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Some Modifications of Gradient Weighted Filters[1]

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Gradient weighted filters are locally adaptive weighted mean filters. In this paper, a general formulation of gradient weighted filters with some characteristic parameters was derived first from existing gradient weighted filters. Then we propose some modifications by varying these parameters. We modify gradient inverse weighted filters, characterize filters into first and second order filters, and propose Π filters. Moreover, an imposed criterion for second order filters to preserve fine details was introduced to promote the existing gradient weighted filters. Finally, a criterion to combine first and second order filters was proposed to remove noise with mixed types. Throughout this paper, rational analysis and experimental results demonstrate the efficiency of the proposed methods. © 1999 Academic Press

Key Words: gradient weighted filters; rational filters; image smoothing; Π filters.

1. INTRODUCTION

Smoothing filter design in image processing basically depends on the types of noise. Rank order based filters such as median filters are good at removing additive impulsive noise and linear filters are good at suppressing Gaussian noise. However, most of them suffer from the trade-off between removing noise and preserving details. Moreover, as the types and the amount of noise are mixed diversely, noise removal continues to provide a challenge to smoothing filter designers.

Weighted mean filters are commonly used spatial filters based directly on the local intensity information. They replace the intensity of the pixel to be processed by the weighted average of the intensities of its neighbors. One major drawback of these filters is that they will blur the sharpness of edges. Adaptive weighted filters are then proposed to avoid this drawback; most of them are based on local gradient information. Examples of such gradient-based filters include gradient inverse weighted filters [8, 9], sigma filters [1], adaptive Gaussian weighted filters [10], and rational filters [3–7].

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Gradient inverse weighted filters use the inverses of the absolute gradients formed by differences between the pixel to be processed and its neighbors as the weighting coefficients. Sigma filters replace the intensity of the pixel to be processed by the average intensity of only those neighbors such that the corresponding gradients are within a fixed sigma range. In adaptive Gaussian weighted filters, the weighting coefficients are determined by a function of gradients and variances. Then in rational filters, the weighting coefficients are reciprocal to the squares of gradients formed by the two extremes of directional masks. A common idea of these filters is that weighting coefficients are chosen to be reciprocal to the corresponding gradients.

Given a 3×3 mask centered at a pixel p. The local gradients associated with p are usually produced by taking the intensity differences of p from its eight neighbors, respectively. However, in rational filters [6], taking the intensity differences of two extremes of the four directional masks produces the employed gradients. To distinguish them, we will call the former *first order local gradients* and the latter *second order local gradients*. Previous works [1, 8, 10] have shown that filters equipped with first order local gradients are suitable for suppressing uniform or Gaussian noise while those equipped with second order gradients are suitable for removing impulsive noise. However, from our observations, filters equipped with first local gradients preserve lines better than those equipped with second order gradients. Therefore, it is our purpose in this paper to propose an approach to combine all good features of the gradient weighted filters.

In Section 2, a general formulation of gradient weighted filters is proposed. According to this formulation, we characterize such filters by three families of parameters. In Section 3, we then modify gradient weighted filters by varying these parameters. In Section 4, we propose a criterion on filters equipped with second order local gradients in order for them to preserve fine details. In the same section, we also propose a combination of filters equipped with first and second order local gradients to remove noise of mixed types. Finally, experimental results are exhibited and discussed in Section 5 and some conclusions are presented in Section 6.

2. FORMULATION OF GRADIENT WEIGHTED FILTERS

Let $f : Z \times Z \to \{0, 1, \ldots, 255\}$ be a gray-level image. In a 3×3 window centered at p, the local gradients are defined as follows. First, we label the eight neighbors of p in an order as shown below.

p_1	p_2	p_3
p_8	p	p_4
p_7	p_6	p_5

Then, the first order local gradients $g_p(k)$, $k = 1, 2, \ldots, 8$, are defined to be

$$g_p(k) = f(p_k) - f(p) \tag{1}$$

and the second order local gradients $g'_p(k)$, $k = 1, 2, \ldots, 8$, are defined to be

$$g'_p(k) = g_p(k) - g_p((k+4) \bmod 8) = f(p_k) - f(p_{(k+4) \bmod 8}). \tag{2}$$

For a sigma filter [1], the output value at p is given by

$$f_{SF}(p) = \frac{1}{1+\sum_{k=1}^{8} w_p(k)} f(p) + \frac{\sum_{k=1}^{8} w_p(k) f(p_k)}{1+\sum_{k=1}^{8} w_p(k)}, \tag{3}$$

where

$$w_p(k) = \begin{cases} 1, & \text{if } |g_p(k)| \le \Delta, \\ 0, & \text{if } |g_p(k)| > \Delta, \end{cases}$$

for $\Delta = 2\sigma$ and σ is a priori assumed or estimated Gaussian noise standard deviation. For a gradient inverse weighted filter [8], the output value at p is given by

$$f_{GIWF}(p) = \frac{1}{2} f(p) + \frac{1}{2} \frac{\sum_{k=1}^{8} w_p(k) f(p_k)}{\sum_{k=1}^{8} w_p(k)}, \tag{4}$$

where

$$w_p(k) = \begin{cases} \frac{1}{|g_p(k)|}, & \text{if } g_p(k) \ne 0, \\ 2, & \text{if } g_p(k) = 0. \end{cases}$$

For an adaptive Gaussian weighted filter [10], the output value is given by

$$f_{AGWF}(p) = \begin{cases} \frac{\sum_{k=1}^{8} w_p(k) f(p_k)}{\sum_{k=1}^{8} w_p(k)}, & \text{if } \sigma_p^2 \ne 0, \\ f(p), & \text{if } \sigma_p^2 = 0, \end{cases} \tag{5}$$

where $\sigma_p^2 = \frac{1}{8}\sum_{k=1}^{8} f^2(p_k) - \frac{1}{64}(\sum_{k=1}^{8} f(p_k))^2$ and $w_p(k) = \exp(-(g_p^2(k))/\sigma_p^2)$. For a rational filter [6], the output is given by

$$f_{RF}(p) = \left(1 - \sum_{k=1}^{8} w_p(k)\right) f(p) + \sum_{k=1}^{8} w_p(k) f(p_k), \tag{6}$$

where $w_p(k) = 1/\alpha(g'_p(k))^2 + A_k$ for some positive constants α and

$$A_k = \begin{cases} \frac{1}{\omega} & k = 2, 4, 6, 8 \\ \frac{\sqrt{2}}{\omega} & k = 1, 3, 5, 7 \end{cases} \quad \text{and } \omega = 0.16.$$

In summary, we can formulate the output of a gradient weighted filter as

$$f_{GWF}(p) = \begin{cases} (1-\gamma_p) f(p) + \gamma_p \frac{\sum_{k=1}^{8} w_p(k) f(p_k)}{\sum_{k=1}^{8} w_p(k)}, & \text{if } \sum_{k=1}^{8} w_p(k) \ne 0, \\ f(p), & \text{if } \sum_{k=1}^{8} w_p(k) = 0, \end{cases} \tag{7}$$

for some local constant γ_p and local weight function w_p. This equation can capture entirely the charactistics of the above gradient weighted filters but it is not unique. Spann and Nieminen [10] and Buf and Campbell [13] also formula gradient weighted filters as

$$\hat{f}(p) = \frac{\sum_{k=-1}^{1}\sum_{l=-1}^{1} h(p+(k,l))\,f(p+(k,l))}{\sum_{k=-1}^{1}\sum_{l=-1}^{1} h(p+(k,l))},$$

where $h(p+(k,l))$ are weighted functions of local gradients. For instance,

$$\gamma_p = \frac{\sum_{k=1}^{8} w_p(k)}{1+\sum_{k=1}^{8} w_p(k)}$$

for sigma filter, $\gamma_p = \frac{1}{2}$ for gradient inverse weighted filters, $\gamma_p = 1$ for adaptive Gaussian weighted filers, and $\gamma_p = \sum_{k=1}^{8} w_p(k)$ for rational filters. The parameter γ_p reflects the specific effect of the center pixel p. The local weight functions are functions of either first order or second order local gradients. If we write $w_p(k) = F(g_p(k))$ or $w_p(k) = F(g'_p(k))$, then such functions F are decreasing with respect to the magnitude of first order or second order local gradients. For instance, $F(x) = 1$ if $|x| \le \Delta$ and 0 if $|x| > \Delta$ for sigma filters, $F(x) = 1/|x|$ if $x \ne 0$ and $F(0) = 2$ for gradient inverse weighted filters, $F(x) = \exp(-x^2/\sigma_p^2)$ for adaptive Gaussian weighted filters, and $F(x) = 1/(\alpha x^2 + A)$ for rational filters.

Therefore, a gradient weighted filter is characterized by the local constants γ_p, the local weight functions w_p, and the global function F. By varying these parameters, a large variety of gradient weighted filters can be obtained. In the following section, we will first modify the gradient inverse weighted filters by varying the local constants γ_p. Second, we will modify existing gradient weighted filters by varying their local weight functions. If they are originally equipped with first order local gradients, then we will make them be equipped with second order local gradients. Then, we will propose a subclass of gradient weighted filters by using π functions as the global functions.

3. THE PROPOSED MODIFICATIONS

A. *Adaptive Gradient Inverse Weighted Filters*

First of all, we will take into account the modification on the characteristic of local constant γ_p. The local weights of a 3×3 window are not fixed; they are adaptive and reciprocal to the local gradients. For sigma and rational filters [1, 6], their γ_p are adaptive determined by local weighted functions. However, in an adaptive Gaussian weighted filter [10], its γ_p is equal to 1 when $\sum_{k=1}^{8} w_p(k) \ne 0$ and results in neglect of the effect of the center pixel. In a gradient inverse weighted filter [8], the weight of the center pixel p is equitable fixed to $\frac{1}{2}$. If the center pixel p is corrupted by noise, the total effect of the vicinity of pixel p should be enlarged and the weight of pixel p should be weakened. The term $1-\gamma_p$ reflects the importance of the center pixel relying on whether it is a noisy point. Therefore, the gradient inverse weighted filters were modified in such a way that the local constants γ_p are proportional to the median magnitude of local gradients. The median magnitude of local gradients is given as $m_p = MED\{0, |g_p(k)|, 1 \le k \le 8\}$ to be a noise estimator about the center point p. For instance, if the median magnitude is obtained, then the local constant γ_p can be chosen to be

$$\gamma_p = \begin{cases} 2\left(\frac{m_p}{\alpha}\right)^2 & \text{if } 0 \le m_p < \frac{\alpha}{2}, \\ 1 - 2\left(\frac{m_p}{\alpha} - 1\right)^2 & \text{if } \frac{\alpha}{2} \le m_p < \alpha, \\ 1 & \text{if } \alpha \le m_p, \end{cases}$$

for some constant α. The parameter α can be regarded as a conditional threshold about noise and depends on the amount of noise. The resulting filters will be called *adaptive gradient inverse weighted filters* (AGIWF). We observe that the constant α should be proportional to the local variance formed by $p_k, k = 1, 2, \dots 8$. Thus, in our experiment, we choose $\alpha = \text{sqrt}(\frac{1}{8}\sum_{k=1}^{8} f^2(p_k) - \frac{1}{64}(\sum_{k=1}^{8} f(p_k))^2)$.

B. *Second Order Gradient Weighted Filters*

Most aforementioned gradient weighted filters are equipped with first order local gradients. Exceptions are rational filters [4–6]. They are equipped with second order local gradients and perform more comfortably than the others. Here, we will further investigate filters such as GIWF and AGWF by equipping them with second order local gradients. For convenience, gradient weighted filters equipped with second order local gradients will be called *second order gradient weighted filters*. That is, gradient inverse weighted filters equipped with second order local gradients will be called second order gradient inverse weighted filters (abridged as GIWF2) and adaptive Gaussian weighted filters equipped with second order local gradients will be referred to as second order adaptive Gaussian weighted filters (abridged as AGWF2), etc.

C. Π *Filters*

Let $f(x_1), f(x_2), f(x_3), \dots f(x_{n-k}), \dots f(x_n), \dots f(x_{n+k}), \dots f(x_m)$ be a sequence of a 1-D signal. A generalized 1-D adaptive gradient weighted filter derived from [6] can be expressed as

$$f'(x_n) = \left(1 - \sum_{i=n-1, i\neq 0}^{n+1} F(g_{x_i}) f(x_n) + \sum_{i=n-1, i\neq 0}^{n+1} F(g_{x_i}) f(x_i)\right), \tag{8}$$

where $g_{x_i} = f(x_i) - f(x_n)$ are the local gradients and F is the weighted function. In gradient weighted filters, a common characteristic of the global functions F is that the values of $F(x)$ are reciprocal to $|x|$ for noise removing. There are many possible candidates for such functions such as those mentioned in Section 2. As a point x_n crossed on an edge with height H delineated in Fig. 1 where N is the window size, $f(x_{n-l}) = a$ for $-1 \le l \le k - 2$; $f(x_{n+r}) = b$ for $2 \le r \le N - k + 1$; and $b - a = H$. The filtered output is $a + \frac{(N-k)}{N} H$ for the mean filter. The resultant value amplifies $\frac{(N-k)}{N} H$ result; therefore, signal points crossed on edges are blurred. By way of responding zero or small weights to x_n's neighbors with large local gradients, adaptive gradient weighted filters would be effective for edge preserving. Thus, the advantage of the adaptive gradient weighted filters is that

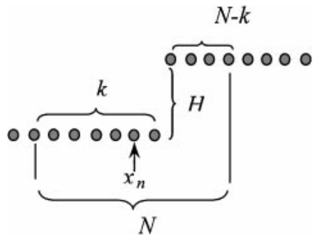

FIGURE 1

they are able to smooth a noisy signal whilst maintaining sharp transitions in the signal. Our purpose is to design a function that is reciprocal to the gradients for removing noise and preserving edges. In our approach, π functions are chosen to substitute the weighted function $F(x)$. The reasons for choosing π functions are that (1) they match our purposes; (2) they are simple, only second degree with respect to x; (3) the parameter α provides the adaptability for the filters. In this paper, we propose to use the following one-parameter π functions,

$$\pi(x) = \begin{cases} 1 - 2\left(\frac{x}{\alpha}\right)^2 & \text{if } |x| \leq \alpha/2 \\ 2\left(\frac{x}{\alpha} - 1\right)^2 & \text{if } \alpha/2 \leq |x| \leq \alpha \\ 0 & \text{if } |x| \geq \alpha, \end{cases}$$

for some constant α.

The resulting gradient weighted filters will be simply called Π *filters*. More precisely, the output value of a first order Π filter is given by

$$f_{\Pi}(p) = \left(1 - \sum_{k=1}^{8} w_p(k)\right) f(p) + \sum_{k=1}^{8} w_p(k) f(p_k), \tag{9}$$

where $w_p(k) = F(g_p(k)) = \frac{1}{8}\pi(g_p(k))$. Similarly, the output value of a second order Π filter is given by

$$f_{\Pi 2}(p) = \left(1 - \sum_{k=1}^{8} w_p(k)\right) f(p) + \sum_{k=1}^{8} w_p(k) f(p_k), \tag{10}$$

where $w_p = F(g'_p(k)) = \frac{1}{8}\pi(g'_p(k))$.

Let $X : Z \times Z \to \{0, 1, \ldots, 255\}$ be a gray-scale image and $X_{g_p(k)}$ be a translation of X along the direction of local gradient $g_p(k)$ within a 3×3 window. To quantify the distribution of $g_p(k)$ $k = 1, 2, \ldots, 8$ in X, the means $\mu(X_{g_p(k)} - X)$ and the variances $\mathrm{Var}(X_{g_p(k)} - X)$ for $k = 1, 2, \ldots, 8$ can be evaluated. The observations concerning $\mu(X_{g_p(k)} - X)$ are close to zero for all $k = 1, 2, \ldots, 8$. According to the terms $\mu(X_{g_p(k)} - X)$ and $\mathrm{Var}(X_{g_p(k)} - X)$, we can roughly estimate the occurrence of local gradients $g_p(k)$. Most of gradient $g_p(k)$ occurs within the range of $\pm 2a$, where $a = \frac{1}{8}\sum_{k=1}^{8}\mathrm{Var}(X_{g_p(k)} - X)$. Thus, we choose $\alpha = 2a$ in the definition of π function. This α provides some information about edge heights of an image and the amount of corrupted noise.

150	150	150
150	50	150
150	150	150

(a)

150	50	50
50	150	50
50	50	150

(b)

FIG. 2. (a) An impulsive noise pattern. (b) A line pattern.

4. DETAIL PRESERVING AND MIXED TYPE NOISE REMOVING

A. Detail Preserving

Consider the situation shown in Fig. 2a, where the center pixel is corrupted by impulsive noise. At this pixel, the output value of a GIWF is

$$\frac{1}{2} \times 50 + \frac{1}{2} \sum_{1}^{8} \left(\frac{1}{8} \times 150 \right) = 100$$

and the output value of an AGIWF is

$$(1 - 1) \times 50 + \sum_{1}^{8} \left(\frac{1}{8} \times 150 \right) = 150$$

when the parameter α is chosen to be less than or equal to 100. For rational filters, the output value is 150 only when $\omega = \frac{1}{4+4\sqrt{2}} \approx 0.1036$. In rational filters [6], parameter ω is fixed at 0.16. Under such conditions, a bias result yielded. For a second order Π filter, the output value is 150 as well, but a first order Π filter retains impulsive noise as $\alpha \leq 100$. Now consider the other situation, shown in Fig. 2b, where a line passes through the center pixel. A first order Π filter produces the right output value, 150, but a second order one produces the wrong output value, 75, at that pixel when $\alpha \leq 100$. In other words, a second order Π filter will smear fine details. To preserve fine details, we slightly modify any second order gradient weighted filter (GWF2) as follows,

$$\text{if } \min_{1 \leq k \leq 4} \{|g_p(k) + g_p((k+4) \bmod 8)|\} \leq \beta, \text{ then output } f(p);$$

$$\text{otherwise, output } f_{GWF2}(p) \text{ as usual,}$$

where β is a threshold of a small integer. For Gaussian and uniform noise, the cases in Fig. 1 scarcely happen. Empirically, as previous works [1, 8, 10] have shown, first order local gradients are suitable for Gaussian and uniform noise removals. Therefore, we apply first order Π filters in cases of Gaussian and uniform noise and apply second order ones in cases of impulsive noise.

B. Mixed Type Noise Removing

Noise types commonly used for testing noise removals include long tail noise (impulsive) and short tail noise (Gaussian and uniform). Most smoothing filters do well on either one of them. In this subsection, we propose a method which combines first and second order Π filters so that the associated filters are nonsensitive to noise distributions. First, we locally classify noise types according to the sum $\Sigma_p = \sum_{k=1}^{8} w_p(k)$ of local weights in a first order Π filter. We regard pixel p as being corrupted by short tail noise if the sum Σ_p is larger than a threshold value; otherwise, we regard it as being corrupted by long tail noise. Then we combine Π filters by the following rule,

$$\text{If } \Sigma_p > \Delta, \text{ then output } f_{\Pi}(p);$$
$$\text{Otherwise, output } f_{\Pi 2}(p),$$

where Δ is a predefined threshold value.

5. EXPERIMENTS AND DISCUSSIONS

In this section, we will compare the performances of proposed filters with some other gradient weighted filters. Many quantitative evaluations of the filtering performance have been shown, such as spatial measurement on natural images (MSE, MAE, NMSE, and PSNR) and predefined spatial measurement on artificial images (edge preserving criteria

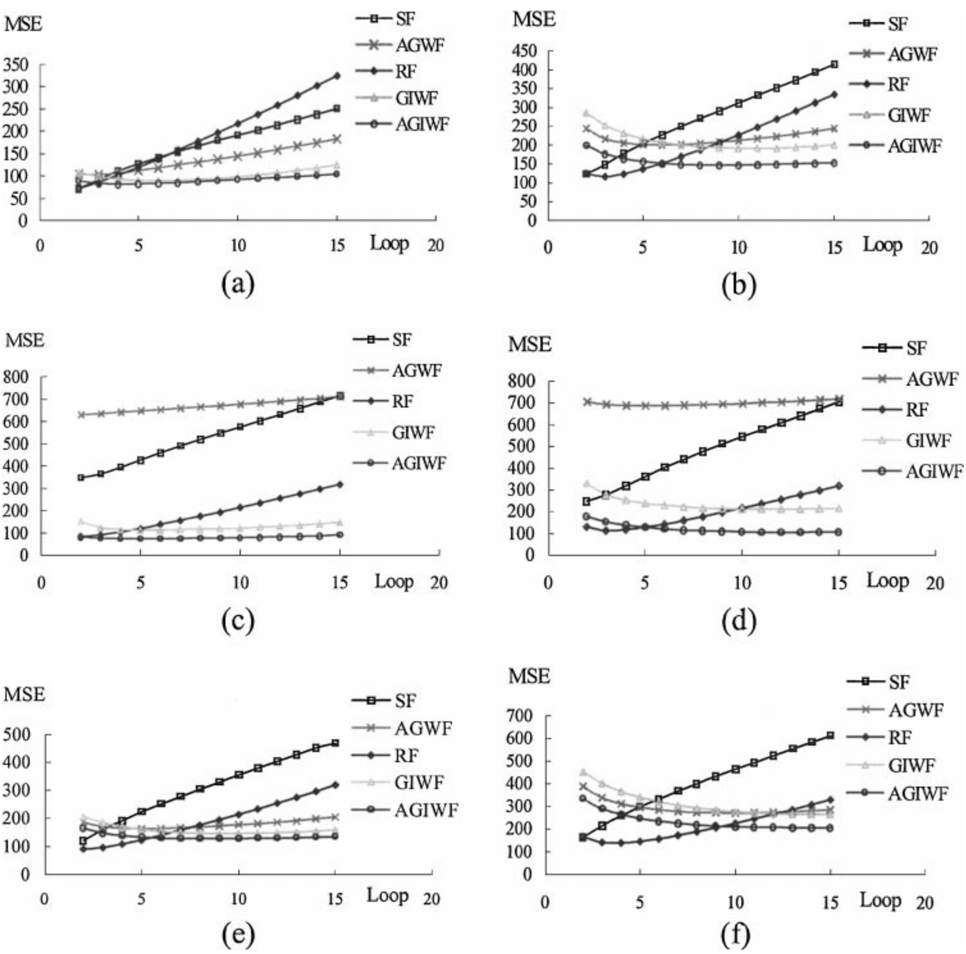

FIG. 3. The merit assessments of AGIWF and GIWF on Lena images damaged by different type noises. (a) Gaussian noise variance = 15, (b) Gaussian noise variance = 25, (c) impulse noise rate = 15%, (d) impulse noise rate = 25%, (e) uniform noise variance = 32%, (f) uniform noise variance = 48%.

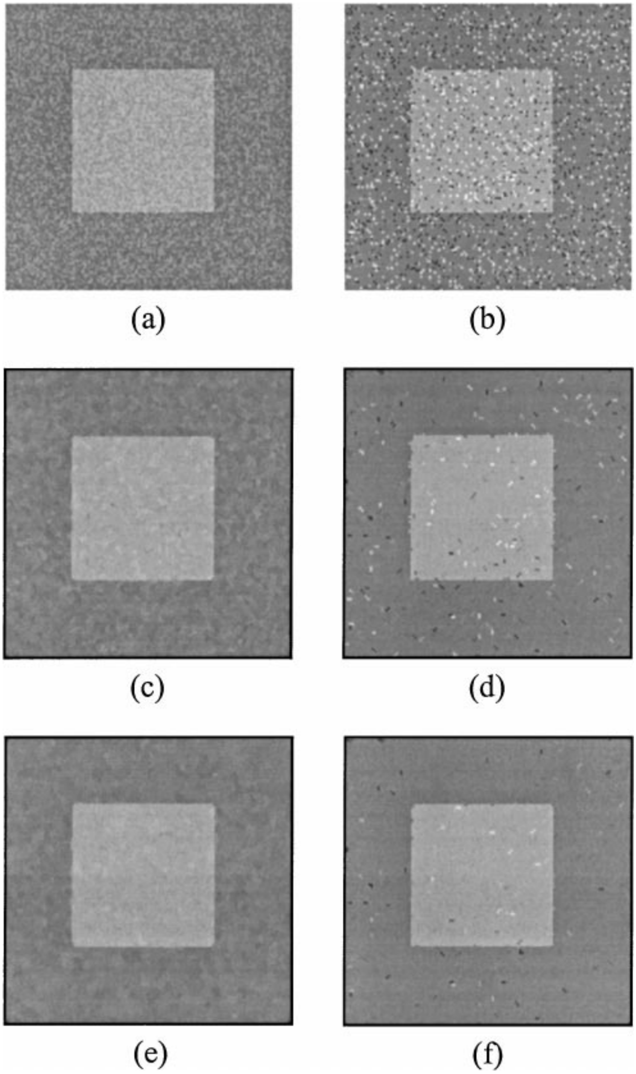

FIG. 4. The comparison between AGIWF and GIWF: (a) Gaussian noisy image with $\sigma = 20$; (b) Impulse noisy image with ratio 20%; (c, d) The results of GIWF with loop = 10 for Figs. 4a and 4b: (c) MSE = 66.48, PSNR = 29.90, and f-statistic = 183931; (d) MSE = 114.19, PSNR = 27.55, and f-statistic = 84465; (e, f) The results of AGIWF with loop = 10 for Figs. 4a and 4b: (e) MSE = 38.51, PSNR = 32.27, and f-statistic = 35147; (f) MSE = 32.12, PSNR = 33.06, and f-statistic = 286811.

[13] and f-statistic [8]). In our experimental results, MSE, PSNR, and f-statistic are considered for demonstration. In the following, we first compare adaptive gradient inverse weighted filters with original ones to demonstrate the efficiency of the first modification on local constant γ_p. Two testing images are investigated. The natural Lena image under consideration is of size 256×256 and the other artificial image is of size 128×128 with eight bits of resolution.

The MSE measures the discrimination between the original images and corresponding filtered images. The f-statistic [8] is based on the fact that the more the uniformity inside each region, the greater the discrimination between regions. Both measurements are the most commonly used and famous quantitative evaluations on smoothing and edge preserving.

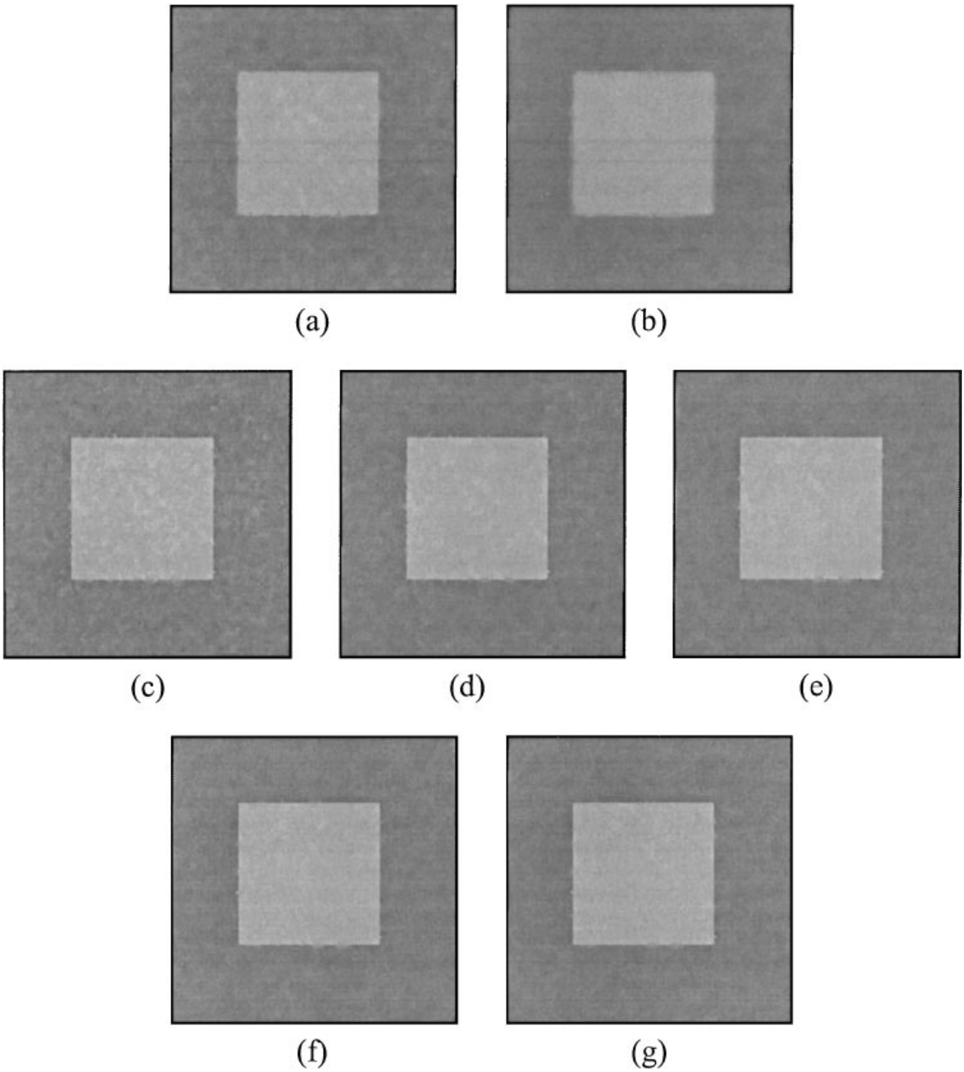

FIG. 5. (a, b) The results of rational filter with loop = 3 and 6: (a) MSE = 18.19, PSNR = 35.53, and f-statistic = 492234; (b) MSE = 29.57, PSNR = 33.42, and f-statistic = 404250; (c–g) The results of Π filter by first local gradient on Fig. 4a with loop = 3, 4, 5, 6, 10, respectively. (c) MSE = 18.31, PSNR = 35.50, and f-statistic = 442659; (d) MSE = 11.86, PSNR = 37.39, and f-statistic = 764373; (e) MSE = 10.70, PSNR = 37.84, and f-statistic = 1020329; (f) MSE = 11.17, PSNR = 37.65, and f-statistic = 1233819; (g) MSE = 19.65, PSNR = 35.20, and f-statistic = 1832383.

Figures 3a–3f evaluate the MSE of an AGIWF and the existing four methods under a diversified noise environment on the Lena image, respectively. The assessments in Fig. 3 show that the AGIWF needs a lower iteration number than the GIWF to achieve optimal MSE values. Observe that optimal MSE values of the AGIWF are also smaller than the corresponding values of the GIWF in all experimental cases. We also assess each merit of sigma filter (SF), AGWF, and RF. The MSE evaluations in Figs. 3c and 3d disclose that AGWF and SF are the most sensitive to impulsive noise. Over all, RF and AGIWF perform better than GIWF and AGWF. AGIWF performs better than RF for the case in Figs. 3a, 3c, 3d and RF performs better than AGIWF for the case in Figs. 3b, 3e, 3f. To simplify, on an artificial image, Figs. 4c–4f show the visual effectiveness, PSNR, and corresponding

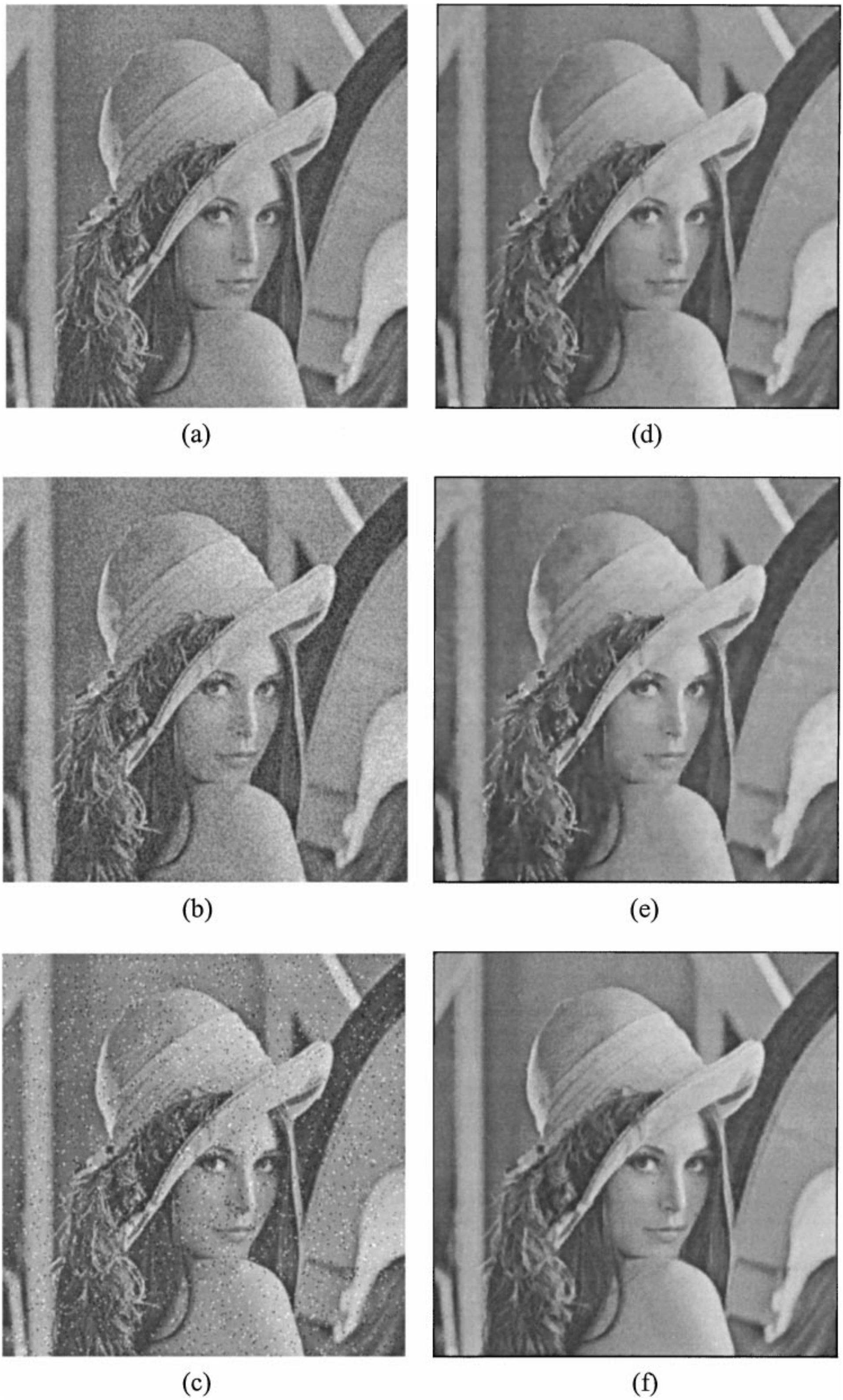

FIG. 6. (a) Gaussian noise ($\sigma = 10$); (b) Uniform noise (20%); (c) Impulsive noise (10%); (d), (e), and (f) are the corresponding smoothing result of their left image.

TABLE 1

Algorithms	Noise	Parameters	MSE
		a. Gaussian noise	
α-tr. filter [2]	$\sigma = 10$	3×3, cut $= 3/9$, $l = 2$	76.92
AGWF [10]		3×3, $l = 2$	57.64
GIWF [8]		3×3, $l = 2$	50.91
RF [6]		$\omega = 0.16$, $\alpha = 0.01$, $l = 2$	59.40
Π		$\alpha = 48$, $l = 2$	35.48
α-tr. filter [2]	$\sigma = 20$	3×3, cut $= 3/9$, $l = 2$	103.62
AGWF [10]		3×3, $l = 4$	145.31
GIWF [8]		3×3, $l = 3$	105.62
RF [6]		$\omega = 0.16$, $\alpha = 0.01$, $l = 3$	99.01
Π		$\alpha = 72$, $l = 3$	89.46
		b. Uniform noise	
α-tr. filter [2]	32%	3×3, cut $= 2/9$, $l = 2$	120.75
AGWF [10]		3×3, $l = 6$	152.91
GIWF [8]		3×3, $l = 6$	152.96
RF [6]		$\omega = 0.16$, $\alpha = 0.01$, $l = 3$	95.75
Π		$\alpha = 80$, $l = 3$	75.80
α-tr. filter [2]	48%	3×3, cut $= 2/9$, $l = 2$	164.99
AGWF [10]		3×3, $l = 10$	249.40
GIWF [8]		3×3, $l = 10$	278.07
RF [6]		$\omega = 0.16$, $\alpha = 0.01$, $l = 3$	141.79
Π		$\alpha = 96$, $l = 3$	120.53
Π		$\alpha = 96$, $l = 4$	115.85
		c. Classical gradient weighted filters for impulsive noise	
AGWF [10]	10%	3×3, $l = 2$	439.60
GIWF [8]		3×3, $l = 6$	72.74
RF [6]		$\omega = 0.16$, $\alpha = 0.01$, $l = 2$	72.54
AGWF [10]	20%	3×3, $l = 3$	785.44
GIWF [8]		3×3, $l = 10$	163.94
RF [6]		$\omega = 0.16$, $\alpha = 0.01$, $l = 3$	104.57
		d. Second order filters with impulsive condition	
AGWF2	10%	3×3, $l = 2$, $\beta = 12$	53.19
GIWF2		3×3, $l = 6$, $\beta = 12$	54.17
RF2		$\omega = 0.16$, $\alpha = 0.01$, $l = 2$, $\beta = 12$	49.36
Π 2		$\alpha = 76$, $l = 2$, $\beta = 12$	49.0
AGWF2	20%	3×3, $l = 3$, $\beta = 12$	73.79
GIWF2		3×3, $l = 10$, $\beta = 12$	78.57
RF2		$\omega = 0.16$, $\alpha = 0.01$, $l = 3$, $\beta = 12$	72.72
Π 2		$\alpha = 100$, $l = 3$, $\beta = 12$	73.70

f-statistic. Here, images corrupted by short tail (Gaussian) and long tail (impulse) noise are considered. Figures 4a and 4b expose the damaged images. The results of original GIWF are shown in Figs. 4c and 4d, and Figs. 4e and 4f show the results of AGIWF. Obviously, AGIWF smoothes noise in a uniform region better than GIWF and preserves edges as well as GIWF.

Second, we take care of the modification about the weighted functions. By comparison with the rational filter [6], we investigate whether the Π filters equipped with first local gradient do better than the second gradient in short tail noise such as Gaussian and uniform noise. We suggest that the parameter α for the π function is set to be twice as large as the standard deviation of local gradient in the corrupted image. Figures 5a and 5b show the results employed with three and six times the rational filter on Fig. 4a. As the loop number increases, the rational filter diffuses the smoothing effect on uniform regions but results in blurred edges. However, the Π filter with first local gradient will not. Figures 5c–5g exhibit the results of the Π filter with first local gradient for loop numbers 3, 4, 5, 6, and 10, respectively. The difference between Figs. 5c–5f and 5g is only the smoothing effect on the uniform region. Actually, in our case of Fig. 5a, the rational filter possesses a better smoothing effect than GIWF [6], but AGIWF performs as well as the rational filter in Fig. 4e. Among them, the Π filter equipped with first local gradient yields the best result. Images shown in Figs. 6d–6f are the smoothing results when applying a Π filter to corrupted images by different kinds of noise in Figs. 6a–6c for natural images.

Then, we compare Π filters with some other gradient weighted filters. Filters under comparison include GIWF [8], AGWF [10], α-trimmed filters [2], and RF [6]. Note that SF was excluded due to the similar MSE behavior with RF and bad performance by contrast with RF. All filters in the current study are iterative and parameter l denotes the loop number in each experiment. Tables 1a and 1b quantize the optimal performance measures of removing noise by first order filters with different amounts of Gaussian and uniform noise. The results exhibit that Π filters are more nonsusceptible to noise than other competitors. Tables 1c and 1d compare the performances of classical gradient weighted filters with their second

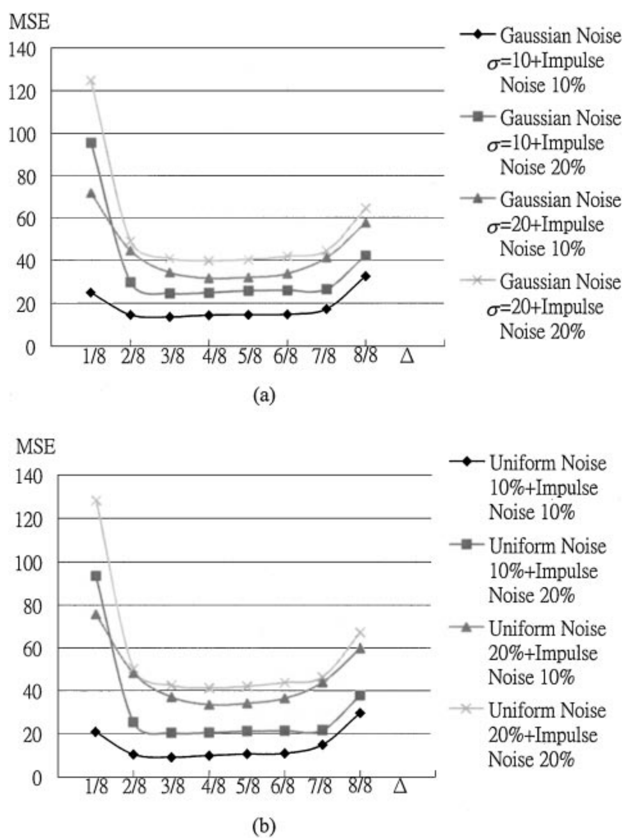

FIG. 7. The testing about threshold Δ on mixed noise with artificial image.

TABLE 2
Mixed Noise ($\Delta = 3/8$)

Algorithms	Noise	Parameters	MSE
α-tr. filter	Implusive 10% +	3×3, cut $= 2/9$, $l = 2$	89.58
RF [6]	Uniform 10%	$\omega = 0.16$, $\alpha = 0.01$, $l = 3$	84.49
Π		$\alpha = 62$, $l = 2$, $\beta = 12$	66.26
α-tr. filter	Implusive 10% +	3×3, cut $= 2/9$, $l = 2$	102.78
RF [6]	Uniform 20%	$\omega = 0.16$, $\alpha = 0.01$, $l = 3$	92.40
Π		$\alpha = 90$, $l = 2$, $\beta = 12$	80.61

order counterparts. Note that in this comparison all considered filters impose a conditional operator for preserving fine detail with $\beta = 12$.

In summary, from the results shown in Tables 1a–1d, we find that first order filters are more suitable for removing Gaussian and uniform noise while second order filters are more suitable for removing impulsive noise. That is because the second order gradients convey information which will be insensitive to impulsive noise in gray-scale image filtering. Under mingled conditions, we also apply combined Π filters which utilized the characteristics of our observation to remove mixed types noise where Δ is obtained empirically and set to $3/8$. Figures 7a and 7b investigate the MSE criteria with different Δ on mixed types noise. By our experiment, if Δ ranges from $3/8$ to $5/8$, we obtain a lower MSE error and a larger PSNR and f-statistic. Finally, Table 2 shows our experimental results.

6. CONCLUSIONS

In this paper, we first characterize gradient weighted filters by three families of parameters: local constants, local weight functions, and a global function. Then we propose some modified gradient weighted filters by varying these parameters. For instance, a Π filter is a gradient weighted filter using a π function as the global function. Moreover, we propose a criterion for second order filters to preserve fine details and a criterion to combine first and second order filters in order to remove noise of mixed types. Most of our proposed filters have very satisfactory performances.

Note that π functions are very commonly used membership functions for fuzzy sets [11, 12]. Therefore, it should be interesting to apply fuzzy theoretical techniques to fine-tune the parameters for a gradient weighted filter in order to achieve better performances.

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