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A general piecewise necessity regression analysis based on linear programming

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Abstract

In possibilistic regression analysis proposed by Tanaka and Ishibuchi (1992), linear programming (LP) formulation of necessity analysis has no feasible solution under the enormous variation of the given data. This work proposes a general piecewise necessity regression analysis based on LP rather than a non-linear interval model that they recommended to obtain the necessity area of the given data. In addition to maintaining a linear property, the proposed method prevents the necessity analysis from having no feasible solution. The problematic univariate example and a multivariate example with respect to different number of change-points are demonstrated by the general piecewise necessity regression. The proposed method characteristic is that, according to data distribution, practitioners can specify the number and the positions of change-points. The proposed method maintains the linear interval model and the order of necessity regression function does not need to be determined. © 1999 Elsevier Science B.V. All rights reserved.

Keywords: Piecewise regression; Necessity area; Fuzzy linear regression

1. Introduction

The possibility theory on possibility distributions has been proposed by Zadeh [14] and advanced by Dubois and Prade [1]. In the early 1980s, Tanaka et al. [11] have introduced a linear programming (LP) based regression method using a linear fuzzy model with symmetrics triangular fuzzy parameters. Since Tanaka et al. introduced fuzzy linear regression, previous literature dealing with fuzzy linear regression has grown rapidly. Then possibility and necessity analyses have been clearly defined in Tanaka [8]. Recently, Sakawa and Yano [5, 6] have generalized the minimization, maximization and conjunction formulation developed by Tanaka et al. [11], Tanaka [8] and Tanaka et al. [9], respectively. Tanaka and Ishibuchi [10] proposed the possibility and necessity analyses on possibilistic regression analysis. However, a weakness of the fuzzy regression model has arised. In necessity analysis, the necessity area cannot be obtained owing to the large variation data [8, 10]. Tanaka [8] pointed out that if a

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polynomial is taken, then there is an optimal solution in the necessity problem. In addition, Tanaka and Ishibuchi [10] suggested using an adequate non-linear interval model to obtain the necessity area for this kind of data. However, two related issues arise. First, does the non-linear or polynomial interval model adhere to large fluctuations in the given data? The model must be analyzed with respect to data property rather than merely to add the terms of the polynomial. In addition, how many degrees are required in the function of non-linear interval model? The above issues show that it is difficult to manage the large variation data by applying a polynomial or non-linear form. That is the reason we suggest piecewise concept to manage the large variation data.

This work proposes another option of solving necessity problem, a general piecewise linear-interval regression model-based LP that can solve their problem easily. Change-points which are the joints of the pieces are quoted from conventional statistical piecewise regression [7]. This terminology is employed throughout the work. Initially, this method assumes every ordered datum except the final one or the suspicious datum is a change-point. Results obtained herein demonstrate how our method works by employing the problematic data [10] and multivariate data [8]. Principally, the less the number of change-points, the more parsimonious the model attained in the necessity problem.

The organization of the work is as follows. Section 2 reviews traditional necessity regression analysis with the interval model. Based on the presentation, Section 3 proposes necessity analysis with a piecewise linear interval model including univariate and multivariate analysis. Finally, univariate and multivariate examples illustrate the approach in necessity problems. These two examples indicate a rather useful approach in the treatment of suspicious outliers.

2. Necessity regression analysis with interval model

This section reviews fuzzy regression analysis via the interval model proposed by Tanaka and Ishibuchi [10]. Their efforts cause failure since the necessity analysis of LP problem has no feasible solution for the given data. This finding suggests that the linear-interval model of the necessity area cannot be obtained under some conditions. The reason why no feasible solution for necessity area exist is sometimes due to various fluctuations of data. Obviously, the solution of such kind of data is not merely a linear-interval model.

2.1. Interval arithmetic

A linear-interval model with q independent variables is represented using interval parameters A_i as

$$Y(x_j) = A_0 + A_1 x_{1j} + \cdots + A_q x_{qj}, \tag{1}$$

where $Y(x_j)$ is the predicted interval corresponding to the input vector x_j and j is the jth sample $(j = 1, 2, \ldots, n)$ and $x_j = (x_{1j}, x_{2j}, \ldots, x_{qj})$. Throughout this work, closed intervals are denoted by upper case letters A and B. An interval is defined by an ordered pair in brackets as

$$A = [a_L, a_R] = \{a: a_L \leqslant a \leqslant a_R\}, \tag{2}$$

where a_L is the left limit and a_R is the right limit of A. Interval A is also denoted by its center and radius as

$$A = (a_c, a_w) = \{a: a_c - a_w \leqslant a \leqslant a_c + a_w\}, \tag{3}$$

where a_c is the center and a_w is the radius, i.e., half of the width of A. From (2) and (3), the center and the radius of interval A can be calculated as

$$a_c = (a_R + a_L)/2, \tag{4}$$

$$a_w = (a_R - a_L)/2. \tag{5}$$

The following additions and multiplications are employed herein:

$$A + B = (a_c, a_w) + (b_c, b_w) = (a_c + b_c, a_w + b_w), \tag{6}$$

$$rA = r(a_c, a_w) = (ra_c, |r| a_w), \tag{7}$$

where r is the real number.

2.2. Linear-interval model

The following linear model (1) is reperesented in detail:

$$Y(x_j) = A_0 + A_1 x_{1j} + \cdots + A_q x_{qj}$$
$$= (a_{0c}, a_{0w}) + (a_{1c}, a_{1w})x_{1j} + \cdots + (a_{qc}, a_{qw})x_{qj}$$
$$= (Y_c(x_j),\ Y_w(x_j)), \tag{8}$$

and

$$Y_c(x_j) = a_{0c} + a_{1c}x_{1j} + \cdots + a_{qc}x_{qj}, \tag{9}$$

$$Y_w(x_j) = a_{0w} + a_{1w}|x_{1j}| + \cdots + a_{qw}|x_{qj}|, \tag{10}$$

where $Y_c(x_j)$ is the center and $Y_w(x_j)$ is the radius of the predicted interval $Y(x_j)$.

2.3. Necessity regression analysis

$$Y_*(x_j) = A_{0*} + A_{1*}x_{1j} + \cdots + A_{q*}x_{qj},$$
$$= (a_{0c*},\ a_{0w*}) + (a_{1c*},\ a_{1w*})\ x_{1j}$$
$$+ \cdots + (a_{qc*},\ a_{qw*})x_{qj},$$
$$= (Y_{c*}(x_j),\ Y_{w*}(x_j)), \tag{11}$$

which satisfies the following conditions:

$$Y_*(x_j) \subseteq Y_j, \quad j = 1, 2, \ldots, n, \tag{12}$$

where Y_j is the jth observation.

2.3.1. Maximization problem for interval-valued data

$$\text{Maximize } Y_{w*}(x_1) + Y_{w*}(x_2) + \cdots + Y_{w*}(x_n) \tag{13}$$

$$\text{Subject to } Y_*(x_j) \subseteq Y_j, \quad j = 1, 2, \ldots, n, \tag{14}$$

$$a_{iw*} \geqslant 0, \quad i = 0, 1, 2, \ldots, q. \tag{15}$$

This LP problem is written as follows:

$$\text{Maximize } z = \sum_{j=1}^{n} (a_{0w*} + a_{1w*}|x_{1j}| + \cdots + a_{qw*}|x_{qj}|) \tag{16}$$

$$\text{Subject to } a_{0c*} + \sum_{i=1}^{q} a_{ic*}x_{ij} + a_{0w*} - \sum_{i=1}^{q} a_{iw*}|x_{ij}| \geqslant y_{jL}, \tag{17}$$

$$a_{0c*} + \sum_{i=1}^{q} a_{ic*}x_{ij} + a_{0w*} + \sum_{i=1}^{q} a_{iw*}|x_{ij}| \leqslant y_{jR}, \tag{18}$$

$$a_{iw*} \geqslant 0, \quad j = 1, 2, \ldots, n, \tag{19}$$

where z is the total vagueness of $Y_*(x_j)$.

Owing to the various fluctuations of the given data, the LP problem (16)–(19) does not always have feasible solutions. There are cases where the linear-interval model $Y_*(x_j)$ cannot be obtained from the given data. However, a general piecewise linear interval model can treat these cases easily. This problem is resolved herein by employing the proposed method in Section 3.

Furthermore, Sakawa and Yano [5] proposed using regression analysis for fuzzy input-output data. Their approach encounters the same problem with necessity analysis. Improving the same problem by employing our method is preferable.

3. Necessity analysis with a general piecewise linear-interval model

An LP formulation is presented to determine the necessity area by the piecewise linear-interval model. For simplicity, univariate piecewise linear regression in the necessity problems is performed first and then multivariate piecewise linear regression in the necessity problems is described.

3.1. Univariate piecewise linear regression in the necessity problems

$$Y_*(x_j) = h(x_j) + \sum_{t=1}^{k-1} B_{t*}(|x_j - P_t| + x_j - P_t)/2, \tag{20}$$
$$h(x_j) = A_{0*} + A_{1*}x_j.$$

If P is a change-point, then

$$(|x_j - P| + x_j - P)/2 = \begin{cases} x_j - P & \text{if } x_j > P, \\ 0 & \text{if } x_j \leqslant P. \end{cases}$$

$P = \{P_1, P_2, \ldots, P_k\}$ are the values of variable x and are subject to an ordering constraint $P_1 < P_2 < \cdots < P_k$, $k \leqslant n$. For easy illustration, the following formulation assumes every datum is a change-point except P_k. Therefore, $k - 1$ change-points in the initial necessity regression model are available.

The difference between (20) and (11) is (21), i.e., the piecewise expression for the given data.

$$\begin{aligned} &\sum_{t=1}^{k-1} B_{t*}(|x_j - P_t| + x_j - P_t)/2 \\ &\quad = \sum_{t=1}^{k-1} b_{tc*}(|x_j - P_t + x_j - P_t)/2 \\ &\qquad + \sum_{t=1}^{k-1} b_{tw*}(|x_j - P_t| + x_j - P_t)/2, \\ &\qquad j = 1, 2, \ldots, n. \end{aligned} \tag{21}$$

The piecewise LP formulation for the necessity analysis is as follows:

Maximize

$$z = \sum_{j=1}^{n} \left[a_{0w*} + a_{1w*}|x_j| + \sum_{t=1}^{k-1} b_{tw*}(|x_j - P_t| + x_j - P_t)/2 \right]$$

Subject to

$$\begin{aligned} &a_{0c*} + a_{1c*}x_j + \sum_{t=1}^{k-1} b_{tc*}(|x_j - P_t| + x_j - P_t)/2 \\ &- [a_{0w*} + a_{1w*}x_j + \sum_{t=1}^{k-1} b_{tw*}(|x_j - P_t| + x_j - P_t)/2] \\ &\geqslant Y_{jL}, \\ &a_{0c*} + a_{1c*}x_j + \sum_{t=1}^{k-1} b_{tc*}(|x_j - P_t| + x_j - P_t)/2 \\ &+ [a_{0w*} + a_{1w*}x_j + \sum_{t=1}^{k-1} b_{tw*}(|x_j - P_t| \\ &+ x_j - P_t)/2] \leqslant Y_{jR}, \quad j = 1, 2, \ldots, n. \end{aligned}$$

The above assumption is not unique. Accommodating the interval-valued data, practitioners can merely specify the suspicious outliers as change-points. The formulation can be extended as multivariate piecewise linear regression model as long as the number of independent variables is increased.

3.2. Multivariate piecewise linear regression in the necessity problems

The output Y may be generally related to q input variables. According to the raw data, the ith input variable has k_i different values. Therefore, in the beginning, $x_1, x_2, \ldots, x_q$ have $k_1 - 1, k_2 - 1, \ldots, k_q - 1$ change intervals, respectively. $Y_*(x_j)$ is the necessity model

$$\begin{aligned} Y_*(x_j) = A_{0*} &+ \sum_{i=1}^{q} A_{i*}x_{ij} \\ &+ \sum_{i=1}^{q} \sum_{t=1}^{k_i-1} B_{it*}(|x_{ij} - P_t| + x_{ij} - P_t)/2. \end{aligned}$$

Maximize

$$\begin{aligned} z = \sum_{j=1}^{n} \Bigg[& a_{0w*} + \sum_{i=1}^{q} a_{iw*}|x_{ij}| \\ & + \sum_{i=1}^{q} \sum_{t=1}^{k_i-1} b_{itw*}(|x_{ij} - P_t| + x_{ij} - P_t)/2 \Bigg] \end{aligned}$$

Subject to

$$\begin{aligned} &a_{0c*} + \sum_{i=1}^{q} a_{ic*}x_{ij} + \sum_{i=1}^{q} \sum_{t=1}^{k_i-1} b_{itc*}(|x_{ij} - P_t| \\ &+ x_{ij} - P_t)/2 - \Bigg[a_{0w*} + \sum_{i=1}^{q} a_{iw*}|x_{ij}| \\ &+ \sum_{i=1}^{q} \sum_{t=1}^{k_i-1} b_{itw*}(|x_{ij} - P_t| + x_{ij} - P_t)/2 \Bigg] \geqslant Y_{jL}, \\ &a_{0c*} + \sum_{i=1}^{q} a_{ic*}x_{ij} + \sum_{i=1}^{q} \sum_{t=1}^{k_i-1} b_{itc*}(|x_{ij} - P_t| \\ &+ x_{ij} - P_t)/2 + \Bigg[a_{0w*} + \sum_{i=1}^{q} a_{iw*}|x_{ij}| \\ &+ \sum_{i=1}^{q} \sum_{t=1}^{k_i-1} b_{itw*}(|x_{ij} - P_t| + x_{ij} - P_t)/2 \Bigg] \leqslant Y_{jR}, \\ &j = 1, 2, \ldots, n. \end{aligned}$$

4. Numerical example

Two examples are shown in this section. From the following examples, the problem of enormous variation data and multivariate piecewise linear regression are done by the proposed method. In these two examples, all ordering data except the last one are assumed as change-points. As mentioned above, practitioners can select some suspicious outliers as change-points rather than all ordering data except the last one.

Example 1. The data used herein provide an example which shows that a necessity analysis of Tanaka–Ishibuchi's method [10] cannot obtain the necessity area. By applying the piecewise concept, the necessity area represented by a piecewise linear-interval model can be obtained rather than a nonlinear-interval regression model. The significant advantage of our model is that practitioners do not need to determine the order term of $Y_*(x)$.

Let us consider the following interval-valued data:

$$\{(x_j;\ y_j)\} = \{(3;\ [12, 17]), (6; [10,\ 13]), (9;\ [13, 18]),$$
$$(12, [14, 18]), (15; [19, 24]), (18; [16, 19])\}.$$

Sample size is 6. The distribution of x ranges from 3 to 18. We use $P_1 = 3$, $P_2 = 6$, $P_3 = 9$, $P_4 = 12$, $P_5 = 15$ as change-points. The model is as follows:

$$\begin{aligned} Y_*(x_j) = {} & (a_{0c^*}, a_{0w^*}) + (a_{1c^*}, a_{1w^*})x \\ & + (b_{1c^*}, \mathrm{b}_{1w^*})(|x_j - 3| + x_j - 3)/2 \\ & + (b_{2c^*}, b_{2w^*})(|x_j - 6| + x_j - 6)/2 \\ & + (b_{3c^*}, b_{3w^*})(|x_j - 9| + x_j - 9)/2 \\ & + (b_{4c^*}, b_{4w^*})(|x_j - 12| + x_j - 12)/2 \\ & + (b_{5c^*}, b_{5w^*})(|x_j - 15| + x_j - 15)/2. \end{aligned}$$

The following piecewise linear model is subsequently obtained:

$$\begin{aligned} Y_*(x_j) = {} & (0, 1.5) + (4.5, 0)x_j \\ & + (-\ 5.167, 0)(|x_j - 3| + x_j - 3)/2 \\ & + (1.667, 0)(|x_j - 6| + x_j - 6)/2 \\ & + (-\ 0.333, 0)(|x_j - 9| + x_j - 9)/2 \\ & + (0.667, 0)(|x_j - 12| + x_j - 12)/2 \\ & + (-\ 2.333, 0)(|x_j - 15| + x_j - 15)/2, \end{aligned}$$

which satisfies the conditions

$$Y_*(x_j) \subseteq Y_j, \quad j = 1, 2, \ldots, 6$$

and depicts the analysis results of the necessity analysis. From Fig. 1, the area which the two solid lines contain is the necessity area. The area that the two dashed lines contain is the observation area.

On the other hand, if just the influential data $x = 6$, 15 are chosen as change-points, then

$$\begin{aligned} Y_*(x_j) = {} & (15, 1) + (-\ 0.67, 0)x_j \\ & + (1.667, 0)(|x_j - 6| + x_j - 6)/2 \\ & + (-1.83, 0.17)(|x_j - 15| + x_j - 15)/2. \end{aligned}$$

The comparison with the raw data is depicted in Fig. 2. It is flexible to set the number and the positions of change-points. Basically, $k - 1$ is the maximum. This method resolves the problematic example that necessity area cannot be obtained by the Tanaka–Ishibuchi method. The reason for such an occurrence is the large variation of the given data.

Example 2. Multivariate fuzzy data are from Tanaka [8].

$$\begin{aligned} & \{(x_{1j};\ x_{2j};\ x_{3j};\ y_j)\} \\ & = \{(3;\ 5;\ 9;\ [96, 42]),\ (14;\ 8;\ 3;\ [120, 47])\ (7;\ 1;\ 4; \\ & [52,\ 33])\ (11; 7; 3; [106, 45], (7; 12; 15; [189, 79]), \\ & (8; 15; 10; [194, 65]), (3; 9; 6; [107, 42]), (12; 15; 11; \\ & [216, 78]), (10; 5; 8; [108, 52]), (9; 7; 4; [103, 44])\}, \\ & n = 10. \end{aligned}$$

The distributions of variable x_{1j}, x_{2j} and x_{3j} have 8 different crisp values $\{3, 7, 8, 9, 10, 11, 12, 14\}$, 7 crisp values $\{1, 5, 7, 8, 9, 12, 15\}$ and 8 crisp values

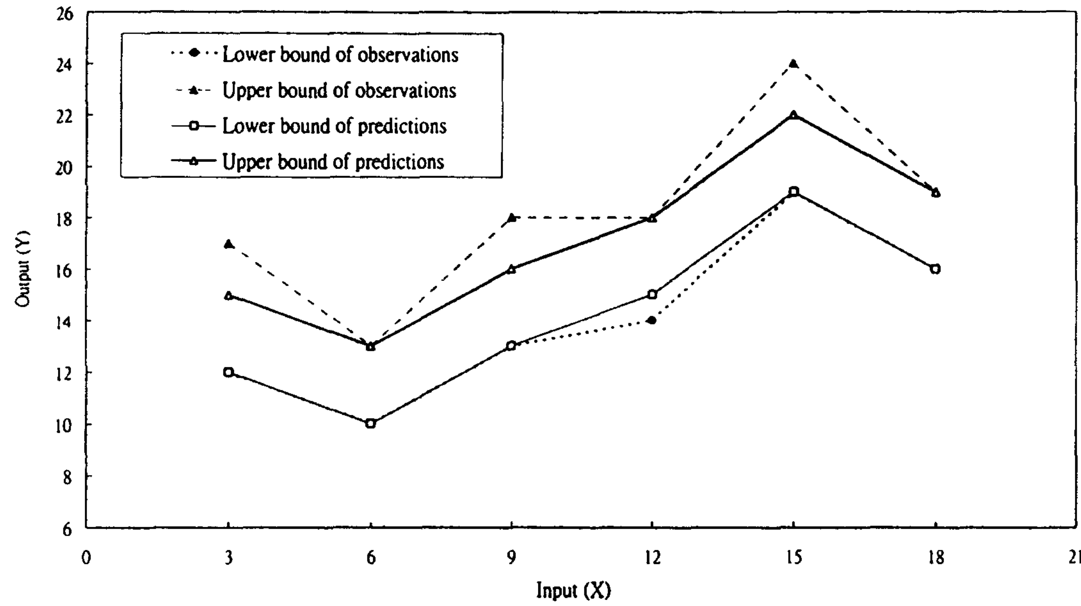

Fig. 1. Piecewise linear-interval model $y(x)$.

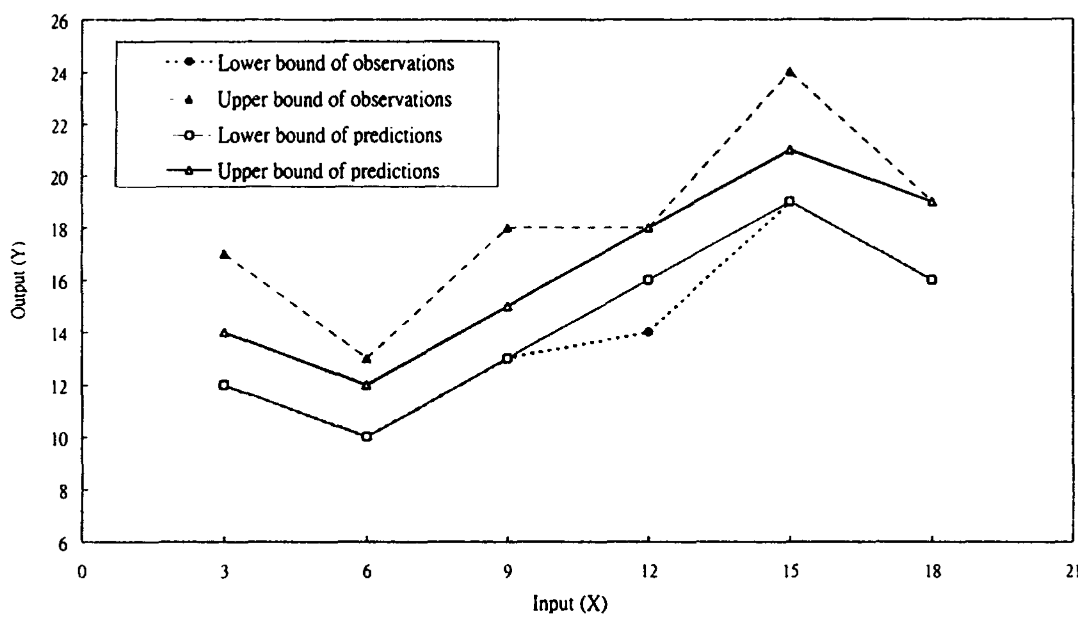

Fig. 2. Piecewise model with two change-point.

$\{3,4,6,8,9,10,11,15\}$, respectively. In the extreme case, variable x_{1j}, x_{2j} and x_{3j} have 7 change-points $\{3,7,8,9,10,11,12\}$, 6 change-points $\{1,5,7,8,9,12\}$ and 7 change-points $\{3,4,6,8,9,10,11\}$, respectively. There are 20 change-points in the beginning. According to the method in Section 3, the multivariate necessity model with 10 change-points is the following:

$$
\begin{aligned}
&Y_*(x_{1j}, x_{2j}, x_{3j}) \\
&= (0, 2.35) + (1.87, 0.6)x_{1j} + (4.08, 0)x_{2j} \\
&\quad + (6.9, 0.7)x_{3j} + (0, 0.21)(|x_{1j} - 7| + x_{1j} - 7|) \\
&\quad + (1.75, 0)(|x_{1j} - 8| + x_{1j} - 8|) \\
&\quad + (-1.36, 0)(|x_{1j} - 9| + x_{1j} - 9|) \\
&\quad + (-3.57, 0.3)(|x_{1j} - 12| + x_{1j} - 12|) \\
&\quad + (0.21, 0)(|x_{2j} - 5| + x_{2j} - 5|) \\
&\quad + (2.61, 2.02)(|x_{2j} - 12| + x_{2j} - 12|) \\
&\quad + (-2.28, 0)(|x_{3j} - 3| + x_{3j} - 3|) \\
&\quad + (-1.59, 0.87)(|x_{3j} - 4| + x_{3j} - 4|) \\
&\quad + (0, 0.38)(|x_{3j} - 6| + x_{3j} - 6|) \\
&\quad + (2.99, 0)(|x_{3j} - 11| + x_{3j} - 11|). \qquad (22)
\end{aligned}
$$

The total vagueness of Eq. (22) $z_1 = 378.7$.

For a parsimonious form, only suspicious outlier can be set as change-points. For instance, from the raw-data distribution and Eq. (22), (7; 1; 4; [52, 33]) is a relative influential interval. If $x_1 = 7$, $x_2 = 1$, $x_3 = 4$ are chosen as change-points, then

$$
\begin{aligned}
&Y_*(x_{1j}, x_{2j}, x_{3j}) \\
&= (5.6, 0) + (3.28, 0.83)x_{1j} + (4.84, 0)x_{2j} \\
&\quad + (2.11, 0.72)x_{3j} + (-1.73, 0)(|x_{1j} - 7| \\
&\quad + x_{1j} - 7|) + (0, 3.11)(|x_{2j} - 1| + x_{2j} - 1|) \\
&\quad + (1.43, 0.37)(|x_{3j} - 4| + x_{3j} - 4|). \qquad (23)
\end{aligned}
$$

The total vagueness of Eq. (23) $z_2 = 365.8$. The concept of assuming every change-point is a change-point except the final one is like interpolation but not really. In fact, the case of every change-point is a change-point except the final one is an extreme assumption. From Table 1, Eq. (23) with fewer change-points than Eq. (22) achieves a promising result which approximates the result of Eq. (23) and the total vagueness z_2 with 3 change-points is slightly smaller than z_1 with 10 change-points. Hence, for an effective and parsimonious form, considering the number of change-points must depend on the raw-data distribution and lower the number of change-point the better.

Table 1

Observed output		Prediction of Eq. (22) $z_1 = 378.7$	Prediction of Eq. (23) $z_2 = 365.8$
1	[96, 42]	[90.2, 48]	[89.1, 42.5]
2	[120, 47]	[120, 47]	[120, 47]
3	[52, 33]	[52, 33]	[50.5, 33.1]
4	[106, 45]	[106, 45]	[105, 45]
5	[189, 79]	[189, 79]	[189, 79]
6	[194, 64]	[194, 64]	[192.4, 72.4]
7	[107, 42]	[107, 42]	[107, 42]
8	[216, 78]	[216, 78]	[206.2, 78]
9	[108, 52]	[108, 52]	[108, 52]
10	[103, 44]	[101.8, 44]	[103, 44]

From these two examples, we have seen in detail how the general piecewise necessity regression analysis work. In this section, $h(x)$ denotes a linear interval function. By adjusting the order terms of $h(x)$, the proposed method can also be presented as the non-linear-interval model. If the property of data is non-linear, we can adjust the order term of $h(x)$ and then $Y_*(x)$ turns into a non-linear piecewise interval regression model.

5. Concluding remarks

Under the circumstance of a large variation of data, a general piecewise linear interval model can obtain the necessity area that Tanaka–Ishibuchi's method cannot. The advantage reserves the linear-interval characteristic. By using a non-linear interval model, the practitioner must address the problem of deciding the degrees of the function. Assuming that every datum except the final one or the suspicious outlier is a change-point, the proposed method resolves Tanaka and Ishibuchi's problem easily. From Example 2, obviously, Eq. (23) with 3 change-points is more parsimonious and effective than Eq. (22) with 10 change-points. Therefore, considering the number and the change-points positions must depend on raw data distribution. Generally, a less number of change-points would achieve a better performance. In the above section, $h(x)$ denotes a linear interval function. If the property of data is non-linear, practitioners can adjust the order term of $h(x)$ and then $Y_*(x)$ turns into a non-linear piecewise interval regression model.

As the sample size increases, the number of change-points increases and the piecewise linear-interval model also become complex, thus, preventing from considering it as interpolation. Therefore, to effectively control the change-point number is a serious problem. For this purpose, you may refer the concept of Li and Yu [2] who proposed a method to adequately control the number of change-points by using a modified goal programming. Change-points detecting and controlling is a topic for our future study. Furthermore, if the interval-valued data distribution shows structure change, then fuzzy piecewise regression is more appropriate than conventional fuzzy regression.

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