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Errata to "The Shift-Inverted J-Lanczos Algorithm for the Numerical Solutions of Large Sparse Algebraic Riccati Equations"

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An error occurred in the above-mentioned paper. The error occurs on page 32, after Lemma 4.1. The paragraph reads "For simplicity, we assume that both H_n and H_m here are J-diagonalizable, that is, $H_n = X^H \Lambda Y$ and $H_m = P^H \Theta Q$, where

$$\Lambda = \operatorname{diag}(\lambda_1, \dots, \lambda_n \mid -\lambda_1, \dots, -\lambda_n) \equiv \begin{bmatrix} \Lambda_1 & 0 \\ 0 & -\Lambda_1 \end{bmatrix} \tag{4.1}$$

and

$$\Theta = \operatorname{diag}(\theta_1, \dots, \theta_m \mid -\theta_1, \dots, -\theta_m) \equiv \begin{bmatrix} \Theta_1 & 0 \\ 0 & -\Theta_1 \end{bmatrix}." \tag{4.2}$$

This assumption should be modified. Notations and results on pages 32 and 33 will be affected and need some minor modifications. Therefore, the paragraphs on pages 32 and 33 should be replaced by the following paragraphs.

For simplicity, we assume that both H_n and H_m here are J-block diagonalizable, that is, $H_n = X^T \Lambda Y$ and $H_m = P^T \Theta Q$, where $X, Y \in \mathbb{R}^{2n \times 2n}$, $P, Q \in \mathbb{R}^{2m \times 2m}$ are real symplectic with $X^T Y = I_{2n}$, $P^T Q = I_{2m}$, and

$$\Lambda \equiv \begin{bmatrix} \Lambda_1 & 0 \\ 0 & -\Lambda_1^T \end{bmatrix} \tag{4.1}$$

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and

$$\Theta \equiv \begin{bmatrix} \Theta_1 & 0 \\ 0 & -\Theta_1^T \end{bmatrix}. \tag{4.2}$$

Λ_1 and Θ_1 are block diagonal real matrices with either 1-by-1 or 2-by-2 diagonal blocks. And there are block diagonal nonsingular matrices U, $V \equiv U^{-1} \in \mathbb{C}^{n\times n}$, Φ, $\Psi \equiv \Phi^{-1} \in \mathbb{C}^{m\times m}$ such that

$$\Lambda_1 = U \cdot \operatorname{diag}(\lambda_1, \dots, \lambda_n) \cdot V,$$
$$\Theta_1 = \Phi \cdot \operatorname{diag}(\theta_1, \dots, \theta_m) \cdot \Psi.$$

Let $E_n = [e_1, e_{n+1}] \in \mathbb{R}^{2n\times 2}$ and $E_m = [e_1, e_{m+1}] \in \mathbb{R}^{2m\times 2}$. With the decompositions of H_n and H_m above and applying Lemma 4.1, one can verify that

$$E_n^T f(H_n) E_n = E_m^T f(H_m) E_m,$$

or, equivalently,

$$E_n^T X^T f(\Lambda) Y E_n = E_m^T P^T f(\Theta) Q E_m,$$

for all $f \in \wp^{4m-1}$. Since X and P are real symplectic, hence $X^T = J_n Y^T J_n^T$, $P^T = J_m Q^T J_m^T$. Substituting this property into the equality above, one obtains

$$\begin{aligned} &\begin{bmatrix} y_{n+1}^T \\ -y_1^T \end{bmatrix} \begin{bmatrix} 0 & -I_n \\ I_n & 0 \end{bmatrix} \begin{bmatrix} f(\Lambda_1) & 0 \\ 0 & f(-\Lambda_1^T) \end{bmatrix} [y_1 y_{n+1}] \\ &= \begin{bmatrix} q_{m+1}^T \\ -q_1^T \end{bmatrix} \begin{bmatrix} 0 & -I_m \\ I_m & 0 \end{bmatrix} \begin{bmatrix} f(\Theta_1) & 0 \\ 0 & f(-\Theta_1^T) \end{bmatrix} [q_1 q_{m+1}]. \end{aligned} \tag{4.3}$$

Here y_i and q_i are the i^{th} column of Y and Q, respectively. Denote $y_1 \equiv \left[\begin{smallmatrix} y_1^{(1)} \\ y_1^{(2)} \end{smallmatrix}\right]$, where $y_1^{(1)}, y_1^{(2)} \in \mathbb{R}^n$. Using the similar notations for y_{n+1}, q_1, and q_{n+1}, equation (4.3) becomes

$$\begin{aligned} &\left(y_1^{(2)} + y_{n+1}^{(2)}\right)^T (f(\Lambda_1) - f(-\Lambda_1)) \left(y_1^{(1)} + y_{n+1}^{(1)}\right) \\ &= \left(q_1^{(2)} + q_{m+1}^{(2)}\right)^T (f(\Theta_1) - f(-\Theta_1)) \left(q_1^{(1)} + q_{m+1}^{(1)}\right), \end{aligned} \tag{4.4}$$

for all $f \in \wp^{4m-1}$. By the property of $f(\Lambda_1) - f(-\Lambda_1)$, there is an odd polynomial g with degree $\le 4m-1$, such that $g(\Lambda_1) = f(\Lambda_1) - f(-\Lambda_1)$. Let

$$\hat{x} = U^H \left(y_1^{(2)} + y_{n+1}^{(2)}\right), \qquad \hat{y} = V\left(y_1^{(1)} + y_{n+1}^{(1)}\right),$$
$$\hat{p} = \Phi^H \left(q_1^{(2)} + q_{m+1}^{(2)}\right), \qquad \hat{q} = \Psi\left(q_1^{(1)} + q_{m+1}^{(1)}\right).$$

Then (4.4) can be rewritten as

$$\sum_{i=1}^{n} g(\lambda_i) \left(\bar{\hat{x}}_i + \hat{y}_i\right) = \sum_{i=1}^{m} g(\theta_i) \left(\bar{\hat{p}}_i + \hat{q}_i\right), \tag{4.5}$$

where $\hat{x}_i$ denotes the i^{th} component of $\hat{x}$, and $\bar{\hat{x}}_i$ is the complex conjugate. Now, let $\sigma_1 = \{\lambda_2, \dots, \lambda_n\}$ and $\hat{\sigma}_1 = \{\theta_2, \dots, \theta_m\}$. Suppose $\sigma_1 \cup \hat{\sigma}_1 = S_1 \cup S_2$ with $S_1 \cap S_2 = \phi$. Define

$$\delta_1(S_2) = \max\left\{ \left|\zeta^2 - \theta_1^2\right| \prod_{\mu\in S_2} \frac{\left|\zeta^2 - \mu^2\right|}{\left|\lambda_1^2 - \mu^2\right|} : \zeta \in \sigma_1 \cup \hat{\sigma}_1 \right\}, \tag{4.6}$$

and

$$\varepsilon_1^{(k)}(S_1) = \inf_{p\in\wp^k, p(\lambda_1^2)=1} \max_{\zeta\in S_1} \left|p\left(\zeta^2\right)\right|. \tag{4.7}$$

With above definitions and notations, we establish an error bound for the J-Ritz values.

THEOREM 4.1. *Assume that* $|\lambda_1| > |\lambda_i|$, $i = 2,\dots,n$, $|\lambda_1| > |\theta_j|$, $j = 2,\dots,m$, *and* $|\lambda_1 - \theta_1| = \min_{1\le j\le m} |\lambda_1 - \theta_j|$. *If* $s = |S_2| \le m-2$ *holds, then*

$$|\lambda_1 - \theta_1| \le \frac{\varepsilon_1^{(2m-s-2)}(S_1)\delta_1(S_2)}{|\lambda_1+\theta_1|\,|\hat{x}_1\hat{y}_1|} \times \left(\sum_{i=2}^{n} |\bar{\hat{x}}_i\hat{y}_i| + \sum_{i=2}^{m} |\bar{\hat{p}}_i\hat{q}_i|\right). \tag{4.8}$$

PROOF. Let

$$g(\zeta) = \zeta\left(\zeta^2-\theta_1^2\right)p\left(\zeta^2\right)\prod_{\mu\in S_2}\left(\zeta^2-\mu^2\right),$$

where $p \in \wp^{2m-s-2}$ with $p(\lambda_1^2) = 1$. Substituting $g(\zeta)$ into (4.5) we obtain

$$\begin{aligned}
&\lambda_1\left(\lambda_1^2-\theta_1^2\right)p\left(\lambda_1^2\right)\prod_{\mu\in S_2}\left(\lambda_1^2-\mu^2\right)\left(\bar{\hat{x}}_1\hat{y}_1\right)\\
&= -\sum_{\lambda_i\in S_1}\lambda_i\left(\lambda_i^2-\theta_1^2\right)p\left(\lambda_i^2\right)\prod_{\mu\in S_2}\left(\lambda_i^2-\mu^2\right)\left(\bar{\hat{x}}_i\hat{y}_i\right)+\sum_{\theta_i\in S_1}\theta_i\left(\theta_i^2-\theta_1^2\right)p\left(\theta_i^2\right)\prod_{\mu\in S_2}\left(\theta_i^2-\mu^2\right)\left(\bar{\hat{p}}_i\hat{q}_i\right).
\end{aligned}$$

From (4.6), we have

$$\begin{aligned}
|\lambda_1-\theta_1| &\le \frac{1}{|\lambda_1|\,|\lambda_1+\theta_1|\,|\bar{\hat{x}}_1\hat{y}_1|}\left(\sum_{\lambda_i\in S_1}|\lambda_i|\,|\lambda_i^2-\theta_1^2|\prod_{\mu\in S_2}\frac{|\lambda_i^2-\mu^2|}{|\lambda_1^2-\mu^2|}p\left(\lambda_i^2\right)|\bar{\hat{x}}_i\hat{y}_i|\right.\\
&\quad\left.+\sum_{\theta_i\in S_1}|\theta_i|\,|\theta_i^2-\theta_1^2|\prod_{\mu\in S_2}\frac{|\theta_i^2-\mu^2|}{|\lambda_1^2-\mu^2|}p\left(\theta_i^2\right)|\bar{\hat{p}}_i\hat{q}_i|\right)\\
&\le \frac{1}{|\lambda_1|\,|\lambda_1+\theta_1|\,|\bar{\hat{x}}_1\hat{y}_1|}\left(\sum_{\lambda_i\in S_1}|\lambda_i|\,|\bar{\hat{x}}_1\hat{y}_1| + \sum_{\theta_i\in S_1}|\theta_i|\,|\bar{\hat{p}}_1\hat{q}_1|\right)\max_{\zeta\in S_1}p\left(\zeta^2\right)\delta_1(S_2).
\end{aligned}$$

Since $|\lambda_1| > |\lambda_i|$, $|\lambda_1| > |\theta_j|$, for $i = 2,\dots,n$, $j = 2,\dots,m$, and $p \in \wp^{2m-s-2}$ with $p(\lambda_1^2) = 1$, from definition (4.7) we obtain the error bound (4.8). ∎