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The fractal properties of sea surface topography derived from TOPEX/POSEIDON (1992–1996)

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Abstract

Fractal dimension with various algorithms has increasing applications in characterizing both linear and areal features. In this study, the variabilities of sea surface topography derived from TOPEX/POSEIDON are studied. The variogram method is applied to compute fractal dimensions for each scene. Cumulatively, 29 scenes taken from 1992, 111 scenes taken from 1993, 110 scenes from 1994, 98 scenes taken from 1995 and 38 scenes taken from 1996 are analyzed. The spatial resolution of each scene is two degrees along both longitudinal and latitudinal directions. The annual averages of fractal dimensions are 2.528, 2.527, 2.523, 2.523 and 2.524 for 1992, 1993, 1994, 1995 and 1996, respectively. © 1999 Elsevier Science Ltd. All rights reserved.

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1. Introduction

Fractal dimension is considered an elegant technique for describing rugged systems (Kaye, 1989). Represented as a real number, fractal dimension can function as an index reflecting the cursiveness of lines, roughness of surfaces and other 'irregularities' in Euclidean space. In an extensive review, Cox and Wang (1993) categorized fractal dimension applications in the field of earth sciences into several general categories. These categories include (a) testing whether or not a feature is fractal, (b) characterizing surface geometry to determine internal properties and (c) using fractal geometry to study formation and degradation processes and (d) using fractal slopes to determine multiple processes and the scales over which they are dominant. In this study, the variability of sea surface topography derived from TOPEX/POSEIDON is analyzed with fractal dimension. The objectives of this study are to characterize the global pattern of the studied sea surface topography (SST) to determine whether it is fractal in nature, whether this pattern is isotropic and how this global pattern changes with time.

2. Methods

Seven schemes are available to compute the fractal dimensions for surfaces: (1) the divider method, (2) the box method, (3) the triangular method, (4) the slit-island method, (5) the power spectral method, (6) the variogram method and (7) the size distribution method. The first four methods apply directly to a simple geometric pattern and the latter three methods to a functional representation (Cox and Wang, 1993). For practical implementation, a number of algorithms are reported. For instance, Jaggi et al. (1993) im-

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plemented the line-divider method, the variogram method and the triangular method. Sarkar and Chaudhuri (1992) proposed a computationally simple algorithm and compared it with four other implementations. Ouchi and Matsushita (1992) devised area-scaling, a scheme similar to the triangular method.

Fractal dimensions computed with different methods tend to vary systematically. This has been well documented in Cox and Wang (1993) and also reported in Lam (1990) and Tate (1998). Actually, different implementations based on the same fractal dimension estimation method may result differently in a systematic manner owing to differences in handling problems, such as the remainder problem, curve-fitting, orientation of the measurement plane, size and direction of the sample, etc. Estimates of fractal dimension for real-world topography are not yet reproducible, between either investigators or methods (Evans and McClean, 1995).

After reviewing the seven methods listed in Cox and Wang (1993), the variogram method is selected in this study. Semivariance is the primary tool of modern geostatistics, a field of analysis developed from regionalized variable theory for the modeling of continuous, non-deterministic surfaces exhibiting spatial dependence (Burrough, 1986; Isaaks and Srivastava, 1989). This technique is based on the idea that the statistical variation of data is a function of distance. The variogram relates distances between sample points to the variance of the differences in the data. The semivariances are used to fit an approved mathematical function. The parameters of a fitted model may include a range (a), a nugget (C_0) and a sill ($C+C_0$). The range indicates a spatial scale of the pattern, the nugget reveals information on variability between adjacent pixels, the sill gives information on the total variability of the area considered and the type of variogram model or the shape of the variogram reveals information on the spatial behavior of the data (Burrough, 1986; Jong and Burrough, 1995). Variograms are used as a description of texture for images (Lark, 1996) and applied to analyze resolution related issues (Atkinson, 1997). In the variogram method, the fractal dimension is estimated based on a geostatistical analysis, i.e. the variogram of semivariance function. For a profile along an array $z(x_i)$, the semivariance $v(h)$ can be estimated as

$$v(h) = \frac{1}{2n}\sum_{i=1}^{n}(z(x_i) - z(x_i + h))^2 \tag{1}$$

where n is the number of pairs of discrete points separated by a distance h. The fractal dimension D is obtained by measuring the slope of the log–log plot of estimated semi-variance against the sampling interval.

$$D = 3 - \frac{\text{slope}}{2}. \tag{2}$$

Whereas topographic surfaces do not exhibit pure fractal behavior, they often exhibit self-similarity across a limited range of scales. The calculations presented here are limited to variation within a range of 20 units (40°), within which fractal behavior was expected. Although the choice of the sampling interval and the slope determination of the log–log plot remains a difficult task, variogram method can be easily adopted for measures of anisotrophic data sets. In this study, data is analyzed with two sets of orthogonal directions: the grid directions (north–south, east–west) and the diagonal directions (north-east to south-west, south-east to north-west). Fractal dimensions are computed for all four directions and all data pairs are used in the case of no direction differentiation.

The computational procedure of the variogram method without direction differentiation is listed as follows:

1. Read the data to be analyzed, in this study, the SST.
2. Specify the maximum distance between two points, `Pairs_distance`. In this study, the `Pairs_distance` is specified as 90, because the longest distance between two points on the globe is 180° and the pixel resolution is 2°. The minimum distance used is one.
3. Initialize the log–log plot array, `x[Pairs_distance][3]`. In `x[.][0]`, the value of the logarithm of distance, `log d`, is stored. The height difference variances of the corresponding distance and the total occurrence of that distance will be computed and stored in `x[.][1]` and `x[.][2]` respectively in the next step.

```
k=0;
for (i=0; i<Pairs_distance; i++)
for (j=i; j<Pairs_distance; j++){
x[k][0]=sqrt(i×i+j×j);
if (x[k][0]>0.0)
x[k][0]=log(x[k][0]);
x[k][1]=0.0;
x[k][2]=0.0;
k++;}
```

4. Accumulate the height differences between two pixels, then compute the variances. There are 180 columns and 71 pixels in each column for each SST data set used in this study. That is, `n_lat=71` and `n_long=180`.

```
for (i=0; i<n_lat; i++){
for(j=0; j<n_long; j++){
k=0;
for(ii=0; ii<Pairs_distance;
ii++)
```

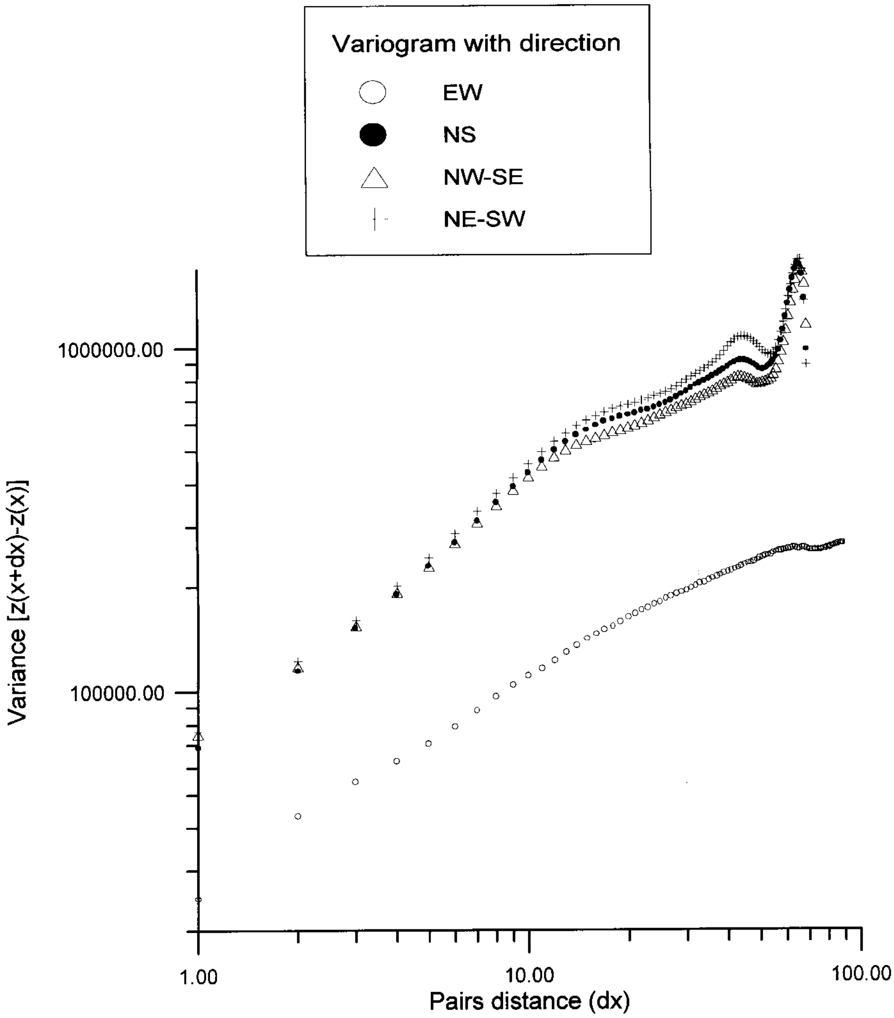

Fig. 1. Log–log plot, scene 01/01/1993, with directions.

```
for(jj=ii;     jj<Pairs_distance;
jj++)
{
if(((i+ii)<71)&&((j+jj)<180))
{
difference=SST[i+ii][j+jj]-SST[-
i][j];
x[k][1]=x[k][1]+differen-
ce×difference;
x[k][2]=x[k][2]+1;
}
if(((i+jj)<71)&&((j+ii)<180))
{
difference=SST[i+jj][j+ii]-
SST[i][j];
x[k][1]=x[k][1]+differen-
ce×difference;
x[k][2]=x[k][2]+1;
}
k++;
}}}
k=0;
for (i=0; i<Pairs_distance; i++)
for(j=i; j<Pairs_distance; j++){
x[k][1]=x[k][1]/x[k][2];
if(x[k][1]>0.0)
x[k][1]=log(x[k][1]);
k++;}
```

5. Conduct a least squares regression between `log d` and the variances. The slope of this regression is then used to calculate the fractal dimension, `D=3-slope/2`.

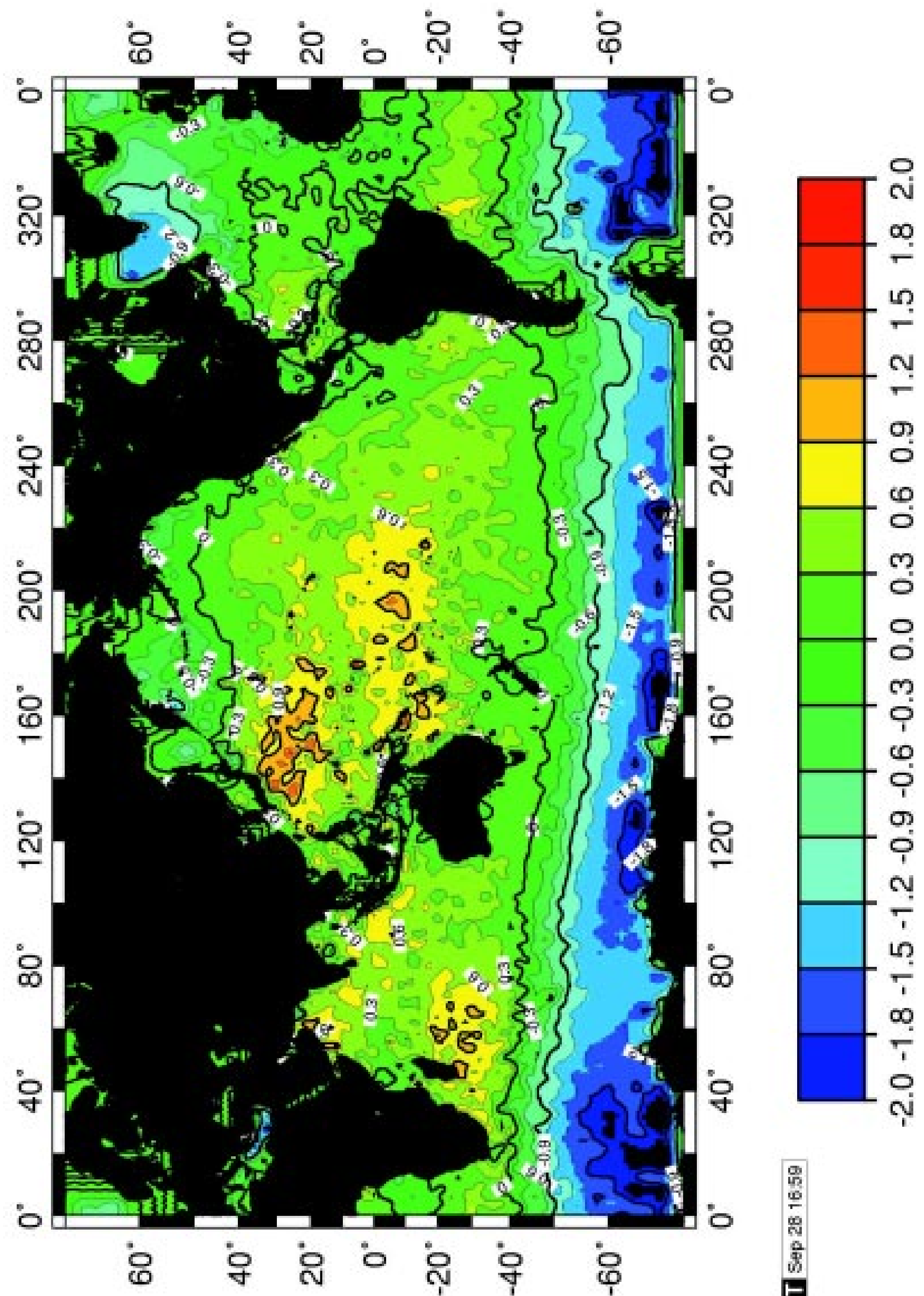

Fig. 2. Contour plot of scene 1993-01-01. Plotted with GMT (Wessel and Smith, 1991).

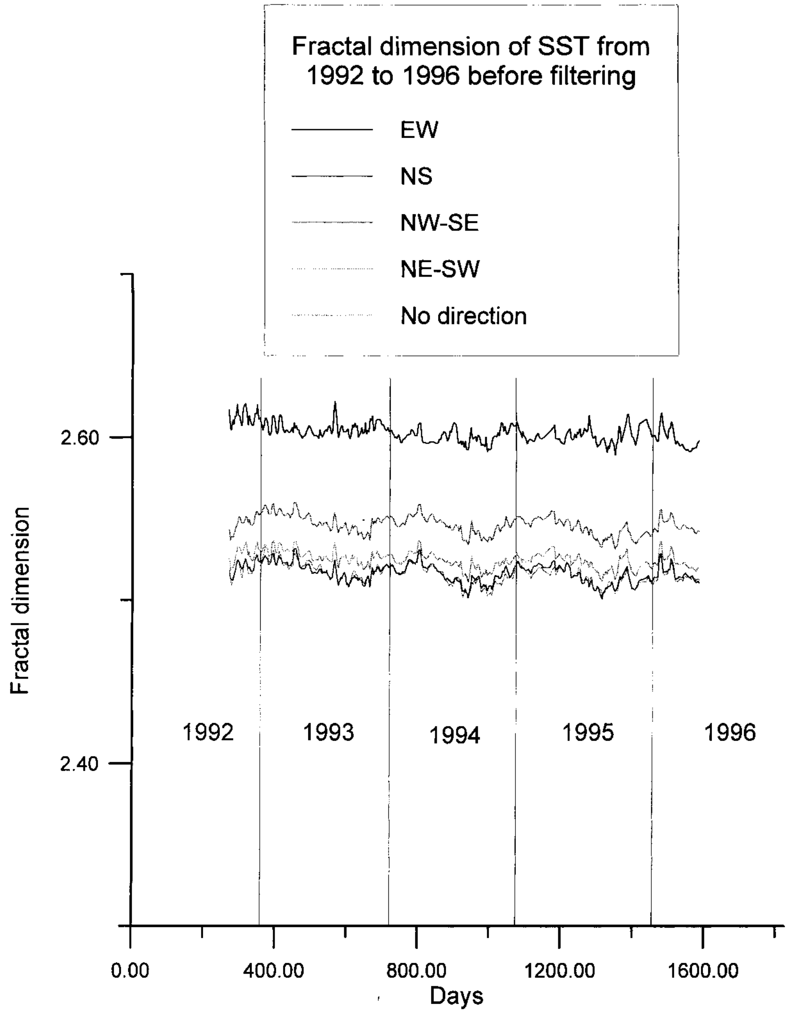

Fig. 3. Fractal dimension of unfiltered scenes.

3. Data

TOPEX/POSEIDON is a joint project conducted by the United States' National Aeronautics and Space Administration (NASA) and the French Space Agency, Centre National d'Etudes Spatiales (CNES), or studying global circulation from space (AVISO, 1992). The primary sensor for the TOPEX/POSEIDON mission is a dual frequency Ku/C band NASA radar altimeter (NRA). The measurements taken simultaneously at two frequencies, 13.6 GHz (Ku band) and 5.3 GHz (C band), are combined to obtain altimeter height of the satellite above the sea surface, including the wind speed, wave height and ionospheric corrections. The data used in this study are the dynamic sea surface heights derived from NRA measurements. Besides wind speed and water vapor corrections, the measured sea surface heights are further applied with the ocean tide model correction and geoid height. Restated, the tide and geoid components are removed from the surface height. Further discussions on data processing and quality of derived sea surface height can be found in Hwang (1996).

The duration period of the TOPEX/POSEIDON satellite is around ten days. However, the frequencies of the frames used in this study are one frame for each 3.33 day period. Each frame contains ten days of data, is spatially smoothed with average filters and is condensed to 2 degrees by 2 degrees resolution. Data condensation is performed using the weighted average method. Data within a 300 km radius of each grid

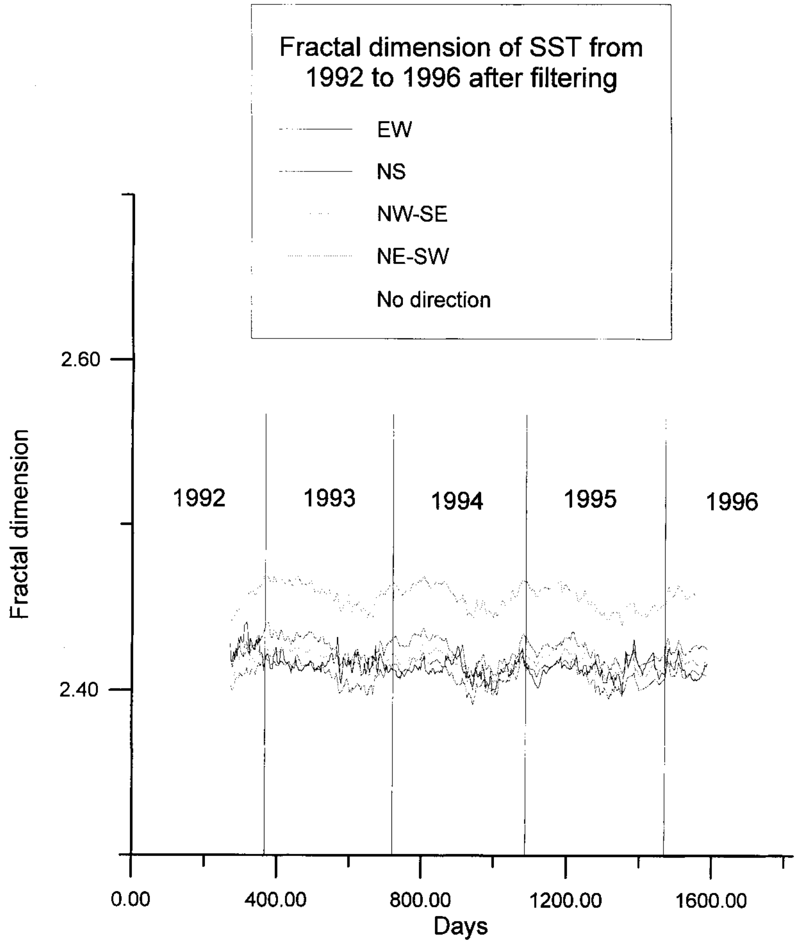

Fig. 4. Fractal dimension of filtered scenes.

node are averaged using an exponential weighting function. Some screening processes are also conducted during the gridding procedure, such as the deletion of measurements exceeding the mean surface height by more than three meters (Christensen, E.J., 1995, pers. comm.).

This data of sea surface heights is obtained from JPL with ftp via network. Each frame is stored as a separate digital file. The grid files are arranged in rows of latitude, with latitude and the 180 nodes for that latitude written in a single record. There are ninety-one such records for latitude ranging from $+90°$ to

Table 1
Statistical indices of fractal dimension for original SST data (29 scenes in 1992, 111 scenes in 1993, 110 scenes in 1994, 98 scenes in 1995, 38 scenes in 1996)

Direction conditions	1992		1993		1994		1995		1996	
	mean	S.D.	mean	S.D.	mean	S.D.	mean	S.D.	mean	S.D.
EW	2.612	0.005	2.610	0.004	2.605	0.005	2.605	0.004	2.602	0.007
NS	2.520	0.004	2.518	0.005	2.516	0.006	2.514	0.006	2.516	0.005
NW–SE	2.548	0.005	2.549	0.005	2.546	0.006	2.543	0.006	2.546	0.004
NE–SW	2.519	0.005	2.519	0.005	2.514	0.006	2.513	0.005	2.515	0.004
No direction	2.528	0.005	2.527	0.004	2.523	0.005	2.523	0.005	2.524	0.005

Table 2
Statistical indices of fractal dimension for filtered SST data (29 scenes in 1992, 111 scenes in 1993, 110 scenes in 1994, 98 scenes in 1995, 38 scenes in 1996)

Direction conditions	1992		1993		1994		1995		1996	
	mean	S.D.	mean	S.D.	mean	S.D.	mean	S.D.	mean	S.D.
EW	2.429	0.007	2.426	0.005	2.422	0.006	2.422	0.005	2.423	0.005
NS	2.410	0.005	2.411	0.007	2.410	0.008	2.408	0.007	2.411	0.003
NW–SE	2.456	0.007	2.459	0.007	2.457	0.007	2.454	0.007	2.457	0.003
NE–SW	2.426	0.005	2.428	0.006	2.424	0.008	2.423	0.006	2.426	0.003
No direction	2.418	0.006	2.420	0.005	2.417	0.007	2.416	0.006	2.418	0.003

Table 3
Fractal dimensions with different vertical scaling (test data: 01/01/1993)

Rescaled cases	Range of data		EW	NS	NW–SE	NE–SW	No direction
	max. (mm)	min. (mm)					
Original	1642.909	−2459.195	2.608	2.524	2.555	2.526	2.544
Case 1	255	0	2.608	2.524	2.555	2.526	2.544
Case 2	200	0	2.608	2.524	2.555	2.526	2.544
Case 3	100	0	2.608	2.524	2.555	2.526	2.544

−90°. The ninety-one data records are followed by ninety-one records containing the flag values, which describes the node as land, sea, or ice (Norman, R., 1995, pers. comm.). These grid files are then processed to extract the sea surface heights by over-writing all land and ice pixels with a default value. In this study, the default value is zero. The latitude of the selected study area ranges from +70° to −70° because the region poleward of 70° is covered primarily by ice and land. Because of the nature of the data sets provided, in this research, the unit used for planimetric coordinates is 2° and height is measured in mm.

4. Results and discussion

For each frame, five fractal dimensions (one for each direction condition) are computed via the variogram method. A typical log–log plot is shown in Fig. 1. Due to the nature of the earth, the meridian of longitude 180° East is the same as the longitude 180° West meridian. That is, the longest distance between two meridians is 180°. If this fact is not considered, the distance-variance plot would be symmetric. On a log–log plot, the right-hand side of the curve exhibits a sharp drop. At the end, the variance equals zero. That is, the distance of 360° is essentially 0°. In this study, the symmetric case is avoided by taking the roundness of the earth into account.

A least squares procedure is applied to obtain the slope. Only data points within a lag of 20 units (40°) are introduced in the curve fitting. The correlation index r^2 obtained from all curve fitting cases ranges from 0.975 to 0.999.

Table 4
The intercepts with different vertical scaling (test data: 01/01/1993)

Rescaled cases	Range of data		EW	NS	NW–SE	NE–SW	No direction
	max. (mm)	min. (mm)					
original	1642.909	−2459.195	4.442	4.939	5.010	4.999	4.789
case 1	255	0	1.664	2.161	2.232	2.221	2.011
case 2	200	0	1.421	1.918	1.990	1.978	1.768
case 3	100	0	0.728	1.225	1.296	1.285	1.075

As shown in Fig. 2, there are some high frequency signals in the original $2 \times 2^\circ$ data frames. Therefore, a 3×3 median filter is applied to each frame to further smoothen the sea surface heights. Fractal dimensions are computed for the filtered data sets as well. As expected, the fractal dimensions for the filtered datasets are relatively lower than those of the original datasets. In Figs. 3 and 4, the fractal dimensions are plotted against the days in a year. The characteristics of seasonal changes are very similar for all five direction conditions. The trends in the filtered and original cases are also similar. The annual average and the associated standard deviation of fractal dimensions are listed in Tables 1 and 2. A comparison of the fractal dimension time sequence for the filtered SST with the one for the original dataset reveals a similar fluctuation pattern. The fractal dimensions for the filtered data are less than those for the original, which can be accounted for by the reduced roughness caused by the removal of noise.

In this study, the unit used for planimetric coordinates is 2° and height is measured in mm. A previous study indicated that fractal dimensions are insensitive to the vertical or horizontal exaggerations and the intercept value in the log–log plot reflects the degree of vertical exaggeration. (Ouchi and Matsushita, 1992). An experiment was performed with the scene 01/01/1993 to verify this phenomenon. The fractal dimensions and the intercepts in the log–log plot are listed in Tables 3 and 4. The results are precisely as expected.

5. Concluding remarks

Results obtained in this study demonstrate that the sea surface heights derived from TOPEX/POSEIDON present a fractal nature. The average fractal dimension for the omni-direction case is 2.525 for the original datasets and 2.418 for the filtered datasets. Slight differences occur between the fractal dimensions computed for different direction conditions. The time series for both filtered and original datasets present similar fluctuations, implying that there are structural patterns in the time domain. However, this could be due to the scale of the analyzed grid data. The distance measure adopted in this research is the Eucledian distance with latitude and longitude. Fractal dimensions with different distance measures, such as the length of the great circle, are also computed. The characteristics of the fractal dimension time series remain, but the lower correlation index value (typically in the range of 0.5 to 0.7) indicates that other non-fractal processes, most likely the numerical effect in the computation of variogram, have larger influences on the geodesics.

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