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Time-Dependent Ginzburg-Landau Equation for $d_{x^2-y^2}$-wave Superconductors: Hall Effect in the Low Field Regime

Tzong-Jer Yang and Mei-Cheng Dai

Department of Electrophysics, National Chiao-Tung University
Hsinchu, Taiwan 30050, Repuplic of China

The Hall effect is studied by using a phenomenological time-dependent Ginzburg-Landau equation with mixing of s-and d-wave components in $d_{x^2-y^2}$-wave superconductors within a low field regime. An equation of motion for a single vortex is derived and the Hall angle is obtained under an external driving current along the crystal axis. We find that not only the imaginary parts of s- and d-wave relaxation time but also the mixed gradient terms may change the sign of the Hall effect.

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1. INTRODUCTION

Anomalous Hall effect in low magnetic fields and at temperatures close to but below the superconducting transition temperature has been observed in most high-T_c superconductors (HTSC) and some conventional superconductors.[1] It seems that this effect is rather general and just needs an explanation in terms of general properties of the vortex dynamics of the mixed state not specific for s-wave or d-wave superconductors. The vortex structure of $d_{x^2-y^2}$-wave superconductors has been verified to be completely different from that of s-wave superconductors. To find the general properties of the vortex dynamics of the mixed state in superconductors is a difficult problem. An approach based on the time dependent Ginzburg-Landau (TDGL) equation has been shown to be good for describing the Hall effect in the superconducting state.[2,3] In the recent experimental results for the Hall anomaly in the superconducting state of HTSC from the underdoped

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to the overdoped regime, Nagaoka et al.[4] argue that the TDGL equation based on the s-wave weak coupling theory fails to predict the Hall sign to be universal and determined by the doping level. Thus, the dynamics of d-wave vortices with a TDGL equation is necessarily studied. Alvarez, Dominguez and Balseiro[5] recently did this approach and numerical simulations under an external driving current oriented with an angle φ with respect to the b-axis for very high Ginzburg–Landau parameter κ are presented to have an intrinsic Hall effect depending on $\sin(4\varphi)$. Zhu, Kim, Ting and Hu[6] derived microscopically a set of coupled TDGL equations for superconductors with mixed d- and s-wave order parameters based on the approach of Gor'kov and Eliashberg[7] But their TDGL equations are hard to solve and very complicated. Here, we use the approach of Dorsey[2] and Kopnin et al.[3] to derive the Hall angle from the TDGL equations for superconductors with mixed s- and d-wave order parameter in low magnetic field. The equation of motion for single vortex and the Hall angle will be presented in the subsequent section. The discussion and conclusion will be described in the final.

2. VORTEX MOTION AND THE HALL ANGLE

The phenomenological TDGL equations in the dimensionless form for $d_{x^2-y^2}$ wave superconductors may be written as ($\hbar = c = 1$ used)

$$\Gamma_d\left(\frac{\partial}{\partial t} + i2e\phi\right)d = \alpha_d d + \frac{8}{3}\left(\frac{\beta_2}{\beta_1}\right)|d|^2 d + \frac{4}{3}\left(\frac{\beta_3}{\beta_1}\right)|s|^2 d + \frac{8}{3}\left(\frac{\beta_4}{\beta_1}\right)d^* s^2 + 2\left(\frac{\gamma_d}{\gamma_s}\right)\vec{\pi}^2 d + 2\left(\frac{\gamma_m}{\gamma_s}\right)(\pi_x^2 - \pi_y^2)s, \tag{1}$$

$$\Gamma_s\left(\frac{\partial}{\partial t} + i2e\phi\right)s = \alpha_s s + \frac{8}{3}|s|^2 s + \frac{4}{3}\left(\frac{\beta_3}{\beta_1}\right)|d|^2 s + \frac{8}{3}\left(\frac{\beta_4}{\beta_1}\right)s^* d^2 + 2\vec{\pi}^2 s + 2\left(\frac{\gamma_m}{\gamma_s}\right)(\pi_x^2 - \pi_y^2)d \tag{2}$$

where $\vec{\pi} = -(i/\kappa)\nabla - \vec{A}$ is defined in the ab plane. $\vec{A}$ is the vector potential. s and d are the order parameters of s-wave and d-wave, respectively. Parameters α_s and α_d depend on temperature; β_1, β_2, β_3 are positive, $\gamma_i = \hbar^2/2m_i$ with $i = s, d, m$ and m_i^* is effective mass. The dimensionless coefficients Γ_d and Γ_s, describing complex relaxation time, are defined to be $\Gamma_d = \eta_{d_1} + i\eta_{d_2}$, $\Gamma_s \equiv \eta_{s_1} + i\eta_{s_2}$ respectively. The imaginary part of the relaxation time can give rise to the particle–hole asymmetry, vortex traction, and nonvanishing Hall current[2,3] in s-wave superconductivity. Here, we limit our consideration in the low field limit $h \ll H_{c_2}$. Except the coupled set of TDGL equations, the Amper's law $\nabla \times (\nabla \times \vec{A}) = 4\pi(\vec{J}_n + \vec{J}_s)$ is required. The continuity equation $\nabla \cdot (\vec{J}_n + \vec{J}_s) = 0$ should also be used. The supercurrent $\vec{J}_s$ is

$$\begin{aligned}\vec{J}_s &= 2s^*\vec{\pi}s + 2\left(\frac{\gamma_d}{\gamma_s}\right) d^*\vec{\pi}d \\ &+2\left(\frac{\gamma_m}{\gamma_s}\right)\{\hat{x}[s^*\pi_x d + d(\pi_x s)^*] - \hat{y}[s^*\pi_y d + d(\pi_y s)^*\} + c.c. \quad (3)\end{aligned}$$

while the normal current density $\vec{J}_n$ is given by

$$\vec{J}_n = \sigma^{(n)} \cdot \vec{E} = \sigma^{(n)}\left(-\frac{1}{\kappa}\nabla\phi - \partial_t \vec{A}\right) \quad (4)$$

where $\bar{\phi} = 2e\phi$ and $\sigma^{(n)}$ is the normal-state conductivity tensor.

Now, the complex order parameters s and d may be expressed in terms of an amplitude and a phase, $d(\vec{r},t) = f(\vec{r},t)e^{i\theta_d(\vec{r},t)}$ and $s(\vec{r},t) = g(\vec{r},t)e^{i\theta_s(\vec{r}_1 t)}$. These complex order parameters are substituted into eqs.(1)–(4), then we separate the real and imaginary parts from the TDGL equations. To solve these nonlinear equations, the method developed by Gorkov and Kopnin[8] in their study of flux flow is used. Three essential steps are made in the calculation. First, the vortices translate uniformly are assumed. All quantities $\psi(x,t)$ such as the order parameter; vector potential, chemical potential, characterizing the vortex system are functions of $\vec{r} - \vec{V}_L t$, where $\vec{V}_L$ is the vortex velocity. Next, $\psi(\vec{r},t)$ may be expanded in first order of V_L

$$\psi(\vec{r},t) = \psi^{(0)}(\vec{r} - \vec{V}_L t) + \psi^{(1)}(\vec{r} - \vec{V}_L t)$$

where $\psi^{(1)}$ is small relative to V_L. Then ψ may be expanded in powers of V_L. The terms of $O(1)$ and $O(V_L)$ correspond to the equilibrium GL equations and a set of inhomogeneous linear differential equations, respectively. The final step is to get the equation of motion for the vortices, which is equivalent to get the solvability condition. Here we are interested in a very large κ limit.

In order to evaluate the solvability condition, we choose the z direction of the coordinate system to be along the uniform magnetic field. The applied transport current J_t is in the x direction (i.e. direction of the a crystal axes). The vortex moves at an angle θ_H with respect to the $-y$ direction and the origin of cylindrical coordinates (r, θ, z) at the center of a vortex. The displacement vector $\vec{\ell}$ makes an angle φ with respect to the x-axis. In order to investigate how the mixed gradient terms affect the sign of the Hall effect, the region close to vortex core is focused. Because the far away from the vortex core, $|s| \ll |d|$, so that mixed gradient terms may be neglected. The vortex-vortex interaction can also be neglected as $r \gg \lambda_d$, where λ_d is the magnetic penetration depth of the d-wave superconductivity. The integration regions in the solvability condition are set the cut-off at λ_d. Finally, we obtain the equation of motion for the vortex

$$\vec{J}_t \times \hat{z} = \frac{\alpha_1 \kappa}{2}\vec{V}_L + \frac{\alpha_2 \kappa}{2}\vec{V}_L \times \hat{z} \tag{5}$$

where the parameters α_1 and α_2 are given by

$$\begin{aligned}
\alpha_1 &= \frac{8\sigma_{xx}^{(n)}}{\kappa^2\lambda_d^2} p_{d_1}^{(1)} + \eta_{d_1} c_1^2 + \eta_{s_1} c_2^2 \\
&\quad -\frac{4}{\kappa^2}[b_2 c_2 + \left(\frac{\gamma_d}{\gamma_s}\right) a_2 c_1 - \left(\frac{\gamma_m}{\gamma_s}\right)(b_2 c_1 + a_2 c_2)], \qquad (6)\\
\alpha_2 &= 8\left(-\sigma_{xx}^{(n)} p_{d_2}^{(1)} + \sigma_{xy}^{(n)} h_o\right)/(\kappa^2\lambda_d^2) - \eta_{d_2} c_1^2 + \eta_{s_2} c_2^2 \\
&\quad -\frac{4}{\kappa^2}[b_1 c_2 + \left(\frac{\gamma_d}{\gamma_s}\right) a_1 c_1 - \left(\frac{\gamma_m}{\gamma_s}\right)(b_1 c_1 + a_1 c_2)], \qquad (7)
\end{aligned}$$

where

$$\begin{aligned}
a_1 &= \kappa^2[6\eta_{d_2} c_1 - 8(\gamma_m/\gamma_s)\eta_{s_2} c_2]/\Delta_o, \\
a_2 &= \kappa^2[-6\eta_{d_1} c_1 - 8(\gamma_m/\gamma_s)\eta_{s_1} c_2]/\Delta_o, \\
b_1 &= \kappa^2[6(\gamma_d/\gamma_s)\eta_{s_2} c_2 - 8(\gamma_m/\gamma_s)\eta_{d_2} c_1]/\Delta_o, \\
b_2 &= \kappa^2[6(\gamma_d/\gamma_s)\eta_{s_1} c_2 + 8(\gamma_m/\gamma_s)\eta_{d_1} c_1]/\Delta_o, \\
\Delta_o &= 36(\gamma_d/\gamma_s) - 64(\gamma_m/\gamma_s)^2, \\
c_2 &= \left(\frac{|\alpha_d|}{\alpha_s}\right)(\gamma_m/\gamma_d)c_1/2.
\end{aligned}$$

Here h_o is magnetic field at the center of vortex core, the scalar potentials p_{d_1}, p_{d_2}, p_{s_1} and p_{s_2} satisfy a set of homogeneous equation for $O(V_L)$ related to $r \to 0$. The coefficient c_1 is the coefficient of the amplitude of d-wave order parameter near the center of the vortex core when the system is in the equilibrium.

3. DISCUSSION AND CONCLUSION

For convenience of the discussion of the Hall angle we present eq. (7) as $\alpha_2 = \alpha_{20} + \alpha_{2I}$ where

$$\begin{aligned}
\alpha_{20} &= 8(-\sigma_{xx}^{(n)} p_{d_2}^{(1)} + \kappa\sigma_{xy}^{(n)} h_o)/(\kappa\lambda_d)^2, \\
\alpha_{2I} &= -\eta_{d_2} c_1^2 + \eta_{s_2} c_2^2 - 4[b_1 c_2 + (\gamma_d/\gamma_s) a_1 c_1 - (\gamma_m/\gamma_s)(b_1 c_1 + a_1 c_2)]/\kappa^2.
\end{aligned}$$

Parameter α_{20} is independent of the imaginary parts of the relaxation time. If $\alpha_{2I} < 0$ and $|\alpha_{2I}| > \alpha_{20}$, then the Hall angle changes sign.

If $\gamma \neq 0$, $\eta_{d_2} = \eta_{s_2} = 0$, then $a_1 = b_1 = 0$, $\alpha_{2I} = 0$ and $\tan\theta_H = \alpha_{20}/\alpha_1$. This shows no sign change of the Hall angle. Now let us look at the situation $\gamma_m = 0$, i.e. no mixed gradient terms, $\eta_{d_2} \neq 0$, $\eta_{s_2} \neq 0$ then $c_2 = 0$, $b_1 = b_2 = 0$, $a_1 = \eta_{d_2} c_1 \kappa^2/6(\gamma_d/\gamma_s)$, $a_2 = -\eta_{d_1} c_1 \kappa^2/6(\gamma_d/\gamma_s)$, $\Delta_o = 36(\gamma_d/\gamma_s)$. Therefore

$$\tan\theta_H = -\frac{\sigma_{xx}^{(n)} p_{d_2}^{(1)} - \kappa\sigma_{xy}^{(n)} h_o + 5\eta_{d_2} c_1^2 \kappa^2 \lambda_d^2/24}{\sigma_{xx}^{(n)} p_{d_1}^{(1)} + 5\eta_{d_1} c_1^2 \kappa^2 \lambda_d^2/24}$$

This is the same form as Dorsey[2] got in his s-wave superconductor. For $\gamma_m \neq 0$, the expression of $\tan\theta_H$ is very complicated, but the mixed gradient terms plays some role to change sign of Hall angle, except the imaginary parts of relaxation time.

In conclusion, the imaginary part of relaxation time of s-wave and d-wave order parameter plays the key role in the anomalous Hall effect. Also, the mixed gradient term plays some role in the sign change of the Hall angle. If no mixed gradient term, then the expression of Hall angle is reduced to the form as that of s-wave superconductor. For further physical content, a microscopic model of the mechanism of HTSC is needed to formulate TDGL equations.

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