Non-Reinitialized Fully Distributed Power Control Algorithm

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***Abstract*—A fully distributed power control (FDPC) algorithm has been recently proposed for cellular mobile systems. In the algorithm, the connection which has the smallest initial carrier-to-interference (CIR) ratio is removed if CIR requirements are not satisfied after L iterations of power control. The transmitter power levels of surviving connections are then reset to the maximal allowed values and the algorithm is executed again. We prove in this paper that, if the transmitter power levels are not reset after a connection is removed, then a feasible power set can be found faster and the power levels employed are smaller.**

***Index Terms*—Distributed algorithms, power control.**

I. Introduction

TRANSMITTER power control is a common technique which can be used to reduce interference and allow as many receivers as possible to obtain satisfactory reception. Many power control algorithms have recently been proposed and analyzed [1]–[9]. In general, one can categorize power control algorithms into centralized and distributed. Centralized power control can achieve optimum outage probability [1], [5], [7] but requires link gains between all mobile users and the base station. Thus centralized power control is not feasible for a large network or an environment where link gains change rapidly. Some distributed power control algorithms which use only local carrier-to-interference ratio (CIR) information were studied [2], [3], [5], [6]. Among these algorithms, the fully distributed power control (FDPC) algorithm was reported in [6] to outperform others in finding a feasible power set, i.e., a power set which can meet the CIR requirements. In the FDPC algorithm, all users start with the maximal allowed transmitting power levels. If no feasible power set is found after L iterations, the connection with the minimal initial CIR is removed. After the connection is removed, the algorithm is reinitialized, i.e., all surviving connections reset their transmitting power levels to the maximal allowed values and the algorithm is executed again. In this paper, we formally prove that, if the transmitter power levels are not reset after a connection is removed, then a feasible power set can be found faster and the power levels employed are smaller.

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II. System Model

We assume that there are N connections in a cellular mobile network and consider the reverse link. (The results can also be used in the forward link.) Let P_i represent the transmitting power of the ith mobile user and η_i denote its thermal noise. Assume that b_i is the base station it is assigned to. As a result, the received CIR for the ith user is given by

$$CIR_i = \frac{P_i G_{ib_i}}{\sum_{j \neq i} P_j G_{jb_i} + \eta_i}$$

where G_{jb_i} represents the link gain between the jth mobile user and the base station b_i. Let Γ_i denote the CIR requirement for the ith user. For all the users to meet their CIR requirements, we must find a power set $\mathbf{P} = \{P_i\}$ $(1 \le i \le N)$ such that $P_i > 0$ and $CIR_i \ge \Gamma_i$ for all $i, 1 \le i \le N$. As in [4], such a power set is called a feasible power set. Given a configuration specified by $\mathbf{G} = \{G_{jb_i}\}$ $(1 \le i, j \le N)$, if there exists a feasible power set $\mathbf{P} = \{P_i\}$, then this configuration is said to be feasible.

III. Power Control Procedure

Let $\mathbf{P}^0 = \{P_i^0\}$ denote the initial transmitter power set. Also, let $\mathbf{P}^m = \{P_i^m\}$ and $\mathbf{CIR}^m = \{CIR_i^m\}$ denote the transmitter power set and the set of received CIR in the mth iteration, respectively. The power control procedure of the FDPC algorithm is described below. In the procedure, T_i represents the maximal allowed transmitter power for the ith mobile user.

The power control procedure of FDPC Algorithm is

$$P_i^0 = T_i$$

and

$$P_i^{m+1} = a_i^m * P_i^m \quad \text{for all } i, \qquad 1 \le i \le N,$$

where

$$a_i^m = \min(CIR_i^m, \Gamma_i)/CIR_i^m.$$

□

Proofs of the following properties of the FDPC power control procedure can be found in [6].

Property 1: $P_i^{m+1} \le P_i^m$ for all i and m.

Property 2: If $CIR_i^m \ge \Gamma_i$, then $CIR_i^l \ge \Gamma_i$ for all $l \ge m$.

IV. Removal Criteria

It is possible that, after L iterations of power control, no feasible power set is found. In this case, the connection with the smallest received initial CIR is removed. Let $\mathbf{ICIR} = \{ICIR_i\}$ represent the set of initial CIR. For convenience, every L iterations are counted as a round and the round number is denoted by n. A new round is begun each time a reset occurs. The FDPC algorithm including removal procedure can be described as follows.

Step 1: Let $n = 1$, $\Omega = \{1, 2, \cdots, N\}$, $P_i^0 = T_i$ and $ICIR_i = CIR_i^0$ for all i.

Step 2: Execute at most L iterations of the FDPC power control procedure.

Step 3: Stop if a feasible power set is found. Else, remove connection i which has the smallest initial CIR (i.e., $ICIR_i \le ICIR_k$ for all $k \in \Omega, k \ne i$).

Step 4: Let $n = n + 1, \Omega = \Omega - \{i\}$, and $P_k^0 = T_k$ for all connection $k \in \Omega$ and go to Step 2.

Notice that the transmitting power levels are reset to the maximal allowed values for all surviving connections in Step 4. For convenience, we call such an algorithm the reinitialized FDPC (R-FDPC) algorithm. An alternative choice, which will be referred to the nonreinitialized FDPC (NR-FDPC) algorithm, is to let $P_k^0 = P_k^L$ $(k \in \Omega)$ and go to Step 2. It has to be pointed out that the connection removed by both R-FDPC and NR-FDPC algorithms in Step 4, if necessary, are the same in every round. We prove in the following that the NR-FDPC algorithm performs better than the R-FDPC algorithm.

Let $\mathbf{P}_r^{n,m} = \{P_{r,i}^{n,m}\}$ and $\mathbf{CIR}_r^{n,m} = \{CIR_{r,i}^{n,m}\}$ denote respectively the transmitter power set and the set of received CIR in the mth iteration of round n for the R-FDPC algorithm. Similarly, let $\mathbf{P}_{nr}^{n,m} = \{P_{nr,i}^{n,m}\}$ and $\mathbf{CIR}_{nr}^{n,m} = \{CIR_{nr,i}^{n,m}\}$ represent those sets for the NR-FDPC algorithm.

Lemma 1: Assume that a connection has to be removed at the end of round n. If $CIR_{nr,i}^{n,L} \ge \Gamma_i$, then $CIR_{nr,i}^{n+1,0} \ge \Gamma_i$ for all i and n.

Proof: Since $P_{nr,i}^{n,L} = P_{nr,i}^{n+1,0}$ for any surviving connection in the NR-FDPC algorithm, we have

$$CIR_{nr,i}^{n,L} = \frac{P_{nr,i}^{n,L} G_{ib_i}}{\sum_{j \ne i} P_{nr,j}^{n,L} G_{jb_i} + \eta_i} \le CIR_{nr,i}^{n+1,0}$$
$$= \frac{P_{nr,i}^{n_1,0} G_{ib_i}}{\sum_{j \ne i,k} P_{nr,j}^{n+1,0} G_{jb_i} + \eta_i}$$

where k represents the connection removed at the end of round n. Therefore, Lemma 1 is true.

Lemma 2: Assume that, at the beginning of round n, the following two conditions hold:

(i) $P_{nr,i}^{n,0} \le P_{r,i}^{n,0}$ for all users i;

(ii) $CIR_{nr,j}^{n,0} \ge \Gamma_j$ if $CIR_{r,j}^{n,0} \ge \Gamma_j$ for any user j.

We have, for all iterations $m \le L$ of round n:

(iii) $P_{nr,i}^{n,m} \le P_{r,i}^{n,m}$ for all users i;

(iv) $CIR_{nr,j}^{n,m} \ge \Gamma_j$ if $CIR_{r,j}^{n,m} \ge \Gamma_j$ for any user j.

Proof: We prove Lemma 2 by mathematical induction. By assumption, (iii) and (iv) are true for $m = 0$. Assume that the lemma is true for $m = M < L$. Consider the case $m = M + 1$. If $CIR_{r,i}^{n,m} \ge \Gamma_i$, then, according to the FDPC algorithm, we have

$$P_{r,i}^{n,M+1} = \frac{\Gamma_i}{CIR_{r,i}^{n,M}} P_{r,i}^{n,M} = \frac{\Gamma_i}{G_{ib_i}} \left(\sum_{j \ne i} P_{r,j}^{n,M} G_{jb_i} + \eta_i \right).$$

Besides, since $CIR_{r,i}^{n,M} \ge \Gamma_i$ implies

$$CIR_{nr,i}^{n,M} \ge \Gamma_i, \text{ we get } P_{nr,i}^{n,M+1} = \frac{\Gamma_i}{CIR_{nr,i}^{n,M}} P_{nr,i}^{n,M}$$
$$= \frac{\Gamma_i}{G_{ib_i}} \left(\sum_{j \ne i} P_{nr,j}^{n,M} G_{jb_i} + \eta_i \right).$$

By hypothesis, we have $P_{nr,j}^{n,M} \le P_{r,j}^{n,M}$ for all users j and thus $P_{nr,i}^{n,M+1} \le P_{r,i}^{n,M+1}$ for all users i. On the other hand, if $CIR_{r,i}^{n,M} < \Gamma_i$, then we have $P_{r,i}^{n,M+1} = T_i \ge P_{nr,i}^{n,M+1}$. Therefore, (iii) is true for $m = M + 1$. The remaining work is to show that (iv) is true for $m = M + 1$.

Assume that $CIR_{r,i}^{n,M} \ge \Gamma_i$ and $CIR_{nr,i}^{n,M} \ge \Gamma_i$. According to Property 2, we have $CIR_{r,i}^{n,M+1} \ge \Gamma_i$ and $CIR_{nr,i}^{n,M+1} \ge \Gamma_i$. Therefore, all we have to prove is that $CIR_{r,i}^{n,M} < \Gamma_i$ together with $CIR_{r,i}^{n,M+1} \ge \Gamma_i$ imply $CIR_{nr,i}^{n,M+1} \ge \Gamma_i$. Assume that $CIR_{r,i}^{n,M} < \Gamma_i$ and

$$CIR_{r,i}^{n,M+1} = \frac{T_i G_{ib_i}}{\sum_{j \ne i} P_{r,j}^{n,M+1} G_{jb_i} + \eta_i} \ge \Gamma_i.$$

Since $P_{nr,j}^{n,M+1} \le P_{r,j}^{n,M+1}$ for all users j, we get

$$CIR_{nr,i}^{n,M+1} = \frac{T_i G_{ib_i}}{\sum_{j \ne i} P_{nr,j}^{n,M+1} G_{jb_i} + \eta_i}$$
$$\ge \frac{T_i G_{ib_i}}{\sum_{j \ne i} P_{r,j}^{n,M+1} G_{jb_i} + \eta_i} \ge \Gamma_i.$$

Consequently, (iv) is also true for $m = M+1$. This completes the proof of Lemma 2.

The meaning of Lemma 2 is that if, at the beginning of a round, the power levels employed in the NR-FDPC algorithm are smaller than or equal to those employed in the R-FDPC algorithm and, moreover, connection j satisfies its CIR requirement in the NR-FDPC algorithm if it is so in the R-FDPC algorithm, then the same conditions hold after every iteration of the round. Based on Lemmas 1 and 2, we obtain the following theorem.

Theorem 1: It holds for all n that:

(i) $P_{nr,i}^{n,m} \le P_{r,i}^{n,m}$ for all i and m;

(ii) if $CIR_{r,j}^{n,m} \ge \Gamma_j$, then $CIR_{nr,j}^{n,m} \ge \Gamma_j$ for all j and m.

The proof for Theorem 1 is similar to that for Lemma 2 and thus is omitted. It is noted that, with the results of Lemma 2, one needs only prove Theorem 1 for $m = 0$.

A consequence of Theorem 1 is that the NR-FDPC algorithm employs smaller power levels and finds a feasible power set faster than the R-FDPC algorithm. Numerical results presented in the following section show that the NR-FDPC algorithm may result in a much smaller outage probability than the R-FDPC algorithm.

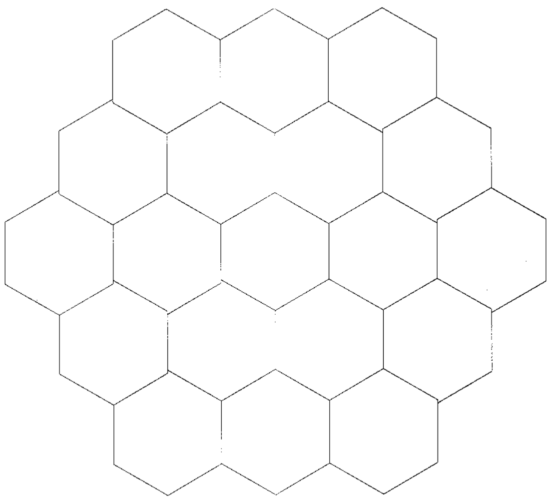

Fig. 1. A 19-cell CDMA cellular network.

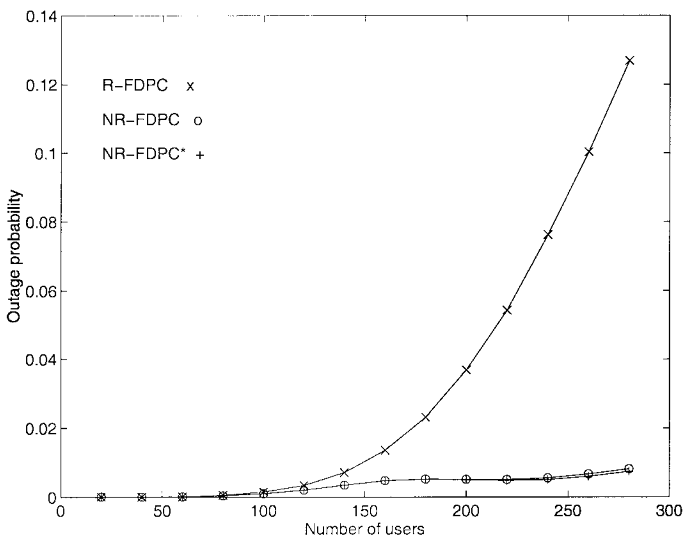

Fig. 2. Outage probability against number of users.

V. Numerical Results

In this section, we study a CDMA cellular network which is composed of 19 cells, as shown in Fig. 1. The locations of users are uniformly distributed in this network and the reverse link is considered. The chip rate is chosen to be 1.2288 Mb/s, the same as that of current IS-95 cellular CDMA system. The data rate is 9.6 kb/s, and hence, the processing gain F is 128. To obtain a bit error probability of 10^{-3}, it was reported [10] that the required E_b/I_0 is 7 dB. Since $CIR = (E_b/I_0)/F$, the CIR requirement for all users is set to -14 dB here.

The link gain G_{jb_i} is modeled as $G_{jb_i} = A_{jb_i}/d_{jb_i}^{\alpha}$, where A_{jb_i} is the attenuation factor, d_{jb_i} is the distance between the jth mobile user and the base station b_i, and α is a constant that models the large scale propagation loss. The attenuation factor models power variation due to shadowing. A_{jb_i} $(1 \leq i, j \leq N)$ is assumed to be independent, log-normal random variables with 0 dB expectation and σ dB log-variance. The parameter value of σ in the range of 4–10 dB and the propagation constant in the range of 3–5 usually provide good models for urban propagation [11]. In our simulations, we choose $\alpha = 4$ and $\sigma = 8$ as in [10].

The number of iterations L is chosen to be eight. The outage probability is defined as the ratio of the number of removed connections to the number of total connections. Numerical results were obtained by means of computer simulation for 20 000 independent configurations. In Fig. 2, we plot the outage probability against the number of users. It can be seen that the NR-FDPC algorithm results in a much smaller outage probability than the R-FDPC algorithm. In this figure, the curve for NR-FDPC* represents the outage probability for the nonreinitialized FDPC algorithm in which the connection removed in round n is the one which has the smallest CIR after one iteration of the round. It can be seen that outage probabilities for NR-FDPC and NR-FDPC* algorithms are close to each other.

VI. Conclusion

We prove in this paper that the nonreinitialized FDPC algorithm employs smaller power levels and finds a feasible power set faster than the reinitialized FDPC algorithm. Simulation results reveal that the NR-FDPC algorithm may result in a much smaller outage probability than the R-FDPC algorithm. One possible further research topic which is currently under investigation is to study the performance of removal algorithms based on other criteria such as the maximum received interference.

References

[1] J. Zander, "Performance of optimum transmitter power control in cellular radio cellular systems," *IEEE Trans. Veh. Technol.*, vol. 41, pp. 57–62, Feb. 1992.
[2] ——, "Distributed cochannel interference control in cellular radio systems," *IEEE Trans. Veh. Technol.*, vol. 41, pp. 305–311, Aug. 1992.
[3] S. A. Grandhi, R. Vijayan, and D. J. Goodman, "Distributed power control in cellular radio systems," *IEEE Trans. Commun.*, vol. 42, nos. 2/3/4, pp. 226–228, Feb./Mar./Apr. 1994.
[4] R. D. Yates, "A framework for uplink power control in cellular radio Systems," *IEEE J. Select. Areas Commun.*, vol. 13, pp. 1341–1347, Sept. 1995.
[5] T. H. Lee, J. C. Lin, and Y. T. Su, "Downlink power control algorithms for cellular radio systems," *IEEE Trans. Veh. Technol.*, vol. 44, pp. 89–94, Feb. 1995.
[6] T. H. Lee and J. C. Lin, "A fully distributed power control algorithm for cellular mobile systems," *IEEE J. Select. Areas Commun.*, vol. 14, pp. 692–697, May 1996.
[7] G. Femenias, F. J. Perez-Briceno, A. Gelonch, and I. Furio, "Transmitter power control for DS/CDMA cellular mobile radio networks," in *Proc. Sixth IEEE Int. Symp. on Personal, Indoor and Mobile Radio Communications (PIMRC'95)*, pp. 46–50.
[8] D. Kim, K. N. Chang, and S. Kim, "Efficient distributed power control for cellular mobile systems," *IEEE Trans. Veh. Technol.*, vol. 46, pp. 313–319, May. 1997.
[9] D. Kim, "Downlink power allocation and adjustment for CDMA cellular systems," *IEEE Commun. Lett.*, vol. 1, pp. 96–98, July 1997.
[10] K. S. Gilhousen, I. M. Jacobs, R. Padovani, A. J. Viterbi, L. A. Weaver, Jr., and C. E. Wheatley, III, "On the capacity of a cellular CDMA system," *IEEE Trans. Veh. Technol.*, vol. 40, pp. 303–312, May 1991.
[11] W. C. Y. Lee, "Elements of cellular mobile radio," *IEEE Trans. Veh. Technol.*, vol. VT-35, pp. 48–56, 1986.