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An overview of a heuristic for vehicle routing problem with time windows

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ABSTRACT

In this paper, a two-stage metaheuristic based on a new neighborhood structure is proposed to solve the vehicle routing problem with time windows. Our neighborhood construction focuses on the relationship between route(s) and node(s). Unlike the conventional methods for parallel route construction, we construct routes in a nested parallel manner to obtain higher solution quality. Computational results for 60 benchmark problems are reported. The results indicate that our approach is highly competitive with all existing heuristics, in particular very promising for solving problems with large size.

Keywords: Routing; Time Windows; Neighborhood; Heuristics

INTRODUCTION

Routing and scheduling problems have been an intensive research for example surveys in (Bodin et al., 1983; Fisher, 1995). The purpose of this paper is to develop a method in solving the vehicle routing problem with time windows (VRPTW) so that either the obtainment of high quality fast-solution or an attempt in finding the best solution can be achieved.

The VRPTW can be formally stated as follows. Let G = (V, A) be a graph with node set $V = N \cup \{0\}$ and arc set $A = \{(i,j)| i \in V, j \in V, i \neq j\}$, where $N = \{1, 2,\dots, n\}$ represents the customer set, and node 0 refers to the central depot. Each node $i \in V$ is associated with a customer demand q_i ($q_0 = 0$), a service time s_i ($s_0 = 0$), and a service-time window $[e_i, l_i]$. For every arc $(i, j) \in A$, a non-negative distance d_{ij} and a non-negative travel time t_{ij} are known. Moreover, vehicles housed at the central depot are identical. Without loss of generality, each customer demand is assumed to be less than the vehicle capacity Q. In addition, the demand of each customer can not be split. The VRPTW is to find an optimal set of routes in such a way that:(i) all routes start and end at the depot;(ii) each customer in N is visited exactly once within its time window;(iii) the total of customer demands for each route can not exceed the vehicle capacity Q. Our primary objective is to minimize the number of vehicles used, and our secondary objective is to minimize the total distance traveled.

It is obvious that the VRPTW is NP-hard due to the NP-hardness of VRP. The exact algorithms recently developed for solving the VRPTW can be found in (Kolen et al., 1987; Desrochers et al., 1992; Fisher et al., 1997; Kohl and Madsen, 1997). However, on the fifty-six 100-customer benchmark problems by Solomon (1987), only a total of 11 problems were solved to optimality. Optimization approach can refer to the surveys by Desrochers et al. (1988) and Desrosiers et al. (1995).

Solomon (1987) was among the first to generalize VRP heuristics for solving the VRPTW. A parallel route building algorithm was contributed by Potvin et al. (1993). Kontoravdis and Bard (1995). Russell (1995) embedded a local improvement procedure into the route construction phase. Literature focusing on the investigation of improvement procedures based on the node-exchange and edge-exchange can be found in (Savelsbergh, 1985, 1992; Thompson and Psaraftis, 1993; Potvin and Rousseau, 1995).

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In this paper, we propose a **Route-NEighborhood-based Two-Stage methaheuristic (RNETS)**. In consideration of the geographical characteristics for VRPs, an appropriate solving approach is to spend time on efficient local operations than on inefficient global operations. Our **RNETS** provides the flexibility by ranging from a very small search domain to the entire domain.

The main contribution of this paper is the development of a new neighborhood structure based on a relationship between routes and nodes. Moreover, two new concepts, nested parallel route construction and end-effect handling, are presented in order to enhance solution quality.

AN OVERVIEW OF RNETS

A typical approach in solving VRPs consists of the following two phases: route construction phase and route improvement phase. Thus, in addition to an improvement procedure designed for the route improvement phase, a local improvement procedure is also considered during our route construction phase. Moreover, previous results have shown that the parallel route construction method is in general superior to the sequential one. **RNETS** considers a mixed strategy consisting of the following two stages. In the first stage, we construct routes in a nested parallel manner from the lower bound direction. With a solution obtained in stage I, stage II is then to construct routes in parallel from an upper bound direction.

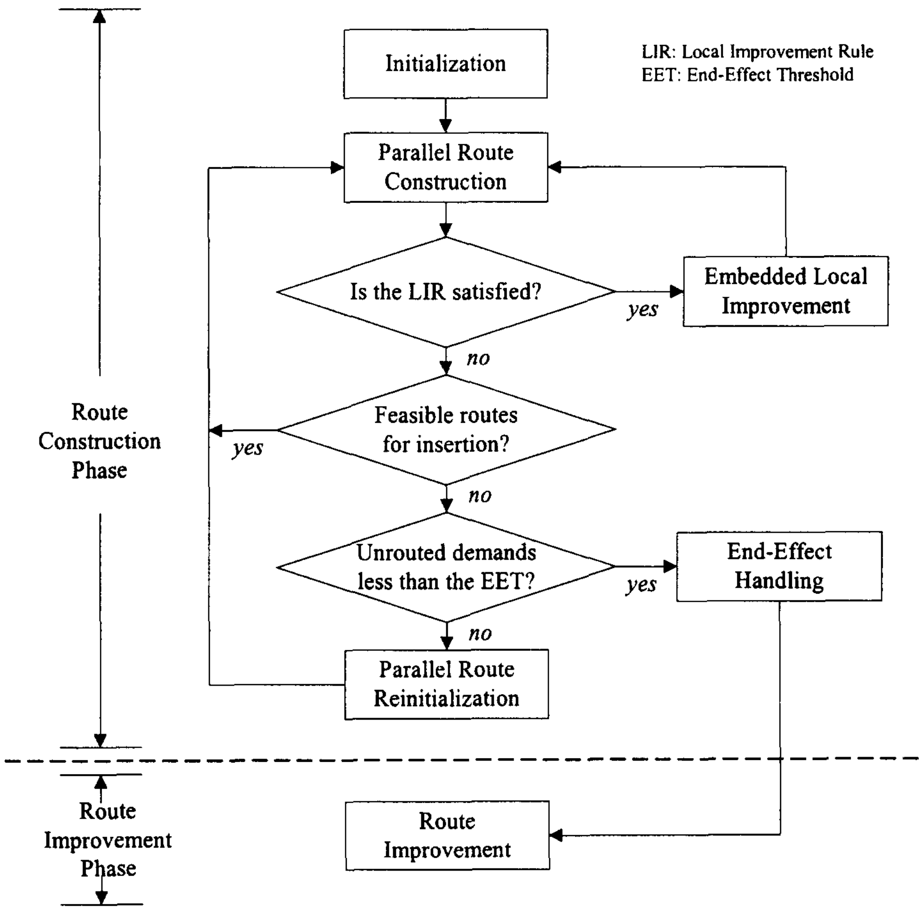

Figure 1. The core framwork for each stage of RNETS

Fig. 1 displays the core framework underlying each stage of our **RNETS**. The *initialization block* refers to the construction of an initial set of route neighborhoods. The *parallel route construction block* is responsible for selecting a candidate and inserting it into a specific route. The *parallel route reinitialization block* is to eliminate a number of constructed routes and reconstruct an appropriate set of partial routes. What we called the nested parallel route construction is accomplished through the *parallel route construction block* and the *parallel route reinitialization block.* The purpose of *end-effect handling block* is to apply a specially designed procedure on "a few" unrouted customers who are left with no feasible vehicles with respect to the current set of routes during the route construction phase. Reasons in addressing the concepts of route neighborhoods, nested parallel route construction, and end-effect handling are now described as follows.

Route Neighborhoods

To efficiently deal with large scale VRPs, an intuitive idea is to decompose the original hard problem into several easier subproblems or to divide the original domain into several smaller subdomains.

For VRPs, the term "neighborhood" usually refers to a set of nodes. To be clear, we discuss the neighborhood in the following two cases. Case 1: Before the start of route construction, we may divide the customer set into several smaller subsets or neighborhoods. Usually, these neighborhoods are disjoint partitions. Case 2: Having finished route construction, we can apply local search techniques on the current solution. In both the cases, the construction of neighborhoods primarily focused on the relationship between node(s) and node(s). The relationship between route(s) and node(s) is almost ignored.

Consequently, from a different point of view, we introduce the concept of *route neighborhood.* It is a reasonable conjecture that a customer holds higher probability of being served by near-by routes than by farther routes. Under this observation, we attempt to construct a route neighborhood containing a set of "routes" for each node. In terms of the timing in constructing neighborhoods, our route neighborhood construction has higher similarities to the neighborhoods described in Case 1. However, our route neighborhoods also play a role like the neighborhoods in Case 2. There are at least two benefits resulting from the construction of route neighborhoods. The first is that the weakness in handling time window constraints faced by the methods like "cluster-first and route-second" can be easily improved; this is indeed why we can generate high quality fast-solution by focusing on a smaller search domain. The second is that to thoroughly explore solutions on the search space can be easily designed by adjusting the number of routes to be contained in a route neighborhood from one to a large enough number.

Nested Parallel Route Construction

Because of the presence of time window constraints, it seems hard to estimate a lower bound tight enough to the number of vehicles actually required. We often encounter that many customers can not be feasibly served by using only those vehicles. Therefore, we perhaps need to generate more new routes to accommodate the remaining unrouted customers. Seeing that the parallel construction method had better performance in average, however, we introduce the concept of *nested parallel route construction* to obtain better solution quality for VRPTW. The nested parallel route construction repeats the following two steps until all customers are routed or a stopping rule is satisfied. At the first step, we estimate a lower bound to the number of vehicles required for the unrouted customers, and construct a corresponding set of partial routes. For the routes just generated, the second step is using these routes to service the unrouted customers until no feasible insertion locations can be found. We will describe the stopping rules in a later.

End-Effect Handling

In the presence of time window constraints, to eliminate a scheduled route entirely might not be an easy thing if we only rely on a refinement procedure in the route improvement phase. When there are only "a few" unrouted customers left after a number of parallel construction runs, we do not immediately create a new route or a new set of routes for them. Since our primary objective is to minimize the number of routes, we design a special procedure to handle the remaining "a few" unrouted customers. We will say that there are only "a few" unrouted customers if the total of unrouted demands is smaller than a specified threshold value. Three measurement criteria, called *end-effect thresholds*, are suggested for the threshold value: vehicle capacity, maximal utilization rate × vehicle capacity, and average utilization rate × vehicle capacity. Here, the maximal and average utilization rates are computed with respect to the routes constructed in all previous parallel construction runs. These thresholds are the stopping rules mentioned earlier.

COMPUTATIONAL STUDIES

To assess the performance of **RNETS**, we first solved four real world problems obtained from George Kontoravids. Among them, two 249-customer problems, D249 and E249, were generated from a 249-customer data set reported in Baker and Schaffer (1988) and two 417-customer problems, D417 and E417, were reported in Russell (1995). In addition, **RNETS** was tested on fifty-six benchmark problems by Solomon (1987); each is 100-customer Euclidean problem.

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