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# MAKING DECISIONS IN ASSESSING PROCESS CAPABILITY INDEX $C_{pk}$

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## SUMMARY

Process capability indices $C_p$, $C_{pk}$ and $C_{pm}$ have been used in manufacturing industries to provide a quantitative measure of process potential and performance. The formulae for these indices are easy to understand and straightforward to apply. However, since sample data must be collected in order to calculate these indices, a great degree of uncertainty may be introduced into capability assessments owing to sampling errors. Currently, most practitioners simply look at the value of the index calculated from the sample data and then make a conclusion on whether the given process meets the capability (quality) requirement. This approach is not reliable, since sampling errors are ignored. Cheng (*Qual. Engng.*, **7,** 239–259 (1994)) has developed a procedure involving estimators of $C_p$ and $C_{pm}$ for practitioners to use to determine whether a process meets the capability requirement or not. However, no procedure for $C_{pk}$ was given, because difficulties were encountered in calculating the sampling distribution of the estimator of $C_{pk}$. In this paper we use a newly proposed estimator of $C_{pk}$ to develop a procedure for practitioners to use so that decisions made in assessing process capability are more reliable. 

KEY WORDS: process capability indices; non-central $t$ distribution; critical values; power of the test; $\alpha$ risk; capability requirement

## INTRODUCTION

Understanding processes and quantifying process performance are essential for any successful quality improvement initiative. The relationship between the actual process performance and the specification limits or tolerance may be quantified using appropriate process capability indices. Three capability indices commonly used in manufacturing industries are $C_p$, $C_{pk}$ and $C_{pm}$. These indices, providing numerical measures of whether a production process meets predetermined specification limits, have been defined as

$$C_p = \frac{\text{USL} - \text{LSL}}{3\sigma}$$

$$C_{pk} = \min\left\{\frac{\text{USL} - \mu}{3\sigma}, \frac{\mu - \text{LSL}}{3\sigma}\right\}$$

$$C_{pm} = \frac{\text{USL} - \text{LSL}}{3\sqrt{\sigma^2 + (\mu - T)^2}}$$

where USL is the upper specification limit, LSL is the lower specification limit, $\mu$ is the process mean, $\sigma$ is the process standard deviation (overall process variability) and $T$ is the target value. The formulae for these indices are easy to understand and straightforward to apply. However, in order to calculate these indices, sample data must be collected. Therefore a great degree of uncertainty may be introduced into capability assessments owing to sampling errors. Currently, most practitioners simply look at the value of the estimators calculated from the sample data and then make a conclusion on whether the given process meets the capability (quality) requirement or not. This approach is highly unreliable, since sampling errors have been ignored. Chen [1] has developed a procedure (using estimators of $C_p$ and $C_{pm}$) for practitioners to use to determine if a process satisfies the targeted quality condition. However, no procedure for $C_{pk}$ was given, because difficulties were encountered in calculating the sampling distribution of the estimator of $C_{pk}$. In this paper we use an estimator of $C_{pk}$ proposed by Pearn and Chen [2] to develop a simple procedure for practitioners to use so that

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decisions made in assessing process capability are more reliable.

## ESTIMATION OF $C_{pk}$

Three estimators have been proposed to estimate the $C_{pk}$ value, namely (a) Bissell's estimator $\hat{C}'_{pk}$ [3], (b) the natural estimator $\hat{C}_{pk}$ [4] and (c) the Bayesian-like estimator $\hat{C}''_{pk}$ [2]. Bissell's estimator assumes the knowledge of $P(\mu \geq m) = 0$ or 1, where $m = (\text{USL} + \text{LSL})/2$. If $\mu \geq m$, then $\hat{C}'_{pk} = (\text{USL} - \bar{X})/3S$; otherwise, $\hat{C}'_{pk} = (\bar{X} - \text{LSL})/3S$. Kotz *et al.* [4] investigated a different estimator of $C_{pk}$ which is defined as $\hat{C}_{pk} = \min\{(\text{USL} - \bar{X})/3S, (\bar{X} - \text{LSL})/3S\}$, where $\bar{X} = (\sum_{i=1}^{n} X_i)/n$ and $S = \{(n-1)^{-1} \sum_{i=1}^{n}(X_i - \bar{X})^2\}^{1/2}$ are conventional estimators of $\mu$ and $\sigma$ which may be obtained from a stable process. Both estimators $\hat{C}'_{pk}$ and $\hat{C}_{pk}$ are biased, but Kotz *et al.* [4] showed that the variance of $\hat{C}_{pk}$ is smaller than that of Bissell's estimator.

Pearn and Chen [2] considered a Bayesian-like estimator $\hat{C}''_{pk}$ to relax Bissell's assumption on the process mean. The evaluation of the estimator $\hat{C}''_{pk}$ only requires the knowledge of $P(\mu \geq m) = p$ or $P(\mu < m) = 1 - p$, where $0 \leq p \leq 1$, which may be obtained from historical information on a stable process. Clearly, if $P(\mu \geq m) = 0$ or 1, then the estimator $\hat{C}''_{pk}$ reduces to Bissell's estimator. The estimator is defined as $\hat{C}''_{pk} = \{d - (\bar{X} - m) I_A(\mu)\}/3S$, where $I_A(\mu) = 1$ if $\mu \in A$, $I_A(\mu) = -1$ if $\mu \notin A$, and $A = \{\mu | \mu \geq m\}$.

Pearn and Chen [2] showed that under the assumption of normality the distribution of the estimator $3n^{1/2}\hat{C}''_{pk}$ is $t_{n-1}(\delta)$, a non-central $t$ with $n-1$ degrees of freedom and non-centrality parameter $\delta = 3n^{1/2} C_{pk}$. The probability density function can be expressed as

$$f(x) = \frac{3n^{1/2}}{2^{n/2}\Gamma\left(\frac{n-1}{2}\right)(\pi(n-1))^{1/2}} \int_0^\infty y^{(n-2)/2} \times \exp\left(-\frac{y + 9n[xy^{1/2}(n-1)^{-1/2} - C_{pk}]^2}{2}\right) \mathrm{d}y$$

Pearn and Chen [2] also showed that by adding the well-known correction factor $b_f$ to the estimator $\hat{C}''_{pk}$, where $b_f = [2/(n-1)]^{1/2}\Gamma[(n-1)/2]\{\Gamma[(n-2)/2]\}^{-1}$, an unbiased estimator $\tilde{C}_{pk} = b_f \hat{C}''_{pk}$ can be obtained. They also showed that the variance of $\tilde{C}_{pk}$ is smaller than those of $\hat{C}'_{pk}$ and $\hat{C}_{pk}$. Therefore in this paper we will use the unbiased estimator $\tilde{C}_{pk}$ to develop a simple procedure, similar to those described in References [1] and [5], for the index $C_{pk}$.

Table 1. Quality conditions and $C_{pk}$ values

| Quality condition | $C_{pk}$ value |
|---|---|
| Inadequate | $C_{pk} < 1.00$ |
| Capable | $1.00 \leq C_{pk} < 1.33$ |
| Satisfactory | $1.33 \leq C_{pk} < 1.50$ |
| Excellent | $1.50 \leq C_{pk} < 2.00$ |
| Super | $2.00 \leq C_{pk}$ |

## TEST HYPOTHESIS

A process is called 'inadequate' if $C_{pk} < 1.00$: this indicates that the process is not adequate with respect to the production tolerances; either the process variation $(\sigma^2)$ needs to be reduced or the process mean $(\mu)$ needs to be shifted closer to the target value. A process is called 'capable' if $1.00 \leq C_{pk} < 1.33$: this indicates that caution needs to be taken regarding the process distribution; some process control is required. A process is called 'satisfactory' if $1.33 \leq C_{pk} < 1.50$: this indicates that the process quality is satisfactory; material substitution may be allowed and no stringent quality control is required. A process is called 'excellent' if $1.50 \leq C_{pk} < 2.00$. Finally, a process is called 'super' if $C_{pk} \geq 2.00$. Table 1 summarizes the five quality conditions and the corresponding $C_{pk}$ values.

To determine whether a given process meets the capability requirement and runs under the desired quality condition, we can consider the following statistical test hypothesis. The process meets the capability (quality) requirement if $C_{pk} > C$, and fails to meet the capability requirement if $C_{pk} \leq C$:

$$H_0: \; C_{pk} \leq C$$
$$H_1: \; C_{pk} > C$$

The critical value $C_0$ is determined by

$$p\{\tilde{C}_{pk} > C_0 | C_{pk} = C\} = \alpha$$
$$p\{b_f \hat{C}''_{pk} > C_0 | C_{pk} = C\} = \alpha$$
$$p\left\{\hat{C}''_{pk} > \frac{C_0}{b_f} \,\middle|\, C_{pk} = C\right\} = \alpha$$
$$p\left\{3n^{1/2}\hat{C}''_{pk} > \frac{3n^{1/2}C_0}{b_f} \,\middle|\, C_{pk} = C\right\} = \alpha$$

$$p\left\{t_{n-1}(\delta_c) > \frac{3n^{1/2}C_0}{b_f}\right\} = \alpha$$

where $\delta_c = 3n^{1/2}C$. Hence we have

$$\frac{3n^{1/2}C_0}{b_f} = t_{n-1,\alpha}(\delta_c)$$

where $t_{n-1,\alpha}(\delta_c)$ is the upper $\alpha$ is the upper $\alpha$ quantile of the $t_{n-1}(\delta_c)$ distribution, or

$$C_0 = \frac{b_f}{3n^{1/2}} t_{n-1,\alpha}(\delta_c)$$

The power of the test can be computed as

$$\begin{aligned}\pi(C_{pk}) &= p\{\tilde{C}_{pk} > C_0|C_{pk}\}\\ &= p\{b_f\hat{C}''_{pk} > C_0|C_{pk}\}\\ &= p\left\{\hat{C}''_{pk} > \frac{C_0}{b_f}\middle| C_{pk}\right\}\\ &= p\left\{3n^{1/2}\hat{C}''_{pk} > \frac{3n^{1/2}C_0}{b_f}\middle| C_{pk}\right\}\\ &= p\left\{t_{n-1}(\delta) > \frac{3n^{1/2}C_0}{b_f}\right\}\end{aligned}$$

where $\delta = 3n^{1/2}C_{pk}$.

## MAKING DECISIONS

Tables 2(a)–2(d) display critical values $C_0$ for $C$ = 1.00, 1.33, 1.50 and 2.00 respectively with sample sizes $n$ = 10(5)250 and $\alpha$ risk = 0.01, 0.025 and 0.05. The computer program (using SAS) generating the tables is available from the authors. To determine if the process meets the capability (quality) requirement, we first determine $C$ and the $\alpha$ risk. Then we calculate the value of the estimator $\tilde{C}_{pk}$ from the sample. From the appropriate table we find the critical value $C_0$ based on $\alpha$ risk, $C$ and sample size $n$. If the estimated value $\tilde{C}_{pk}$ is greater than the critical value $C_0$, then we conclude that the process meets the capability (quality) requirement. Otherwise, we do not have sufficient information to conclude that the process meets the present capability requirement.

### *The procedure*

1. Determine the value of $C$ (normally chosen from Table 1), the desired quality condition, and the $\alpha$ risk (normally set to 0.01, 0.025 or 0.05), the chance of incorrectly accepting an incapable process (which does not meet the quality requirement) as a capable process (which meets the quality requirement).

Table 2(a). Critical values $C_0$ for $C$ = 1.00, $n$ = 10(5)250 and $\alpha$ = 0.01, 0.025, 0.05

| $n$ | $\alpha = 0.01$ | $\alpha = 0.025$ | $\alpha = 0.05$ |
|---|---|---|---|
| 10 | 1.957 | 1.175 | 1.541 |
| 15 | 1.686 | 1.529 | 1.411 |
| 20 | 1.556 | 1.436 | 1.343 |
| 25 | 1.477 | 1.377 | 1.299 |
| 30 | 1.422 | 1.337 | 1.269 |
| 35 | 1.383 | 1.306 | 1.246 |
| 40 | 1.352 | 1.283 | 1.228 |
| 45 | 1.327 | 1.264 | 1.213 |
| 50 | 1.307 | 1.248 | 1.201 |
| 55 | 1.290 | 1.235 | 1.190 |
| 60 | 1.275 | 1.223 | 1.181 |
| 65 | 1.262 | 1.213 | 1.174 |
| 70 | 1.251 | 1.205 | 1.167 |
| 75 | 1.241 | 1.197 | 1.161 |
| 80 | 1.233 | 1.190 | 1.155 |
| 85 | 1.225 | 1.184 | 1.150 |
| 90 | 1.217 | 1.178 | 1.145 |
| 95 | 1.211 | 1.173 | 1.141 |
| 100 | 1.205 | 1.168 | 1.137 |
| 105 | 1.199 | 1.163 | 1.134 |
| 110 | 1.194 | 1.159 | 1.131 |
| 115 | 1.189 | 1.155 | 1.127 |
| 120 | 1.185 | 1.152 | 1.125 |
| 125 | 1.181 | 1.148 | 1.122 |
| 130 | 1.177 | 1.145 | 1.119 |
| 135 | 1.173 | 1.142 | 1.117 |
| 140 | 1.170 | 1.140 | 1.115 |
| 145 | 1.166 | 1.137 | 1.113 |
| 150 | 1.163 | 1.135 | 1.111 |
| 155 | 1.160 | 1.132 | 1.109 |
| 160 | 1.157 | 1.130 | 1.107 |
| 165 | 1.155 | 1.128 | 1.105 |
| 170 | 1.152 | 1.126 | 1.104 |
| 175 | 1.150 | 1.124 | 1.102 |
| 180 | 1.148 | 1.122 | 1.100 |
| 185 | 1.145 | 1.120 | 1.099 |
| 190 | 1.143 | 1.118 | 1.098 |
| 195 | 1.141 | 1.117 | 1.096 |
| 200 | 1.139 | 1.115 | 1.095 |
| 205 | 1.138 | 1.114 | 1.094 |
| 210 | 1.136 | 1.112 | 1.093 |
| 215 | 1.134 | 1.111 | 1.092 |
| 220 | 1.132 | 1.110 | 1.090 |
| 225 | 1.131 | 1.108 | 1.098 |
| 230 | 1.129 | 1.107 | 1.088 |
| 235 | 1.128 | 1.106 | 1.087 |
| 240 | 1.126 | 1.105 | 1.086 |
| 245 | 1.125 | 1.103 | 1.086 |
| 250 | 1.124 | 1.102 | 1.085 |

Table 2(b). Critical values $C_0$ for $C = 1.33$, $n = 10(5)250$ and $\alpha = 0.01, 0.025, 0.05$

| $n$ | $\alpha = 0.01$ | $\alpha = 0.025$ | $\alpha = 0.05$ |
|---|---|---|---|
| 10 | 2.569 | 2.255 | 2.208 |
| 15 | 2.216 | 2.012 | 1.859 |
| 20 | 2.046 | 1.891 | 1.771 |
| 25 | 1.943 | 1.815 | 1.714 |
| 30 | 1.873 | 1.762 | 1.675 |
| 35 | 1.822 | 1.723 | 1.645 |
| 40 | 1.782 | 1.693 | 1.622 |
| 45 | 1.750 | 1.668 | 1.603 |
| 50 | 1.724 | 1.648 | 1.587 |
| 55 | 1.702 | 1.631 | 1.574 |
| 60 | 1.683 | 1.616 | 1.562 |
| 65 | 1.666 | 1.604 | 1.552 |
| 70 | 1.652 | 1.592 | 1.543 |
| 75 | 1.639 | 1.582 | 1.535 |
| 80 | 1.628 | 1.573 | 1.528 |
| 85 | 1.618 | 1.565 | 1.522 |
| 90 | 1.608 | 1.558 | 1.516 |
| 95 | 1.600 | 1.551 | 1.511 |
| 100 | 1.592 | 1.545 | 1.506 |
| 105 | 1.585 | 1.539 | 1.501 |
| 110 | 1.578 | 1.534 | 1.497 |
| 115 | 1.572 | 1.529 | 1.493 |
| 120 | 1.567 | 1.524 | 1.489 |
| 125 | 1.561 | 1.520 | 1.486 |
| 130 | 1.556 | 1.516 | 1.483 |
| 135 | 1.551 | 1.512 | 1.480 |
| 140 | 1.547 | 1.509 | 1.477 |
| 145 | 1.543 | 1.505 | 1.474 |
| 150 | 1.539 | 1.502 | 1.471 |
| 155 | 1.535 | 1.499 | 1.469 |
| 160 | 1.532 | 1.496 | 1.467 |
| 165 | 1.528 | 1.493 | 1.465 |
| 170 | 1.525 | 1.491 | 1.462 |
| 175 | 1.522 | 1.488 | 1.460 |
| 180 | 1.519 | 1.486 | 1.458 |
| 185 | 1.516 | 1.484 | 1.457 |
| 190 | 1.513 | 1.481 | 1.455 |
| 195 | 1.511 | 1.479 | 1.453 |
| 200 | 1.508 | 1.477 | 1.452 |
| 205 | 1.506 | 1.475 | 1.450 |
| 210 | 1.504 | 1.474 | 1.448 |
| 215 | 1.501 | 1.472 | 1.447 |
| 220 | 1.499 | 1.470 | 1.446 |
| 225 | 1.497 | 1.468 | 1.444 |
| 230 | 1.495 | 1.467 | 1.443 |
| 235 | 1.493 | 1.465 | 1.442 |
| 240 | 1.492 | 1.464 | 1.440 |
| 245 | 1.490 | 1.462 | 1.439 |
| 250 | 1.488 | 1.461 | 1.438 |

Table 2(c). Critical values $C_0$ for $C = 1.50$, $n = 10(5)250$ and $\alpha = 0.01, 0.025, 0.05$

| $n$ | $\alpha = 0.01$ | $\alpha = 0.025$ | $\alpha = 0.05$ |
|---|---|---|---|
| 10 | 2.887 | 2.535 | 2.281 |
| 15 | 2.490 | 2.263 | 2.091 |
| 20 | 2.300 | 2.126 | 1.992 |
| 25 | 2.185 | 2.041 | 1.929 |
| 30 | 2.106 | 1.983 | 1.885 |
| 35 | 2.049 | 1.939 | 1.852 |
| 40 | 2.004 | 1.905 | 1.826 |
| 45 | 1.969 | 1.878 | 1.804 |
| 50 | 1.939 | 1.855 | 1.787 |
| 55 | 1.915 | 1.836 | 1.772 |
| 60 | 1.894 | 1.819 | 1.759 |
| 65 | 1.875 | 1.805 | 1.748 |
| 70 | 1.859 | 1.793 | 1.738 |
| 75 | 1.845 | 1.781 | 1.729 |
| 80 | 1.832 | 1.771 | 1.721 |
| 85 | 1.821 | 1.762 | 1.714 |
| 90 | 1.811 | 1.754 | 1.707 |
| 95 | 1.801 | 1.746 | 1.701 |
| 100 | 1.792 | 1.740 | 1.696 |
| 105 | 1.784 | 1.733 | 1.691 |
| 110 | 1.777 | 1.727 | 1.686 |
| 115 | 1.770 | 1.722 | 1.682 |
| 120 | 1.764 | 1.717 | 1.678 |
| 125 | 1.758 | 1.712 | 1.674 |
| 130 | 1.752 | 1.707 | 1.670 |
| 135 | 1.747 | 1.703 | 1.667 |
| 140 | 1.742 | 1.699 | 1.664 |
| 145 | 1.737 | 1.695 | 1.661 |
| 150 | 1.733 | 1.692 | 1.658 |
| 155 | 1.729 | 1.688 | 1.655 |
| 160 | 1.725 | 1.685 | 1.652 |
| 165 | 1.721 | 1.682 | 1.650 |
| 170 | 1.717 | 1.679 | 1.648 |
| 175 | 1.714 | 1.677 | 1.645 |
| 180 | 1.711 | 1.674 | 1.643 |
| 185 | 1.708 | 1.671 | 1.641 |
| 190 | 1.705 | 1.669 | 1.639 |
| 195 | 1.702 | 1.667 | 1.637 |
| 200 | 1.699 | 1.664 | 1.635 |
| 205 | 1.696 | 1.662 | 1.634 |
| 210 | 1.694 | 1.660 | 1.632 |
| 215 | 1.691 | 1.658 | 1.630 |
| 220 | 1.689 | 1.656 | 1.629 |
| 225 | 1.687 | 1.654 | 1.627 |
| 230 | 1.684 | 1.653 | 1.626 |
| 235 | 1.682 | 1.651 | 1.625 |
| 240 | 1.680 | 1.649 | 1.623 |
| 245 | 1.678 | 1.647 | 1.622 |
| 250 | 1.676 | 1.646 | 1.621 |

Table 2(d). Critical values $C_0$ for $C = 2.00$, $n = 10(5)250$ and $\alpha = 0.01, 0.025, 0.05$

| $n$ | $\alpha = 0.01$ | $\alpha = 0.025$ | $\alpha = 0.05$ |
|---|---|---|---|
| 10 | 3.826 | 3.361 | 3.026 |
| 15 | 3.302 | 3.002 | 2.776 |
| 20 | 3.050 | 2.821 | 2.645 |
| 25 | 2.899 | 2.710 | 2.562 |
| 30 | 2.795 | 2.633 | 2.504 |
| 35 | 2.270 | 2.575 | 2.461 |
| 40 | 2.661 | 2.531 | 2.426 |
| 45 | 2.614 | 2.495 | 2.399 |
| 50 | 2.576 | 2.465 | 2.376 |
| 55 | 2.543 | 2.440 | 2.356 |
| 60 | 2.516 | 2.418 | 2.339 |
| 65 | 2.492 | 2.399 | 2.324 |
| 70 | 2.471 | 2.383 | 2.311 |
| 75 | 2.452 | 2.368 | 2.300 |
| 80 | 2.435 | 2.355 | 2.289 |
| 85 | 2.420 | 2.343 | 2.289 |
| 90 | 2.407 | 2.332 | 2.271 |
| 95 | 2.394 | 2.323 | 2.264 |
| 100 | 2.383 | 2.313 | 2.256 |
| 105 | 2.372 | 2.305 | 2.250 |
| 110 | 2.363 | 2.297 | 2.243 |
| 115 | 2.354 | 2.290 | 2.238 |
| 120 | 2.345 | 2.283 | 2.232 |
| 125 | 2.337 | 2.277 | 2.227 |
| 130 | 2.330 | 2.271 | 2.223 |
| 135 | 2.323 | 2.266 | 2.218 |
| 140 | 2.317 | 2.261 | 2.214 |
| 145 | 2.311 | 2.256 | 2.210 |
| 150 | 2.305 | 2.251 | 2.206 |
| 155 | 2.299 | 2.247 | 2.203 |
| 160 | 2.294 | 2.242 | 2.199 |
| 165 | 2.289 | 2.238 | 2.196 |
| 170 | 2.284 | 2.235 | 2.193 |
| 175 | 2.280 | 2.231 | 2.190 |
| 180 | 2.276 | 2.227 | 2.187 |
| 185 | 2.272 | 2.224 | 2.185 |
| 190 | 2.268 | 2.221 | 2.182 |
| 195 | 2.264 | 2.218 | 2.180 |
| 200 | 2.260 | 2.215 | 2.177 |
| 205 | 2.257 | 2.212 | 2.175 |
| 210 | 2.253 | 2.209 | 2.173 |
| 215 | 2.250 | 2.207 | 2.171 |
| 220 | 2.247 | 2.204 | 2.169 |
| 225 | 2.244 | 2.202 | 2.167 |
| 230 | 2.241 | 2.199 | 2.165 |
| 235 | 2.238 | 2.197 | 2.163 |
| 240 | 2.236 | 2.195 | 2.161 |
| 245 | 2.233 | 2.193 | 2.159 |
| 250 | 2.230 | 2.191 | 2.158 |

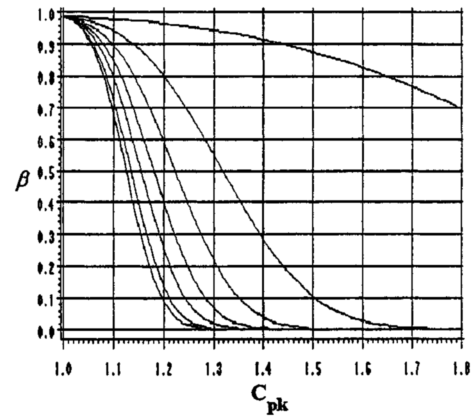


Figure 1. OC curves for $C = 1.00$, $\alpha = 0.01$ and $n = 10(40)250$ (top to bottom in plot)

2. Calculate the value of the estimator $\tilde{C}_{pk}$ from the sample.
3. Check Tables 2(a)–2(d) to find the corresponding $C_0$ based on $\alpha$, $C$ and sample size $n$
4. Conclude that the process meets the capability requirement if $\tilde{C}_{pk}$ is greater than $C_0$. Otherwise, we do not have enough information to conclude that the process meets the capability requirement.

To accelerate the calculations of the estimator $\tilde{C}_{pk}$, we have provided values of the correction factor $b_f$ for various sample sizes $n = 10(5)250$ (see Table 3). Figure 1 plots the OC curves ($\beta = 1 - \pi(C_{pk})$ versus $C_{pk}$ value) for the quality conditions with $C$ set to 1.00, $\alpha$ risk = 0.01 and sample sizes $n = 10(40)250$.

## AN EXAMPLE

Consider the following example taken from *bopro*, a manufacturer and supplier in Taiwan exporting high-end audio speaker components including rubber edge, Pulux edge, Kevlar cone, honeycomb and many others. The production specifications for a particular model of Pulux edge are the following: USL = 5.95, LSL = 5.65, $T = 5.80$. The quality requirement was defined as 'Satisfactory' ($C_{pk} > 1.33$). A total of 90 observations were collected which are displayed in Table 4.

To determine whether the process is 'Satisfactory', we first calculate $d = (\mathrm{USL} - \mathrm{LSL})/2 = 0.15$, $m = (\mathrm{USL} + \mathrm{LSL})/2 = 5.80$, sample mean $\bar{X} = 5.830$ and sample standard deviation $S = 0.023$. To calculate the value of the estimator $\tilde{C}_{pk}$, we need to determine the value of $I_A(\mu)$, which requires the knowledge of $P(\mu \geq m)$ or $P(\mu < m)$. The historical information of the process shows that $P(\mu \geq m) = 0.75$. Thus

Table 3. Values of $b_f$ for various sample sizes $n$

| $n$ | $b_f$ | $n$ | $b_f$ | $n$ | $b_f$ | $n$ | $b_f$ | $n$ | $b_f$ | $n$ | $b_f$ | $n$ | $b_f$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 10 | 0.914 | 45 | 0.983 | 80 | 0.990 | 115 | 0.993 | 150 | 0.995 | 185 | 0.996 | 220 | 0.997 |
| 15 | 0.945 | 50 | 0.985 | 85 | 0.991 | 120 | 0.994 | 155 | 0.995 | 190 | 0.996 | 225 | 0.997 |
| 20 | 0.960 | 55 | 0.986 | 90 | 0.992 | 125 | 0.994 | 160 | 0.995 | 195 | 0.996 | 230 | 0.997 |
| 25 | 0.968 | 60 | 0.987 | 95 | 0.992 | 130 | 0.994 | 165 | 0.995 | 200 | 0.996 | 235 | 0.997 |
| 30 | 0.974 | 65 | 0.988 | 100 | 0.992 | 135 | 0.994 | 170 | 0.996 | 205 | 0.996 | 240 | 0.997 |
| 35 | 0.978 | 70 | 0.989 | 105 | 0.993 | 140 | 0.995 | 175 | 0.996 | 210 | 0.996 | 245 | 0.997 |
| 40 | 0.981 | 75 | 0.990 | 110 | 0.993 | 145 | 0.995 | 180 | 0.996 | 215 | 0.996 | 250 | 0.997 |

Table 4. Collected sample data (90 observations)

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 5.88 | 5.83 | 5.84 | 5.80 | 5.89 | 5.81 | 5.84 | 5.83 | 5.82 | 5.83 |
| 5.81 | 5.82 | 5.85 | 5.81 | 5.81 | 5.81 | 5.84 | 5.82 | 5.80 | 5.84 |
| 5.86 | 5.87 | 5.82 | 5.87 | 5.80 | 5.81 | 5.85 | 5.84 | 5.83 | 5.86 |
| 5.81 | 5.81 | 5.82 | 5.83 | 5.85 | 5.80 | 5.86 | 5.82 | 5.86 | 5.83 |
| 5.80 | 5.77 | 5.82 | 5.85 | 5.84 | 5.82 | 5.85 | 5.81 | 5.86 | 5.79 |
| 5.84 | 5.83 | 5.80 | 5.83 | 5.81 | 5.83 | 5.81 | 5.85 | 5.83 | 5.88 |
| 5.82 | 5.87 | 5.80 | 5.82 | 5.83 | 5.81 | 5.84 | 5.79 | 5.85 | 5.85 |
| 5.84 | 5.84 | 5.80 | 5.82 | 5.84 | 5.85 | 5.86 | 5.81 | 5.81 | 5.85 |
| 5.86 | 5.81 | 5.81 | 5.83 | 5.85 | 5.85 | 5.82 | 5.83 | 5.86 | 5.81 |

we can determine the value of $I_A(\mu) = 1$ or $-1$ using available random number tables.

Suppose the generated two-digit random number is 65, then we have $I_A(\mu) = 1$. Checking the value of $b_f$ from Table 3, we obtain $b_f = 0.992$. Thus $\tilde{C}_{pk} = b_f\hat{C}''_{pk} = b_f(d - \bar{X} + m)/3S = 1.890$. Assume the $\alpha$ risk is 0.05. We find the critical value $C_0 = 1.516$ from Table 2(b) based on $C = 1.33$, $\alpha = 0.05$ and sample size $n = 90$. Since $\tilde{C}_{pk}$ is greater than the critical value $C_0$, we conclude that the process is 'Satisfactory'.

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