hwang [4] recursive $O(n)$ algorithm can be expressed as:

$$R_L(1,n) = R_L(1,n-1) - R_L(1,n-k-1) \cdot p_{n-k} \cdot \prod_{i=n-k+1}^{n} q_i \quad (3)$$

While computing $R_L(1,n)$, we also get $R_L(1,1)$, $R_L(1,2)$, ..., $R_L(1,n-1)$; that is important. Using this property, we can evaluate: $R_L(2,n-k)$, ..., $R_L(2,n-1)$; $R_L(3,n-k+1)$, ..., $R_L(3,n-1)$; ...; $R_L(k+1,n-1)$ with time complexity $O(n-2) + O(n-3) + \ldots + O(n-k-1) = O(n \cdot k)$. And we can compute $\Pi_{i=1}^{1} q_i$, ..., $\Pi_{i=1}^{k-1} q_i$; $\Pi_{i=n-k+2}^{n} q_i$, ..., $\Pi_{i=n}^{n} q_i$ in $O(k)$. Store all derived values in memory. The sum in (2) contains $½k(k+1)$ terms, so the time complexity for computing $R_C(1,n)$ is: $O(n \cdot k) + O(k) + O(k^2) = O(n \cdot k)$, for $n > k$.

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