Nonlinear dynamics of self-pulsating laser diodes under external drive

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A self-pulsating laser diode under external drive shows the nonlinear dynamical effects of two competing frequencies. One can establish a route to chaos by increasing the modulation current. However, when the modulation frequency is varied, the chaotic state is suddenly destroyed at the harmonic frequencies of the self-pulsating frequency. This approach therefore offers a simple chaotic light source for possible optical chaotic communication.

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Dynamical chaos in laser diodes has become an interesting topic because of its potential application in digital communication.[1,2] Chaotic light output from a laser diode can be achieved by external optically or electronically controlled techniques. Optically controlled techniques include optical feedback by use of an external cavity[3] and optical injection from a second laser diode.[4] Electronically controlled techniques are carried out by addition of a sinusoidal signal to a dc bias current.[5,6] In general, high bias and strong sinusoidal current modulation are required for chaos.

Interaction between nonlinear oscillators and applied signals is a problem that has long been of importance.[7] In particular, the self-pulsating laser diode presents itself as an ideal candidate for such a study.[8–11] When this diode is under a dc bias, a self-sustained periodic light output (a frequency of f_0) exists. Egan *et al.*[12] have shown a synchronization effect between the applied signal and the periodic light output. In this Letter we further investigate the nonlinear dynamics of this self-pulsating laser diode under an external sinusoidal input f. A set of rate equations with three dimensions, the photon density and the electron densities in the gain region and the absorption region, is used to describe various properties of this self-pulsating laser diode. Various nonlinear dynamical effects between these two competing frequencies, including independent oscillation, locking, quasi-periodic oscillation, a route to chaos, and crises at the harmonic frequencies, are reported in this Letter. The results are based on the power spectra and the bifurcation diagram of the output photon density.

The three-dimensional rate equations that describe the self-pulsating effects in a saturable-absorption laser diode are[9]

$$\mathrm{d}S/\mathrm{d}t = [k_1\xi_1(N_1 - N_{g1}) + k_2\xi_2(N_2 - N_{g2}) - G_{\mathrm{th}}] \times S + C\,\frac{N_1 V_1}{\tau_s}\,, \tag{1}$$

$$\mathrm{d}N_1/\mathrm{d}t = -\frac{k_1\xi_1}{V_1}(N_1 - N_{g1})S - \frac{N_1}{\tau_s} - \frac{N_1 - N_2}{T_{12}} + \frac{I}{eV_1}\,, \tag{2}$$

$$\mathrm{d}N_2/\mathrm{d}t = -\frac{k_2\xi_2}{V_2}(N_2 - N_{g2})S - \frac{N_2}{\tau_s} - \frac{N_2 - N_1}{T_{21}}\,, \tag{3}$$

where S is the total photon density, N is the electron density, τ_s is the carrier lifetime, I is the injection current, V is the layer volume, ξ is the confinement factor, G_{th} is the threshold gain level, T is the carrier time-diffusion constant between the two layers, k is the linear approximation constant for the gain curve, N_g is the transparent level of electron density, and C is the coupling ratio between the spontaneous field and the lasing mode. The subscripts 1 and 2 represent each term in the active layer and the absorption layer, respectively. Detailed descriptions of the model are given in Ref. 9. In addition, the injection current is given by $I = a + b \sin 2\pi ft$, where a is the bias current, b is the modulation current, and $f = mf_0$ is the modulation frequency.

Table 1 lists all the parameters of the self-pulsating laser diode obtained from Ref. 9 that are used in the simulation. Increasing the bias current yields a dramatic change in output light at 19.8 mA, corresponding to the threshold current of the laser. When it is biased above the threshold, the bias current increases, and so does the self-pulsating frequency f_0. When $a = 30$ mA is injected into the laser diode, the corresponding f_0 is 2.28 GHz. This value of f_0 is used throughout the calculations.

With a constant injection current, the self-pulsating laser diode produces a self-sustained periodic light output with a frequency of f_0. The external drive $b \sin 2\pi ft$ is also added to the diode. Thus there are two fundamental frequencies oscillating in the system.

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Table 1. Parameters Used for Simulation of Self-Pulsating Laser Diodes

Parameter	Value	Units
k_1	3.08×10^{-12}	m^3/S
k_2	1.232×10^{-11}	m^3/S
ξ_1	0.2034	–
ξ_2	0.1449	–
N_{g1}	1.4×10^{24}	m^{-3}
N_{g2}	1.6×10^{24}	m^{-3}
V_1	72	μm^3
V_2	102.96	μm^3
T_{12}	2.65	ns
T_{21}	4.452	ns
G_{th}	3.91×10^{11}	S^{-1}
C	1.573×10^{-23}	μm^{-3}
τ_s	3	ns

The system shows independent oscillation if these two frequencies are far apart. However, frequency locking occurs if the external frequency f is approaching the self-pulsating frequency f_0. Figures 1 and 2 show the power spectra of S in arbitrary units for $m = 0.3$ and $b = 1$ mA and $m = 0.9$ and $b = 1$ mA, respectively. When $m = 0.3$, the two competing frequencies are far apart and coexist in the output spectra. The other peaks in the spectrum are the linear combination of f and f_0. By changing the modulation frequency ($m = 0.9$), we can lock the frequency at f. The spectrum shows only f and its harmonic frequencies. If b is decreased to 0.5 mA, there will be no locking effect because the external drive is too weak to have any effect.

By increasing b, we can establish the route to chaos. Figure 3 shows the power spectrum of S for $m = 0.3$ and $b = 9$ mA. Comparing this figure with Fig. 1, we can see that the two spectra correspond to chaotic and periodic output with the same modulation frequency. Note that for the periodic case a linear combination frequency of f and f_0 is shown in the spectrum, with an overall noise level of much less than 10^0. By adjustment of the modulation current from 1 to 9 mA, the output of this laser diode can be made chaotic. In the chaotic case the peaks at f are present but the spectrum has also developed a broad continuous component. This component is far above the noise level of 10^0, as is evident in Fig. 3. The appearance of such a broad spectrum in the output clearly indicates chaotic vibrations in the laser diode, whereas the input of the system is a single-frequency signal.

The above-described chaotic behaviors can be further illustrated in a bifurcation diagram. Figure 3 shows a bifurcation diagram of the photon density S versus b, where the local maxima of S are plotted beginning from 40 ns. The first 40 ns is not plotted because we intend to show the orbit on the attractor and not the transient motion leading to it. A parameter resolution of 0.01 mA is used, which is computationally demanding. For the region in which b is less than 6 mA, the contribution of the external drive is increasing. The attracting orbit is gradually widened and fills the entire interval. The motion is quasi-periodic. However, there also exists a b value (4.8 mA) for which the attractor is periodic. The occasional values that give rise to periodic attractors are interspersed between those that produce quasi-periodic attractors. As $b = 6.4$ mA, a three-period orbit develops. From 6.4 to 8.5 mA there is a three-period window in the process. As b increases further, the orbit undergoes period doubling and is finally widened abruptly into a single band. It is apparently a chaotic orbit.

The external drive at the harmonic frequencies of f_0 can cause a discontinuous change in the chaotic

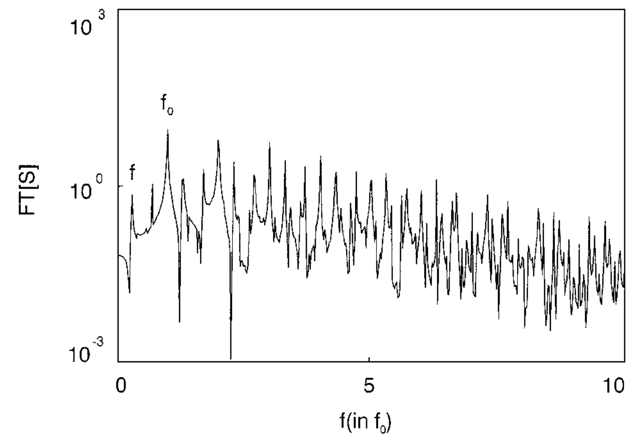

Fig. 1. Power spectrum of S in arbitrary units for $m = 0.3$ and $b = 1$ mA.

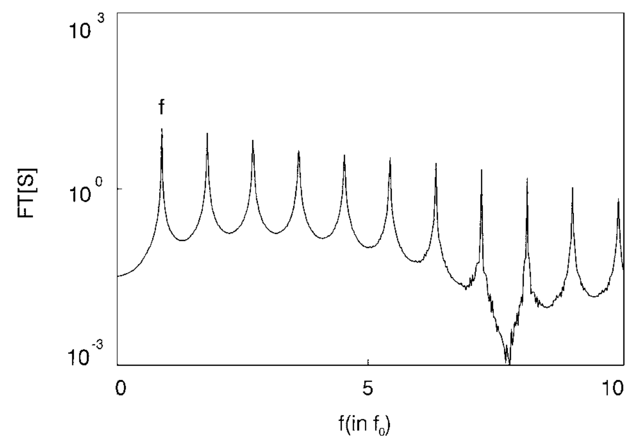

Fig. 2. Power spectrum of S in arbitrary units for $m = 0.9$ and $b = 1$ mA.

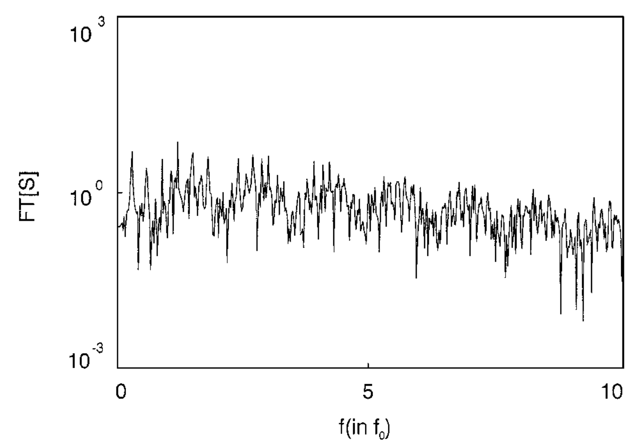

Fig. 3. Power spectrum of S in arbitrary units for $m = 0.3$ and $b = 9$ mA.

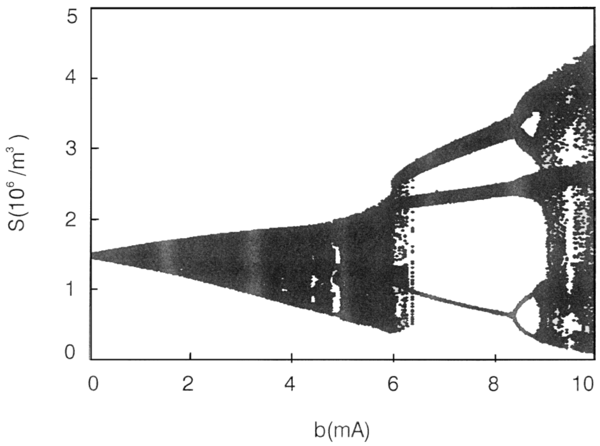

Fig. 4. Bifurcation diagram of S in units of $10^6/\mathrm{m}^3$ for $m = 0.3$ versus b.

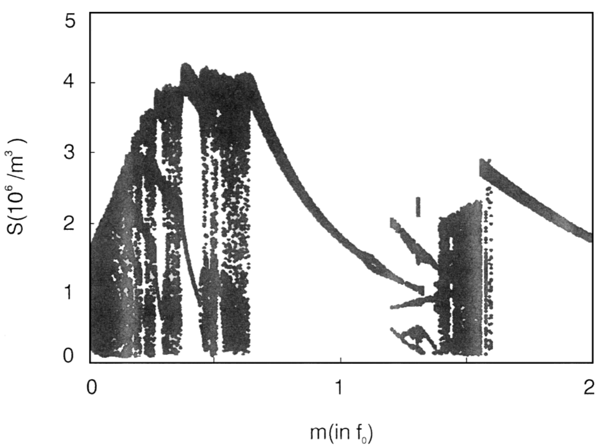

Fig. 5. Bifurcation diagram of S in unit of $10^6/\mathrm{m}^3$ for $b = 9$ mA versus m.

behaviors. Figure 4 shows a bifurcation diagram of S for $b = 9$ mA versus m. The resolution of the diagram is 0.002. Note that m is less than 0.6 and that the photon density has shown a chaotic orbit. When the modulation frequency varies with the same modulation current b, the chaotic orbit is suddenly destroyed, as m passes through the frequency of f_0. Such changes are caused by the collision of the chaotic attractor with a one period-orbit. When m reaches 1.2, the crises are over and the orbits undergo period doubling. Then there is a sudden creation of chaos at $m = 1.4$. In this window from 0.6 to 1.2, the chaotic orbit collides with a strong one-period orbit in which the external frequency is near the first-harmonic frequency, f_0. The same effect happens near the second-harmonic frequency, $2f_0$. This result indicates that strong periodic orbits exist at harmonic frequencies in this system. Therefore these two competing frequencies have returned to a periodic state if the external frequency is near the harmonic frequencies of f_0.

In conclusion, it has been proposed that a self-pulsating laser diode can achieve periodic and chaotic behaviors if the modulation current and the modulation frequency are varied. The power spectra that indicate the interaction between self-pulsating frequency f_0 and external modulation frequency f can develop a broad continuous component that is above the regular noise level. In addition, plots of the bifurcation diagram versus the modulation frequency indicate that the chaotic state is suddenly destroyed if the frequency of the diode is near the harmonic frequencies of the self-pulsating frequency.

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